DOCUMENT RESUME

ED 372 911 SE 053 503

AUTHOR	Atweh, Bill, Ed.; Watson, Jane, Ed.
TITLE	Research in Mathematics Education in Australasia: 1988-1991.
INSTITUTION	Mathematics Education Research Group of Australasia, (Australia).
SPONS AGENCY	Queensland Univ. of Technology (Australia). Centre for Mathematics and Science Education.
REPORT NO	ISBN-0-949477-99-0
PUB DATE	92
NOTE	274p.
AVAILABLE FROM	New South Wales School of Education, Macquarie University, New South Wales 2109, Australia.
PUB TYPE	Information Analyses (070)
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	Algebra; Calculators; Cognitive Development; Context Effect; Educational Research; Elementary Secondary Education; Foreign Countries; Geometry; *Learning Strategies; Literature Reviews; *Mathematics Curriculum; *Mathematics Education; Measurement; Microcomputers; Problem Solving; *Teaching Methods
IDENTIFIERS	*Australia; *Mathematics Education Research; Teacher Researchers

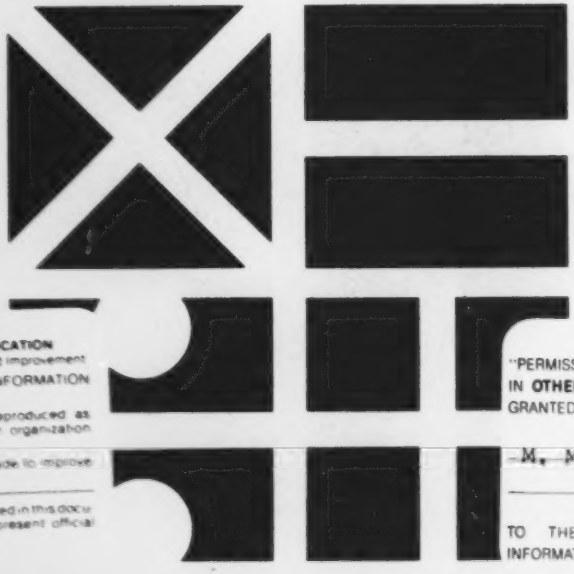
ABSTRACT

The Mathematics Education Research Group of Australia (MERGA) was officially constituted in 1980. In 1984, MERGA produced the first review of the mathematics education research carried out in that region. This book is the third in that series of research reviews. An overview provides the context in which the Australian research was conducted and relates that to an international context for mathematics education research. A total of 12 chapters have been divided into 3 parts with 4 chapters per part. Part 1 considers the social context within which mathematics educators carry out their research. Part 2 considers the role of cognition, language, learning strategies, and technology in learning mathematics. Part 3 focuses on particular areas of mathematics learning. The chapters are as follows: (1) "Politics of Mathematics Education in Australia" (J. Thomas); (2) "The Social and Cultural Context of Mathematics Education" (B. Atweh, T. Cooper, and C. Kanes); (3) "Gender: A Critical Variable in Mathematics Education" (G. Leder and H. Forgasz); and (4) "Research in Practice: Teachers as Researchers" (J. Mousley); (5) "Cognitive Studies in Mathematics Education" (L. English-Halford); (6) "Research in Learning Strategies in Mathematics" (K. Y. Wong and T. Herrington); (7) "Calculators and Computers in Teaching and Learning of Mathematics" (B. Doig, M. Carss, and P. Galbraith); and (8) "Language Factors in Mathematics Education" (N. Ellerton and P. Clarkson); (9) "Research on Early Childhood Mathematics Development" (R. Perry, J. Mulligan, and R. Wright); (10) "Research in Mathematical Problem Solving" (I. Putt and I. Isaacs); (11) "Research in Geometry and Measurement" (G. Davey and J. Pegg); and (12) "Research in Teaching and Learning Algebra" (M. Macgregor and C. Quinlan). A list of contributors is provided. (MDH)

ED 372 911

MATHEMATICS EDUCATION RESEARCH GROUP OF AUSTRALASIA

Research in Mathematics Education in Australasia 1988–1991



U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

X This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL IN **OTHER THAN PAPER COPY** HAS BEEN GRANTED BY

M. Mitchelmore

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

Edited by Bill Atweh and Jane Watson

**Sponsored by the Centre for Mathematics and Science Education
Queensland University of Technology**

SE053 503

BEST COPY AVAILABLE

MERGA

MATHEMATICS EDUCATION RESEARCH GROUP OF AUSTRALASIA

AIMS

The **Mathematics Education Research Group of Australasia** was established as a forum for mathematics educators to disseminate research information on the teaching and learning of mathematics.

The purposes of the Group include:

- encouraging and promoting quality research in the learning and teaching of mathematics
- the dissemination to members of research results through regular publications and conferences
- seeking means of implementing research findings at all decision levels to the teaching of mathematics and to the preparation of teachers of mathematics
- liaising with other organisations with similar interests in mathematics education or educational research.

Membership is open to those who share and wish to promote the general aims of the Group.

MEMBERSHIP

There are two categories of membership: personal and library. Personal members receive:

- all issues of the *Mathematics Education Research Journal* (currently two issues per year)
- the annual *Research Directory*
- regular newsletters on matters of topical interest
- reduced registration rates for the annual MERGA conferences

Library members receive:

- all issues of the *Mathematics Education Research Journal*
- the annual *Research Directory*
- other occasional publications

Membership fees are payable at the beginning of each calendar year. To become a member of MERGA for 1992, please complete the form overleaf and return it with your subscription to:

Michael Mitchelmore
School of Education
Macquarie University
NSW 2109
AUSTRALIA

ENQUIRIES: Telephone (02) 805 8655, Fax (02) 805 8674.

SUBSCRIPTION RATES 1992

The 1992 subscription rates are as follows:

Personal membership	A$40
Library membership	A$75

In addition, A$10 is payable for overseas postage (if applicable).

PERSONAL DATA

First name: _____

Last name: _____

Institution: _____

Department: _____

Name & full postal address (*exactly as it should appear on your mailing label*)

_____

_____

_____

_____

MEMBERSHIP REQUIRED

Personal ____ *or* Library ____ New member ____ *or* Renewal ____

PAYMENT TO BE MADE

1992 subscription A$ _____

Overseas postage A$ _____ TOTAL: A$ _____

METHOD OF PAYMENT

Cheque ____ *Please make your cheque payable to M.E.R.G.A.*

Credit card ____ *Please complete the following authorisation.*

Please debit the amount of $ _____ to my Bankcard ____ /Mastercard ____ /Visa ____:

Card Number: _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

Name on card: _____ Valid from _____ to _____

Signature: _____ Today's date: _____

Centre for Mathematics and Science Education

The Centre was established in 1990 on the Kelvin Grove Campus of the Queensland University of Technology and is dedicated to developing excellence in mathematics and science education in research, teaching and service. It aims to develop international standard research projects in diverse and significant areas, to support higher degrees and doctoral programs, and to disseminate research findings to the profession and the general public.

The main research projects of the Centre are currently in the areas of

* Curriculum and Instruction
* Learning and Cognitive Science
* Social and Cultural Context
* Technology Based Instruction

The Centre conducts conferences, seminars and workshops on research in mathematics and science education, such as the international Number Sense Conference held in 1991, and the forthcoming Algebra Conference in September 1992. It also offers remedial and diagnostic services in mathematics and special programs for the gifted and talented in science.

The Centre welcomes inquires about

* M.Ed and M.Ed (Research) programs and scholarships
* PhD and EdD programs and scholarships
* Visiting Scholars
* Adjunct Professors
* Post Doctoral Fellowships
* Cooperative Research Projects

Centre for Mathematics and Science Education
Queensland University of Technology
Locked Bag No 2, Red Hill Q 4059 Australia
International phone +61 7 864 3718
International fax +61 7 864 3985

Research in Mathematics Education in Australasia 1988-1991

Edited by
Bill Atweh and Jane Watson

Mathematics Education Research Group of Australasia
Sponsored by
Centre for Mathematics and Science Education - Queensland
University of Technology

Copyright © 1992 Mathematics Education Research Group of Australasia

All Rights Reserved. Apart from any fair dealing for the purposes of private study, research, criticism or review as permitted under the Copyright Act, no part of this publication may be produced, stored in a retrieval system or translated in any form or by any means: electronic, electrostatic, magnetic, mechanical, photocopying, recording or otherwise, without the permission of the publisher.

First edition 1992

ISBN 0-949477-99-0

Printed at the Queensland University of Technology - Kelvin Grove

CONTENTS

Introduction

An Overview of Australasian Mathematics Education Research: 1988-1991
Jane Watson and Bill Atweh 2

Part I Mathematics Education in a Wider Context

Politics of Mathematic Education in Australia
Jan Thomas 26

The Social and Cultural Context of Mathematics Education
Bill Atweh, Tom Cooper and Clive Kanes 43

Gender: A Critical Variable in Mathematics Education
Gilah Leder and Helen Forgasz 67

Research in Practice: Teachers as Researchers
Judith Mousley 96

Part II Issues in Learning and Instruction

Cognitive Studies in Mathematics Education
Lyn English-Halford 116

Research in Learning Strategies in Mathematics
Khoon Yoong Wong and Tony Herrington 143

Calculators and Computers in Teaching and Learning of Mathematics
Brian Doig, Margorie Carss and Peter Galbraith 129

Language Factors in Mathematics Education
Nerida Ellerton and Philip Clarkson 153

Part III Specific Topics in Mathematics Learning

Research on Early Childhood Mathematics Development
Robert Perry, Joanne Mulligan, and Robert Wright 180

Research in Mathematical Problem Solving
Ian Putt and Ian Isaacs 208

Research in Geometry and Measurement
Geoff Davey and John Pegg 231

Research in Teaching and Learning Algebra
Mollie Macgregor and Cyril Quinlan 248

Contributors 259

Introduction

AN OVERVIEW OF AUSTRALASIAN MATHEMATICS EDUCATION RESEARCH: 1988 - 1991

Jane M. Watson
School of Education
University of Tasmania

Bill Atweh
Centre for Mathematics and Science Education
Queensland University of Technology

The Mathematics Education Research Group of Australasia (MERGA) was officially constituted in 1980 after informally holding annual conferences since 1977. Hence 1992 marks the 15th year that MERGA has provided support and encouragement for researchers in mathematics education in Australia, southeast Asia and the south-west Pacific region. This year also marks the occasion of the 7th International Congress on Mathematical Education (ICME-7) to be held in Quebec, Canada. Since ICME-5 in Adelaide in 1984, MERGA has produced a book summarizing the research carried out in this region in mathematics education in the years leading up to the ICME. This book is the third in the series and represents the most extensive effort yet to provide a coverage of all areas in which Australian and regional researchers work.

The first *Summary of Research in Mathematics Education in Australia* (Briggs, 1984) was prepared by four writers and the editor. It emphasized the contributions of Australians to mathematics education research from the time of MERGA's inception, both within the organization itself (Jones, 1984) and more widely in the education community (Conroy, 1984). Nearly 300 studies were discussed under general headings, such as curriculum, attitudes, problem solving, cognition, and teacher education. As well 100 articles related to gender and mathematics were reviewed in an annotated bibliography (Leder, 1984). Finally, a research register was produced for 61 of the members of MERGA (Blane, 1984), outlining specific research projects in progress under nine main content headings and 22 more specific content codes. This method of keeping track of the specific research interests of MERGA members was subsequently replaced by a *Research Directory* (Booth, 1988; Watson, 1991, 1992).

The second summary, *Mathematics Education in Australia: A Selection of Recent Research* (Blane & Leder, 1988) was prepared by five authors and the two editors. After a general review of critical variables in mathematics education (Jones, 1988), three annotated bibliographies were presented in the areas of gender and mathematics (Barnes, 1988), psychology of mathematics

learning (Watson, 1988), and problem solving (Groves & Stacey, 1988). In total these three chapters covered 378 articles, over 200 to do with problem solving. Although more limited in scope than originally intended, what was produced provided useful resources for those who would carry out research in any of the three fields of mathematics education, both in Australia and overseas.

The third summary, the present volume, has benefited from the experiences of the previous two editions and has brought together 25 members of MERGA as contributors and editors. Planning began in 1991 and meetings were held with the authors at the annaul conferences of MERGA and the International Study Group for the Psychology of Mathematics Education (PME) at Perth and Assisi in July of that year. Original suggestions for chapter topics were based on Special Interest Groups within MERGA. In addition volunteers were invited to suggest other areas where it would be useful to provide summaries of Australasian research. The overwhelming response of MERGA members has resulted in 12 chapters which present a wide-ranging coverage of research areas.

A decision was made in consultation with the potential authors, to move away from the annotated bibliography format to the writing of summary chapters which included complete referencing. This has allowed the authors the freedom to pick out emphases which they saw as important and in some cases to make suggestions for future research in the area. The editors felt that the readability of the volume would be enhanced by this move to a review format.

Other decisions were made with respect to what sources should be used for the book. Because the publication is sponsored by MERGA, it was felt essential to include where relevant reference to papers presented at MERGA conferences. Since MERGA has not produced conference proceedings in the years covered by this review, it may be difficult to obtain access to such papers. Some authors have supplied addresses for MERGA conference presentations. The existence of the MERGA *Research Directory* (Watson, 1992), which is currently updated annually with information on research projects and interests of members, also should provide contact addresses where readers can request copies of conference papers. In addition to papers from MERGA conferences, institutional reports, papers published in overseas journals, and minutes and letters associated with important organizational decisions in Australia, were referenced. Papers and articles were not restricted to those by MERGA members. The organization wished to further the exposure of all mathematics education research in Australia and the region known to its authors.

In presenting this review of research, MERGA is proud of the contribution made by its members and other researchers in the region at both the national and international levels. Mathematics education researchers offer papers more widely than just at MERGA conferences; for example, papers are often presented at Australian Association for Research in Education and Australian Association of Mathematics Teachers conferences, at Australia and

New Zealand for the Advancement of Science congresses, and at conferences organized by the Regional Centre for Education in Science and Mathematics (RECSAM), in Southeast Asia. As well, on the larger world stage, Australian researchers have been prominent in organizations such as PME and the International Commission on Mathematical Instruction. They also have been strongly represented in the major research journals of the world, for example with nine papers in the *Journal for Research in Mathematics Education* and 11 in *Educational Studies in Mathematics*, during this four-year period. As well 10 chapters in Steffe and Wood's (1990) survey of international perspectives on early childhood mathematics education following ICME-6 were authored or co-authored by Australians. At both national and international levels, MERGA wants to encourage further participation and contribution in the next four years.

Although every attempt has been made in this review to cover the majority of research in mathematics education in the Australian, New Zealand and South-west Pacific regions, inevitably some references may have been missed. In particular, masters' and doctoral dissertations in Australia are not as readily available for review as they are in some overseas countries. Further, since some conferences in the region do not publish proceedings, papers presented at such conferences may have been missed. Lastly, while research publication exchange between Australia and the rest of the region is improving as the following section argues, currently there is no single educational database that covers publications from the whole region. This means that, regrettably, some studies conducted in New Zealand and the Southwest Pacific may have been not available for review. The Editors apologise for these omissions.

It was intended that sources be reported for the period 1988 to 1991 (or early 1992) since the previous summary was published. In the three general areas which were covered in 1988, this was quite reasonable. Flexibility, however, was allowed for the other areas where it may have been necessary to quote earlier sources to paint a more complete picture of the research scene.

Well over 700 papers and articles are referenced in the chapters of this volume. Even allowing for some overlap, this represents the most extensive coverage yet given by MERGA four-yearly research summaries. Some overlap in topic coverage occurred in this volume. This was encouraged to some extent as it was not expected that all readers would read every chapter. It would be unfortunate if some important strand were to be missed because of a deletion due to an overlap of references.

The rest of this chapter is divided into five sections. The first section deals with the context of research in mathematics education. The section is based on the premise that for a total appreciation of research findings, an understanding of the context in which the research has been conducted is important. National and international factors affecting the directions of research in mathematics education in Australia will be examined. Section two presents an overview of the chapters in this book. It outlines the three parts into which the chapters in this review have been grouped as well as presenting

the major conclusions of each chapter. The third section reviews some published research that could not be classified under any of the other headings. The fourth and fifth sections posit some future directions for research in mathematics education as well as some overall conclusions of the review.

AUSTRALASIAN RESEARCH IN PERSPECTIVE

In the period of years covered by this review, questions were raised about the relationship between Australian and international educational research. In a paper entitled "Educational research in Australia: Indigenous or exotic? The case for the indigenous", Porter (1989), argued that the context of research gives research a meaning. Research findings travel over cultural boundaries with a necessary loss in meaning and/or generality. Porter added that the significance of the context of research has not always been acknowledged by policy makers in education.

This point of view was also adopted by the Review Panel of the Strategic Review for Educational Research in Australia, a national inquiry that is expected to set the patterns of funding of educational research for the 90's. The interim report of the panel argued that:

> Educational research must take account of [the] cultural context so that, for many of the questions to be addressed, the form of the questions and the nature of the answers will be culturally bound. This does not mean that educational research cannot be part of an international community of scholarship, but it does mean that external perspectives cannot automatically be translated into the Australian traditional context. (Educational Research in Australia, 1991, p. 6)

The period covered in this review witnessed the emergence of such questions. Undoubtedly, more researchers will have to address these questions in their future designs of research and analysis of their findings.

The Australian Context

Understanding the cultural, social and political context is important for the full understanding of research carried in Australia. It is not the intention here to discuss the whole of the Australian context. Rather, this section will focus on three factors that explain aspects of the studies reported in this summary.

Increase in research projects and funding. The period 1988-1991 was significant in the development of research in mathematics education. The period started in a context where tertiary institutions followed a binary system that differentiated between Colleges of Advanced Education (CAEs) and Universities. Based on a similar model in the United Kingdom, this system was introduced in mid 1960s. In this system, colleges and universities had

differential funding and expectation of their roles in teaching and research. The majority of teacher education was carried out at the CAEs and the majority of research at the universities. For example, the interim report of the Strategic Review (Educational Research in Education, 1991) found that during 1988-1991, on average, 40% of the time of university lecturers was devoted to research in contrast to 10% of the time of college staff.

The period of time covered in this review ended with the abandonment of the binary system and the amalgamation of CAEs, or the transformation of some of the more diversified colleges into universities. This change was, arguably, the most drastic change in the Australian educational scene since the mid 1960s (Meek & Goedegebuure, 1989). This increase in the pool of researchers, corresponds to an increase in funding for research. After a decline between 1981 and 1986, funding for research was significantly increased during 1987-1991. The educational share of these funds also increased (Educational Research in Australia, 1991). With this increase in both funding and expectations of research, the pool of researchers and research projects in mathematics education research have increased significantly. This increase is reflected in the number of articles reviewed in this summary compared to the previous MERGA summaries.

The increased funding for research in education requires further comment. A major finding of the interim report of the Strategic Review (Education Research in Australia, 1991) was that, even though research funding has increased significantly within the past five years, it was still very low in comparison to funding available to other professions. In Australia, educational research consumes about 0.35 per cent of the total educational expenditure. This is far below the 1.40 per cent of the health budget that is spent on research in that area. One recommendation from the Strategic Review was that such funding should be doubled within 10 years, with the ultimate aim of reaching the same ratio as in health.

Considering this recent surge in numbers of reported research studies, many of the studies reported in this review are single studies that have not been replicated or have not led into further studies. The current status of such research was described in the interim report of the Strategic Review (Educational Research in Australia, 1991) as "too uncoordinated". The report added,

> Individual researchers tend to work in isolation, fitting research activities around other commitments that frequently press more urgently, and undertaking, single, relatively short-term studies. Much more can be achieved if teams of researchers with complementary skills work together in a sustained way upon larger issues. (p. viii)

This situation is compounded by the relatively limited number of researchers in Australia. Although the number of researchers and projects has increased during the past four years, Australia's relatively small population places limits

on the number and variety of viable specialised professional journals. In the face of these factors, professional organisations such as MERGA provide important venues in which researchers can meet, share findings and discuss mutual concerns. White (1988) noted that Australian researchers have a tendency to attend more educational conferences than their English counterparts. This factor may explain the relatively large number of publications reviewed in this summary that are papers presented at conferences.

Research methodologies. A second feature of educational research in Australia relates to the background of the majority of researchers. White (1988) noted that "nearly all Australian researchers have been school teachers" (p. 10). Such background explains, in part, the emphasis in Australia on certain types of studies that can be described as field studies (Popkewitz, 1981). Relatively few studies reported in this review were carried out in clinical or laboratory settings. Researchers at the Deakin University in Geelong, Victoria developed and promoted action research as a useful methodology for research. The effect of this background and the interest in classroom research was described by White (1988):

> The teaching experience of Australian researchers disposed them to break away from simple models and a research style that disregarded interactions between pupils and that treated their differences as statistical "errors". They could see the importance of doing long term research in the classroom with (not on) the teacher. Their experience as teachers made it easy for them to do this sort of research, when it would have been difficult if not impossible for a non-teacher. Experience meant that they knew how to behave in classrooms so that students accepted them; it made them alert to nuisances of behaviour by teachers and students; it made it easy for them to establish trust. Without that experience the lengthy action research that we are now engaged in would not have been possible. (p. 10)

Policy making and research. Australia has had a large number of state and national reports on various educational matters. Navaratnam and West (1989), identified at least eight state and national reports on mathematics and science education alone, in addition to at least twelve general education reports and commissions, carried out in the decade preceding 1988. Porter (1989) pointed out that, in Australia, there is a tendency for major educational decisions to be made as a result of government inquiries and commissioned reports rather than as a result of research.

While research may not always inform policy development in Australia (see also Meek & Goedegebuure, 1989), policy development may delineate the limits and directions of research. For example, the Strategic Review of Research in Education (Educational Research in Australia, 1991), undertaken by the Australian Research Council, recommended that priorities be set and

reviewed frequently in relation to research issues that should be encouraged and supported. For the past few years, educational funding for research in Australia has been regulated by lists of priority areas. For example, funding from the Australian Research Council has been allocated according to priority areas (Gallagher, 1989). Cognitive science is one current priority. Likewise, The Department of Education, Employment and Training, the government body responsible for university funding, has identified mathematics and science education as priority areas over other subjects of the curriculum (Department of Employment, Education and Training, 1989). In addition to funding research according to priority areas, the Australian Research Council gives priority to applied research over basic or theoretical research, encourages concentration of research skills in specific research centres, and promotes competitive research funding (Gallagher, 1989).

Considering the limited input of research into policy development and the potential effect of policy decisions on research in education, researchers in mathematics education cannot afford to ignore the implications of changes in government policy on their profession. The inclusion of a chapter on politics of mathematics education within this review is a witness to this situation.

International Context

The educational system and research in Australia have been traditionally open to interaction with systems and researchers from the United States and the United Kingdom. This connection with the Anglo-Saxon world is reflected in the journals and references that Australian researchers use in their publications as well as in the methodologies they employ, if not the type of questions that they raise. Similarly, before Australian publications evolved, the United Kingdom and United States of America were the major places where Australian research was reported. Australians seeking further studies in mathematics education a few years ago, had no alternative but to go overseas. Once again, the United States and the England were the most popular destinations. Lastly, many national conferences in Education had at least one, often more, international guest speakers. American and British speakers were highly represented on the list of invited speakers.

Arguably, this interaction with the Anglo-Saxon world has been valuable in the development of research culture and expertise in Australia. During the last quarter of this century, however, Australian research in mathematics education also has had exposure to influences from continental Europe and South American countries through international conferences such as PME and ICME. Australian researchers are often represented at these meetings and contribute papers at them. It is to be expected that, in the next few years, the effect of such an interaction will lead to further diversifying of research questions, methodologies, and theories.

The late 1980s and the early 1990s were described by Islam (1990) as "a time when political awareness of the need for Australia to be more internationally orientated and more closely integrated with Asia and the Pacific

region is at its peak" (p. 48). Calls for increased cooperation and integration with the region came from political leaders (Hawke, 1990), government reports (Garnaut, 1989) and academics (Viviani, 1989). In addition to the economic rationalisation that most of these calls seem to adopt, Viviani argued for a long term goal of increasing cultural and educational cooperation that should commence by giving a priority for changes in the curriculum at the school and university levels. Mathematic educators have responded to these trends in society by establishing links with similar organisations in the Southwest Pacific and Southeast Asia. RECSAM, located in Penang, Malaysia is an organisation under the auspices of the Southeast Asian Ministers of Education Organisation and represents many countries in Southeast Asia. During its last conference in February, 1992 Australian educators were well represented and presented papers. Another indication of the increase awareness of the importance of research cooperation with the Asian and Pacific countries is the holding in Australia of the first conference on Asian Perspectives in Mathematics Education adjoined to the MERGA annual conference in 1992. It is hoped that this cooperation will expand and that future MERGA reviews and journal will have more Asian content.

THE CONTENTS OF THIS REVIEW

The chapters have been divided into three sections reflecting different emphases in mathematics education research. The choice was made to order the sections from the more general to the more specific. Hence the first considers the broad social issues which are encountered in mathematics education today, the second considers general education issues related to the teaching and learning of mathematics, and the third considers specific mathematical topics of interest to researchers.

Part I - Mathematics Education in a Wide Social Context

Except for the relatively specific topic of gender and mathematics education, the issues dealt with in this section have not been considered in previous MERGA research summaries. The past few years have seen increased attention paid to the political scene within which mathematics educators must carry out their research, the social context within which mathematics learning takes place, and the role of teachers as active researchers in the classroom.

Jan Thomas' chapter on "Politics of Mathematics Education in Australia" helps to define the political milieu within which Australian mathematics education researchers find themselves. While governments desire their education systems to produce better results in the areas of mathematics, science and technology, difficult economic times bring pressure to produce these results with fewer resources. This has forced organizations such as MERGA to band together to form umbrella groups to fight for support for mathematics in all its forms: pure and applied research, education, and educational research.

The collaborative structure has then made possible stronger, more unified protests on issues of importance such as a national mathematics curriculum and national assessment in mathematics. Although many mathematics educators would not wish to become involved in political action, Thomas argues well the case that it cannot be ignored and at least some members of the research community must play an active role to protect the interests of all.

More specially in relation to the learning and teaching of mathematics, research on "The Social and Cultural Context of Mathematics Education" is considered by Bill Atweh, Tom Cooper and Clive Kanes. They have divided their sources into four categories for discussion: culture and language, students' social background, classroom interaction and the role of the teacher. With respect to culture and language, studies related to Aboriginal students, socio-linguistic contexts, bilingualism and ethnomathematics are considered. The largest number of sources relates to the difficulties of learning mathematics for the Aboriginal student, indicating that it is an area of increasing concern for mathematics educators as well as for educators in general. Under the heading of students' social background, family influences, community attitudes, social class and participation are noted as areas receiving attention in Australia. An observation in relation to the apparent exercise of greater control of learning by teachers in lower socio-economic class schools than in higher status schools points to an interesting area for future research and intervention. The section on classroom interactions features studies in relation to activity, environment, and communication. The section on the teacher focuses on teacher practice and the analysis of exemplary teaching. Atweh, Cooper and Kanes put three challenges to researchers: become involved in longitudinal studies, integrate concerns about the social context as part of studies which focus on other areas of mathematics education, and consider research within theoretical frameworks. There can be little doubt that attention to these three issues would enhance the contribution of research in this area.

Issues associated with gender and mathematics have been reviewed in each of the MERGA research summaries, and Gilah Leder and Helen Forgasz produce an update on research over the last four years under the title "Gender: A Critical Variable in Mathematics Education". They note the trend over the three four-year periods covered by MERGA summaries for explanatory and intervention studies to become more prominent than performance studies that highlight inequities. They also summarize (i) recent Australian participation data which show that differences continue to exist in advanced mathematics courses at senior secondary level, and (ii) recent government initiatives in the areas of policy, curriculum development and professional development. The research into possible explanations for gender differences has focused on classroom interactions, the influence of societal values, beliefs and attitudes, and structural problems within schools. Singled out for attention is the issue of coeducation versus single-sex settings. As well, many intervention studies and initiatives are discussed, providing an optimistic outlook for the future. Leder and Forgasz believe that government initiatives in the past four years are

producing results but that economic conditions may act to slow the positive trends. It is not yet time to remove the gender issue from the mathematics education research agenda.

As the notion of action research has become more prevalent in mathematics education in recent years, so has the part to be played by the classroom teacher in research. For some, this is a contentious issue, while for others it provides an opportunity to gain from the vast experience of those who potentially have more direct experience with children's learning of mathematics than those who devote a large proportion of their time to formal research. Judy Mousley confronts this issue in the chapter, "Research in Practice: Teachers as Researchers". She sets the context within which teachers operate precisely as fits this part of the book: a politically and socially changing environment where dictums no longer descend from above and considerable freedom is encouraged. Another factor in increasing teachers' awareness of the possibilities for doing their own research has been the greater opportunity to share outcomes through state and national conferences and teachers' journals. A wide-ranging summary of the research interests of classroom teachers is given under four headings: "what works for us", partnerships in classroom research, curriculum development, and school improvement programs. Mousley concludes on a positive note, reflecting on the contribution which can be made by classroom teachers if given the opportunity, encouragement and help.

Part II Issues of Learning and Instruction

While the chapters in Part I address societal issues in mathematics education, this part relates more narrowly to issues that are closely related to the classroom. The concern is more specifically with the learning of mathematics and the part played by cognition, language, learning strategies, and technology. Some of these fields, such as cognition, have been prominent in mathematics education for some time, while others are relatively new.

Because of the potential overlap with other chapters in the book, Lyn English-Halford has restricted her chapter on "Cognitive Studies in Mathematics Education" to research dealing specifically with cognition, cognitive models and mathematics education. In relation to cognition and problem solving she discusses the continued use of structured interviews to probe students' thought processes using various frameworks for analysis. These involve cognitive, metacognitive, affective, self-assessment and scanning aspects, combined in different manners depending on the specific model. In general increasing levels of sophistication of performance are observed in the studies reported. Other approaches to the study of problem solving include stimulated recall and the study of schema acquisition and rule automation. Two models of mathematical learning developed in Australia are outlined with applications in solving word problems, algebra, volume measurement, common and decimal fractions, and novel problems for preschoolers. Other areas touched upon in relation to cognition are constructivism, socio-cognitive

conflict, success-based learning, and attitudes to teaching mathematics. English-Halford calls for research in specific topic areas, research using a wider range of cognitive models, and research which studies the interplay of process and product.

A more specific issue, associated with "Research in Learning Strategies in Mathematics", is considered by Khoon Yoong Wong and Tony Herrington. Although perhaps broadly related to cognitive studies, a case is made for the importance of learning strategies themselves because they reflect what students do with regard to a given teaching approach, how they develop particular problem solving strategies, and what metacognitive processes they engage in to construct their understandings. Strategy learning is then set in an interactive model of mathematics learning before research into three aspects is reviewed. These are strategy awareness, interactions between awareness and performance, and teaching learning strategies, including through writing, concept mapping, and a learning strategies program. Wong and Herrington finally suggest four areas for further research, associated with the development of instruments to fathom strategy use, the investigation of the influence of contextual factors on beliefs and strategy use, the development and evaluation of intervention strategies, and the consideration of the teacher's perspective in the whole area of learning strategy development.

The next chapter in Part II considers the impact of "Calculators and Computers in Teaching and Learning Mathematics". Brian Doig, Marjorie Carss, and Peter Galbraith review the two major projects associated with calculators in the past four years, both of which are based in Victoria, as well as a variety of issues relating to computer usage in the mathematics classroom. Among these are social issues in computer use, Logo, computers for mathematical development, using computers to teach specific mathematical topics, and teaching computer programming. A brief look is then taken at the effect of calculators and computers on classroom practice. Several areas which hold promise for future research are mentioned by Carss, Doig and Galbraith: the impact of technology on teachers and the curriculum, the changes needed to assessment practices in the light of new technology, the effect of technology on the acquisition of number facts and concepts, and the role to be played by teachers in all aspects of research.

This volume is the first of the three MERGA research summaries to have a complete chapter devoted to "Language Factors in Mathematics Education". In it Nerida Ellerton and Philip Clarkson make a strong case for the long held interest of Australian researchers in this area and in particular the important contributions made in the past four years. The major contributions of Australian mathematics educators are summarized under five headings: development of models relating language and mathematics; expanding the modes of communication in mathematics classrooms; language, culture and mathematics learning; children writing mathematics; and mathematics in context: story maths. Other research of a more specific nature is reported in relation to textbooks, teacher-student interactions, semantic structure and word

problems, and early childhood studies. Among the challenges set by Ellerton and Clarkson in the light of the large number of studies reviewed is the development of programs to 'dejargonise' school mathematics and provide learning environments which create links between children's experiences and the formal language and applications of mathematics.

Part III Specific topics in mathematics learning

In moving from the general to the specific, Part III focuses on particular areas of mathematics learning which have been of major interest to Australasian researchers. The chapter on problem solving follows up specifically on the work reported in the 1988 MERGA research summary. The other three topics, early mathematical development, geometry and measurement, and algebra have not been singled out for individual coverage in the past.

Robert Perry, Joanne Mulligan, and Robert Wright take an extensive look at "Research on Early Childhood Mathematical Development". They first survey research generally in the early childhood area in relation to children constructing mathematics, children learning mathematics through resource production, problem solving, mathematics and music, and early childhood teacher education. The mathematical topics then identified for discussion are spatial concepts, early number learning, addition and subtraction, multiplication and division, and fraction concepts. This chapter shows the continued interest by Australian mathematics educators in the important age-span where mathematics learning is beginning. The close connections of many of the studies to actual classroom situations indicates the importance placed on understanding children's thinking and performance in its educational context. Although Perry, Mulligan and Wright do not specifically outline directions for future research, some of these are noted throughout the chapter as natural extensions of some of the studies described. It would appear in the light of the current interest in constructivism throughout the world, that there is great potential in following the processes which operate in the classroom and through which children construct their early representations of mathematical concepts. Intervention studies which manipulate conditions of teacher-child and child-child interactions may help clarify the debate on how representations are best constructed.

Ian Putt and Ian Isaacs open their chapter on "Research in Mathematical Problem Solving" with a summary of the comprehensive 1988 chapter of the same title, before moving to analyse Australasian research of the past four years in similar broad categories. The first section on the importance of problem solving notes the work of several writers in support of the problem solving goal noted in *A National Statement on Mathematics for Australian Schools* (Australian Education Council (AEC), 1991). Subsequently research is reported in the areas of teaching problem solving, innovative practices, gender differences, cognitive processes, strategies, assessment, and mathematical modelling and investigations. A topic such as problem solving will undoubtedly overlap on some of its dimensions with other chapters in the

book, most notably cognition. The usefulness of having the discussion in context, however, should justify its inclusion. Perhaps insights gained from the two perspectives will be useful to readers. In their conclusion Putt and Isaacs contrast the approaches of teaching domain specific strategies and teaching heuristics to enhance problem solving abilities. This debate will continue in the future and offers fertile ground for research, especially for those who can develop a common ground upon which to provide comparisons. Further they call for research to develop and evaluate innovative professional development programs, especially to change teachers' perceptions of their roles in teaching problem solving, and to develop instruments to study change factors and monitor thinking processes and problem solving achievement.

"Research in Geometry and Measurement" is summarized by Geoff Davey and John Pegg. After setting the rationale for the study of geometry in terms of assisting children to understand and use shape and space in problem solving tasks, and addressing the more formal aspects of structure and proof, they discuss Australian research of the past four years under five headings. These include research on visual abilities and drawing, the generative learning model, formal geometry and proof, levels of growth of understanding according to the Van Hiele and SOLO Taxonomy models, and measurement. The importance of concern among Australian researchers with how students acquire and use skills in geometry is noted, as well as the significance of most research being based in theoretical models. Davey and Pegg suggest that research continue in the areas where progress has recently been made and begin in two new spheres. These are the geometric understanding held by pre-school and early school children and the place of computers in the learning of geometry and measurement.

Whereas "Research in Teaching and Learning of Algebra" was in the past likely to be considered in the context of problem solving or cognition, in this volume Mollie MacGregor and Cyril Quinlan devote a chapter to a topic which appears to be attracting increased attention in Australia. They claim that the current thrusts of research are an evaluation of various teaching approaches to algebra in the early secondary years and the development of theories to explain how students acquire algebraic skills and understanding. The five categories within which the research studies are summarized are teaching experiments, evaluation studies, interaction of language and algebra learning, students' misconceptions and errors, and cognitive foundations. Although much of the research reported could be used to exemplify other broader features of mathematics education research, such as the interaction of language and mathematics learning and cognitive theory, the presentation here combines all available research on the topic to contribute to the understanding of the difficulties associated with learning a subject which is so central to achievement at the higher levels of mathematics. Although MacGregor and Quinlan do not specifically suggest directions for further research, these follow implicitly from the results of the studies reported. Studies to address instruction which will aid the construction of algebraic concepts, remediation

which will help reconstruct misconceived concepts, and the interaction of both language and visual/concrete aids with the symbolism of algebra would all be of benefit to the learner.

At the end of the book brief biographies and addresses are given for each of the contributors. Without them MERGA could not have produced this volume and the organization extends thanks to each.

OTHER AREAS OF RESEARCH

No matter how careful authors or editors are there will be the occasional reference which is overlooked. Several of the authors in this volume have acknowledged with regret this possibility. As well as the possibility of oversight, there are some areas of mathematics education research which knowingly were not covered by chapters in this research review, for example tertiary mathematics education, mathematics instruction and teaching practice, the history and philosophy of mathematics education and the professional development of teachers. The reasons that chapters do not exist on these topics vary. In some cases it was the lack of availability of author. In others it was the perception that not enough research had taken place to warrant the inclusion of a complete chapter. This may have resulted from the lack of a complete research data base in Australia, for example on higher degree theses. A perusal of MERGA's journal, the *Mathematics Education Research Journal*, published since 1989, uncovered three articles which were not referenced elsewhere in this review and these will be noted in this section. As well, several other examples of research topics not mentioned elsewhere will be mentioned. Although not attempting to be comprehensive, this will serve to illustrate some of the areas of Australian research which operate on a smaller scale.

Teaching Practice and Instruction

Many issues surround the practice of teaching mathematics which have been the basis for research studies. Two of these have featured in the *Mathematics Education Research Journal*. The preservice preparation of mathematics teachers was the subject of a study carried out by Pegg (1989). His detailed analysis of a secondary mathematics lesson led to a comparison with a developmental theory of cognition and to suggestions being made for the improvement in the level at which lessons are delivered. His work was based on the view that there is a close analogy between the way students learn mathematics and the way teachers learn to be better teachers.

Further into the careers of practicing teachers it is possible to consider research questions related to the specific effects of instruction. Bobis (1991) did this in a study of computational estimation skills with fifth grade boys. Using classroom teachers to extensively work with estimation training, she found an improvement in an experimental group compared to a control group.

History of Mathematics Education

Although perhaps history could be considered as part of the social or cultural context of mathematics education, this was not done in this volume. One example of research in the area was a paper by Grasset (1991) which reported the results of examining documents from the 1850s to the 1970s in relation to school timetabling in the state of Victoria. A strong case was made that this is a factor contributing to the continued low participation of girls in mathematics courses, with consequent secondary effects. Grimison (1990) presented a detailed account of the considerations which led to the inclusion of calculus in the secondary mathematics curriculum first in England and then in the state of New South Wales. This was discussed in terms of a changing student population and note made of the situation in comparison to today.

Tertiary Mathematics Education

Although educational research has not generally been of interest in tertiary institution's mathematics departments, there have been some researchers who studied student difficulties with tertiary level mathematics concepts. Barnes (1988) interviewed both secondary and tertiary students about their understanding of the function concept and found that many misconceptions occurred at both levels. Some students preferred visualization and others preferred algebraic manipulation in using the concept. This was a very detailed study which offered pointers for future research. White (1990a, 1990b) took a similarly detailed look at students' understanding of the rate of change and derivative concepts. He considered a framework for assessing conceptual knowledge in terms of obtaining an overview, determining the structural complexity, and applying a procedure to generate a response. White (1988) also analyzed tertiary students responses to a question about the second derivative being zero. Ryan (1991) studied both secondary and tertiary students and their misconceptions in early calculus.

The question of anxiety suffered by tertiary mathematics students was considered by Watson (1988). Among her results were an association of higher anxiety with increasing age, decreasing quantitative ability, decreasing enjoyment of mathematics, and decreasing self-confidence. It was also found that women were predominant in the high anxiety group and that in an individualized setting designed to put students at ease, anxiety was still related to performance in a negative fashion. Further with respect to teaching method, Hubbard (1989) reported on the trialling and evaluation of a tertiary mathematics course without lectures.

Assessment in Mathematics

Assessment has not been a major focus of Australian mathematics education research in recent years, probably because of greater interest shown in questions associated with the social context, cognitive models which stress development rather than absolute performance, and the philosophical restraints

of constructivism. The acknowledged need of recent curriculum statements for newer methods of evaluating both students and programs, is likely to produce an increase in research in this area. Collis and Romberg (1990) addressed this question and called upon their background in cognitive psychology to suggest that multi-modal functioning must be taken into account in devising assessment procedures. Stacey (1990) considered the use of projects in senior secondary mathematics classes and suggested an assessment checklist and criteria as a way of using projects to assess students. The common assessment tasks being developed in several states have been the focus of reports, for example by Sentry and Money (1990) in Victoria and Karmelita (1990) in Western Australia. Doig (1990) discussed a new method of testing based on diagnostic mathematics profiles.

Other Topics in Secondary Mathematics

Although several chapters consider the teaching and learning of mathematical subjects at secondary level, not all topics are covered in them. Grimison (1988) carried out a study of the multiplication and division of positive whole numbers which looked at effects caused by question context and measurement units. Clatworthy and Galbraith (1987, 1989) reported a case study of mathematical modelling at the senior secondary level. Student responses to a senior secondary polynomial problem were analyzed by Chick (1988) in the light of the SOLO Taxonomy which provided a means of mapping solutions and classifying misconceptions.

FURTHER DIRECTIONS FOR FUTURE RESEARCH

As has been noted, most authors have generated issues and questions for further research as part of the conclusions to their chapters. The question also arises as to whether there are areas of mathematics education which have not yet received very much attention in Australasia but which should be addressed in the near future. One issue raised in the chapter on politics in this book suggests a field of research which should be entered by MERGA researchers, in particular because of the controversy associated with it: a national curriculum and its implications for teaching, learning and assessment.

Two issues have arisen from the publishing of *A National Statement on Mathematics for Australian Schools* (AEC, 1991) and the associated determination that national profiles be developed in order to follow students' progress. The first is related to the curriculum covered in the *National Statement*. The five content strands in the document are space, number, measurement, chance and data, and algebra. Of these, four are currently receiving research attention in Australia as shown in the chapters in Part III of this book. Research in relation to chance and data, however, has been minimal. The suggestions for ordering of topics and the appropriateness of

topics for students at particular levels have been made without any research base. This deficiency needs to be addressed.

When it comes, secondly, to the development and evaluation of profiles to reflect the *National Statement*, the situation is nearly impossible for chance and data for there are virtually no teachers or students out in the school systems who have had enough exposure to the topics to be able to be used in testing. The situation may be better in the other subject areas but the controversy reported over the manner in which profiles are being introduced suggests that rigourous research into profile development or into the development of acceptable alternatives should be a high priority of Australian researchers.

Returning to the *National Statement*, three other general strands are included in the suggested curriculum: attitudes and appreciations, mathematical inquiry, and choosing and using mathematics. Most mathematics educators would be pleased to see the acknowledgment that these aspects of learning mathematics are important enough to be singled out for special attention. Some of these areas, perhaps most notably the problem solving and expression (language) aspects of mathematical inquiry have received considerable research attention in the past but other areas, such as attitudes, appreciation, justification, applications and modelling require initial or renewed research attention. To date there appear to have been no attempts to provide profiles in these areas. For some educators this may be a relief. There must be a fear, however, that if profiles proceed in the content strands but not in these other three, then these three strands will become the poor relations when schools develop their own mathematics curricula. The question of providing valid profiles in these three areas requires urgent consideration by researchers.

The debate on profiles in relation to the *National Statement* leads to the wider question of assessment and evaluation in mathematics education. Although as noted earlier, some research is taking place in relation to assessment, the question needs to be addressed in various contexts. At the societal level, the places of the assessment of the mathematics student and the evaluation of mathematics programs need to be debated, although not only debated but also argued rationally from well-defined theoretical bases. At the classroom level, research needs to continue to find ways of getting at the constructs which it is believed important for students to have. As these vary from specific content, as reflected in three of the eight strands of the *National Statement*, then different methods of assessment and evaluation must be developed.

The influence of constructivism on mathematics education research in Australasia is likely to continue to grow in the next few years. Some readers may be surprised that there is not a chapter with that title in this volume. The topic is of course mentioned within several chapters, including those on teachers as researchers, cognitive studies and early mathematical development. The discussions surrounding constructivism published overseas (e.g., Cobb, Yackel & Wood, 1992) indicate that all three levels of interactions of learners with their environment are crucial: societal, classroom, and mathematical

content. It will be important in future research in relation to contructivist tenets to keep a balanced perspective among the three components. Much research needs to be completed before conclusions can be reached on the validity of various aspects of constructivism in achieving the goals set within mathematics education, even if those goals are altered by the constructivists themselves. Methods of evaluation of constructivist techniques in the classroom will also need to be developed.

It hardly seems necessary to comment on the desirability of encouraging joint research efforts among mathematics educators in different countries. While some research questions are culture dependent and should be considered on a local level, others are global in nature and could profit from cross-cultural research. Current joint research, such as that involving Australians and Asians through RECSAM in Malaysia, is to be encouraged and extended, as well as that with mathematics educators in the further reaches of the world.

CONCLUSIONS

It is hoped that the readers of this volume will find that mathematics education research in Australasia is alive and vibrant. That it is continuing to grow is illustrated by the increasing number of papers and articles to which references have been made in this MERGA review of research compared with earlier ones. It is MERGA's desire, through its conferences, qwards, journal and other publications to foster all aspects of mathematics education research in the region. The wider range of issues considered in this volume acknowledges that MERGA and its members cannot exist in ivory towers, considering only esoteric research questions, they must also respond to the political and cultural contexts within which they exist. Further they must show the relevance of all research to the betterment of educational understanding of mathematics learning and teaching. It is hoped that this volume contributes to that understanding.

It is appropriate that 1992 marks both another gathering of the international mathematics education community at ICME-7 and the 15th anniversary of the first MERGA conference. This volume represents an opportunity for MERGA to show both itself and others that it is coming of age.

REFERENCES

Australian Education Council. (1991). *A national statement on mathematics for Australian schools*. Carlton, Victoria: Curriculum Corporation.

Barnes, M. (1988). Research in gender and mathematics: An annotated bibliography of Australian research, 1984-1987. In D. C. Blane & G. C. Leder (Eds.), *Mathematics education in Australia: A selection of recent*

research (pp. 22-48). Melbourne, Victoria: Mathematics Education Research Group of Australasia.

Barnes, M. (1988). Understanding the function concept: Some results of interviews with secondary and tertiary students. *Research in Mathematics Education in Australia*, June, 24-33.

Blane, D. C. (1984). Current research and development in mathematics education 1982-1984. In J. Briggs (Ed.), *Summary of research in mathematics education in Australia* (pp. 82-101). Mathematics Education Research Group of Australasia.

Blane, D. C., & Leder, G. C. (1988). *Mathematics education in Australia: A selection of recent research*. Melbourne, Victoria: Mathematics Education Research Group of Australasia.

Bobis, J. (1991). The effect of instruction on the development of computational estimation strategies. *Mathematics Education Research Journal, 3*(1), 17-29.

Booth, L. R. (Ed.). (1988). *MERGA research directory 1988*. Townsville, Queensland: Mathematics Education Research Group of Australasia.

Briggs, J. (Ed.). (1984). *Summary of research in mathematics education in Australia*. Mathematics Education Research Group of Australasia.

Chick, H. L. (1988). Student responses to a polynomial problem in the light of the SOLO Taxonomy. *Australian Senior Mathematics Journal, 2*, 91-110.

Clatworthy, N. J., & Galbraith, P. L. (1987). Mathematical modelling: Innovation in senior mathematics. *Australian Senior Mathematics Journal, 1*, 38-49.

Clatworthy, N. J., & Galbraith, P. L. (1989). Mathematical modelling: An attack on conformity. *Australian Senior Mathematics Journal, 3*, 88-106.

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education, 23*, 2-33.

Collis, K. F., & Romberg, T. (1990). 'The Standards', theme and assessment. In K. Milton & H. McCann (Eds.), *Mathematical turning points - Strategies for the 1990s* (pp. 173-189). Hobart, Tasmania: Australian Association of Mathematics Teachers.

Conroy, J. S. (1984). Beyond MERGA - Research in mathematics education in Australia: 1977-1983. In J. Briggs (Ed.), *Summary of research in mathematics education in Australia* (pp. 36-62). Mathematics Education Research Group of Australasia.

Department of Employment, Education and Training. (1989). *Discipline review of teacher education in mathematics and science* (Vols. 1-3). Canberra, Australian Capital Territory: Australian Government Publication Service.

Doig, B. (1990). New approaches to mathematics assessment. In K. Milton & H. McCann (Eds.), *Mathematical turning points - Strategies for the 1990s* (pp. 220-223). Hobart, Tasmania: Australian Association of Mathematics Teachers.

Educational Research in Australia Interim Report of the Review Panel. (1991). Howthorne, Victoria: Australian Council for Educational Research.

Gallagher, A. P. (1989). Changing directions of research findings: The establishment of the Australian Research Council. *Australian Education Researcher, 16*(1), 37-44.

Garnaut, R. (1989). *Australia and the northeast Asian ascendancy: Report to the Prime Minister and Minister for Foreign Affairs and Trade*. Canberra, Australian Capital Territory: Australian Government Publication Service.

Grasset, N. (1991). Why girls don't count. *Mathematics Education Research Journal, 3*(2), 1-13.

Grimison, L. (1988). Multiplication and division of positive numbers. *Research in Mathematics Education in Australia*, June, 70-79.

Grimison, L. (1990). Some historical considerations relating to the inclusion of calculus in the secondary school mathematics curriculum in England and New South Wales. In K. Milton & H. McCann (Eds.), *Mathematical turning points - Strategies for the 1990s* (pp. 304-314). Hobart, Tasmania: Australian Association of Mathematics Teachers.

Groves, S., & Stacey, K. (1988). Problem solving: An annotated bibliography. In D. C. Blane & G. C. Leder (Eds.), *Mathematics education in Australia: A selection of recent research* (pp. 66-119). Melbourne, Victoria: Mathematics Education Research Group of Australasia.

Hawke, B. (1990). Address by Prime Minister at the opening of the Asian Australian Institute. *Australian Foreign Affair and Trade, 61*, October, 706-710.

Hubbard, R. (1989). Tertiary mathematics without lectures. *Australian Senior Mathematics Journal, 3*, 23-29.

Islam, I. (1990). The Garnaut report: A critical review. *Asian Studies Review, 13*(3), 47-48.

Jones, G. A. (1984). A review of MERGA research in mathematics education in Australia: 1977-1983. In J. Briggs (Ed.), *Summary of research in mathematics education in Australia* (pp. 6-35). Mathematics Education Research Group of Australasia.

Jones, G. A. (1988). Critical variables in mathematics education revisited. In D. C. Blane & G. C. Leder (Eds.), *Mathematics education in Australia: A selection of recent research* (pp. 3-21). Melbourne: Mathematics Education Research Group of Australasia.

Karmelita, W. (1990). Common assessment tasks in WA. In K. Milton & H. McCann (Eds.), *Mathematical turning points - Strategies for the 1990s* (pp. 348-365). Hobart, Tasmania: Australian Association of Mathematics Teachers.

Leder, G. C. (1984). Girls and mathematics: An annotated bibliography of Australian research. In J. Briggs (Ed.), *Summary of research in mathematics education in Australia* (pp. 63-81). Mathematics Education Research Group of Australasia.

Meek, V., & Goedegebuure, L. (1989). Changes in higher education the Australian case. *Australian Educational Researcher, 16*(4), 1 -25.

Navaratnam, K., & West, L. (1989, November). *Trends and issues in teacher preparation in mathematics and science education: The past decade revisited*. Paper presented at the annual conference of the Australian Association of Educational Research, Adelaide, South Australia.

Pegg, J. (1989). Analyzing a mathematics lesson to provide a vehicle for improving teaching practice. *Mathematics Education Research Journal, 1*(2), 18-33.

Popkewitz, T. S. (1981). The study of schooling: Paradigms and field-based methodologies in educational research and evaluation. In T. Popkewitz & B. Tabachnick (Eds.), *The Study of Schooling*. New York: Praeger Publishers.

Porter, P. (1989). Educational research in Australia: Indigenous or exotic? The case for the Indigenous. *Australian Educational Researcher, 16*(3), 13-26.

Ryan, J. (1991). Calculus: With some respect to time. *Australian Senior Mathematics Journal, 5*, 88-97.

Sentry, K., & Money, R. (1990). Common assessment tasks in VCE mathematics. In K. Milton & H. McCann (Eds.), *Mathematical turning points - Strategies for the 1990s* (pp. 478-496). Hobart, Tasmania: Australian Association of Mathematics Teachers.

Stacey, K. (1990). Using projects to teach and assess senior mathematics. In K. Milton & H. McCann (Eds.), *Mathematical turning points - Strategies for the 1990s* (pp. 512-518). Hobart, Tasmania: Australian Association of Mathematics Teachers.

Steffe, L. P., & Wood, T. (Eds.) (1990). *Transforming children's mathematics education: International perspectives*. Hillsdale, NJ: Erlbaum.

Viviani, N. (1989). Debate directions. *Review (Asian Studies association of Australia), 12*(3), 7-8.

Watson, J. M. (1988). Achievement Anxiety Test: Dimensionality and utility. *Journal of Educational Psychology, 80*, 585-591.

Watson, J. M. (1988). Psychology of mathematics education: An annotated bibliography of Australian research, 1984-1987. In D. C. Blane & G. C. Leder (Eds.), *Mathematics education in Australia: A selection of recent research* (pp. 49-65). Melbourne, Victoria: Mathematics Education Research Group of Australasia.

Watson, J. M. (Ed.). (1991). *MERGA research directory 1991*. Geelong, Victoria: Mathematics Education Research Group of Australasia.

Watson, J. M. (Ed.). (1992). *MERGA research directory 1992*. Geelong, Victoria: Mathematics Education Research Group of Australasia.

White, P. (1988). Students' responses to interpreting $f''(x) = 0$. *Australian Senior Mathematics Journal, 2*, 23-26.

White, P. (1990a, July). *An investigation into student responses to problems involving rates of change and derivative*. Paper presented to the

Mathematics Education Research Group of Australasia conference, Hobart. Paper available from author, Australian Catholic University, PO Box 201, Castle Hill 2154, Australia.

White, P. (1990b). Is calculus in trouble? *Australian Senior Mathematics Journal, 4*, 105-110.

White, R. T. (1988, December). *Influences on the development of Australian research in science education.* Paper presented at the annual conference of the Australian Association of Educational Research in Armidale, New South Wales.

PART II

Issues in Learning and Instruction

POLITICS OF MATHEMATICS EDUCATION IN AUSTRALIA

Jan Thomas
Victoria University of Technology (Footscray)

Mathematics education is part of education and is political because it is concerned with access to power and privilege. Research in mathematics education is political because it can not operate outside of the political nature of mathematics education. Neither is it isolated from the influence of government funding and policies. For the purposes of this review, there must be some definition of the politics of mathematics education as it is discussed here.

Thomas (1989a) described the politics of mathematics education as having two dimensions. One is concerned with social context and the factors that assist, or prevent, people from participating in mathematics. These include language, culture, gender and socioeconomic status. It is an established area of research in Australia and is important for effective political action in regard to the other dimension of the politics of mathematics education. This is concerned with government policies and practices and is the focus of this chapter.

It is a recent phenomenon for mathematics education researchers in Australia to engage in direct political action related to government actions through professional associations and lobby groups. Why this has occurred and the current political milieu in which mathematics education research operates are important in understanding both the strengths and weaknesses of mathematics education research in Australia. The first section of this chapter describes some recent changes in the Australian political scene that impinge on mathematics education. The involvement of the Mathematics Education Research Group of Australasia (MERGA) in political action and the formation of the Australian Mathematical Sciences Council (AMSC) are discussed. In the second section, writings that have supported the emergence of the politics of mathematics education as an issue in MERGA are reviewed. In addition, some recent government reports and actions that are ensuring on-going political involvement are discussed. Some final observations conclude the chapter.

THE POLITICISING OF MATHEMATICS EDUCATION

The Australian Political Context

In Australia income tax is collected by the Federal Government with the States having limited revenue raising powers. The States are given a proportion of the revenue raised and assume responsibility for primary and secondary education. The Federal Government assumes the major responsibility for funding tertiary education and research through the Department of Employment, Education and Training (DEET). The Federal Government does fund some special programs and curriculum development in the schools sector. These efforts have tended to lack continuity and special curriculum projects such as the Mathematics Curriculum and Teaching Program (Lovitt & Clarke, 1988) have seldom lasted more than three years.

The Australian economy has been very dependent on natural resources and agriculture with a result that both education and scientific strengths have tended to be undervalued. Government reports in recent years have attempted to address this (e.g., Victorian Government, 1987). In difficult economic times, however, government rhetoric about strengthening the economic base through a better educated work-force is not necessarily reflected in practice. Speedy (1992) has noted in a review of curriculum initiatives that "the economic malaise of Australia has forced a reconsideration of priorities for the economy and consequently for education" (p. 28) but that there has been a decrease in support services available to schools. The percentage of Gross Domestic Product devoted to education has been falling and is low compared with other Organisation for Economic Cooperation and Development countries (OECD, 1990). In 1991 it was 4.2% and it has been predicted that it will be 3.6% in 2001 (AEC Review Committee, 1991).

While improving mathematics education may be a government priority in principle, support for its development has actually declined. Until a few years ago, the States had considerable resources invested in research and development, curriculum projects and the professional development of teachers. The mathematics educators employed in ministries, and in regional education centres, maintained close contact with schools and with tertiary institutions. Some were active in MERGA and probably all were involved with local or national associations of mathematics teachers. In 1986 there were twelve such positions in the central office of the Ministry of Education in Victoria. By the end of 1990, none of these people still held their positions and some had been forced out of mathematics education altogether. In addition, regional staff had also been reduced. Similar reduction of curriculum support and research staff occurred across Australia during this period (M. Carss, H. Reeves, J. Malone & J. Pegg, personal communications).

In the past, the right of the States to determine their own curriculum and assessment strategies has been jealously guarded. The reduction in curriculum development at the State level has, however, been accompanied by a push for a national curriculum and the increasing prominence of the Australian Education

Council (AEC), the meeting of State and Federal ministers of education (Speedy, 1992). There is no doubt that the duplication of curriculum effort across the States, without a national framework, was costly and that it was an appropriate time for more effective national collaboration. Unfortunately, collaboration and consultation that had usually been a feature of curriculum efforts in the individual States, have frequently been less than satisfactory during recent national developments (Australian Mathematical Sciences Council, 1990-1991; National Education Forum, 1991).

Changes in the administration of the schools sector have been paralleled by re-organisation of the tertiary sector (DEET, 1988). Colleges of Advanced Education have been merged with universities or become universities in their own right. This has commonly resulted in large, multi-campus universities, with considerable tension between the merging partners. In the long term there is potential for mathematics education research to be strengthened. In the short term the mergers have been extremely demanding of time and commitment.

MERGA and the Politics of Mathematics Education

Achieving an effective voice for mathematics education research in national decision making within the current mode of operation of the AEC and DEET is proving an elusive task. It has led to alliances with other professional associations which may eventually strengthen mathematics education and is already engendering greater collaboration between mathematics educators, mathematicians and statisticians. As a result it could be argued that as government decision making in education has become more closed and bureaucratic, the interactions between the professional associations concerned with mathematics education have become more open and democratic.

The politicisation of MERGA probably began at the Tenth Annual Conference held in July 1987. A paper given by Thomas (1987) drew attention to the apparent increased attention to mathematics, science and technology education, and research in these fields, at the federal level, and detailed some of the work then being undertaken by the Victorian government (Victorian Government, 1987; Working Group on Education for Science and Technology, 1987). Attention was drawn to the role being played by the Federation of Australian Scientific and Technological Societies (FASTS) as a political lobby group promoting the sciences at the federal level.

FASTS was formed in November 1985 and a full-time executive officer, Dr David Widdup, appointed in June 1986. Widdup, a mathematics educator and lawyer, actively sought the involvement of mathematics and science educators in FASTS (e.g., see Widdup, 1988). He believed they were needed as a counter to the voices of industry and higher education and their support for a `back to basics' approach to mathematics as a way of improving standards. He noted that it was not really what they wanted but they did not know how to articulate other needs (Thomas, 1987). In 1987, the Australian Association of Mathematics Teachers (AAMT) was an associate member of FASTS and the Australian Science Teachers Association was a full member.

AAMT became a full member shortly after this. The Australian Mathematical Society (AMS) and the Statistical Society of Australia (SSA) were both foundation members of FASTS.

The Eleventh Annual MERGA Conference was held in July 1988 at Deakin University, Geelong. It was at this conference that the Special Interest Group on the Politics of Education was formed and a decision taken that MERGA should also join FASTS. At subsequent conferences, the Annual General Meeting of MERGA has dealt extensively with political issues.

In 1988, the AAMT, AMS and SSA met to consider the formation of a Mathematical Sciences Council under the auspices of FASTS. In mid-1989 a further meeting was held where representatives of MERGA and the Mathematics Education Lecturers' Association (MELA) were also present. Subsequent to this meeting, the inaugural meeting of the Australian Mathematical Sciences Council (AMSC) was held in October 1989. The foundation members of the AMSC were MERGA, AMS, SSA, AAMT and MELA.

The President's report to the first Annual General Meeting noted:

> The first year of the Council proved, not surprisingly, to be a period of learning, of familiarisation of each member Society with the others; finding common interests and concerns; and assessing the best ways to respond, as a group, to important national issues. (Gaudry, 1990)

To some extent this continued into 1991 although the Council could point to a number of solid achievements (Gaudry, 1991). It is probable that some tensions are inevitable in the formation of what is essentially the political arm of a group of professional associations with somewhat different agendas. Further, attracting the attention of government may only be possible by using language and means which are not in common usage in professional dialogue and actions. The members of the societies involved can only feel comfortable with this when they see it as political action which may be the only way the voice of the profession is heeded.

There is considerable potential for mathematics education research in Australia to benefit through the formation of the AMSC. MERGA, by itself, is small and its political voice has tended to be lost. Further, mathematics education research in Australia is tied to research in education which has not been afforded the same status and support as research in the sciences (McGaw, 1991). As part of collaborative work with others interested in the health of the discipline of mathematics, mathematics education research can have a political voice and it can influence government policies and practice. The AMSC needed time to make itself known and it is now having some success. In November 1991 the Council organised a very successful Symposium in Canberra which highlighted the importance of mathematics to Australia, and the way in which mathematics was contributing to important issues such as the

control of AIDS, water pollution, business and quality control in industry. It was widely reported in major newspapers (FASTS, 1991).

The importance of the AMSC must be seen in the light of the shift of focus of much educational policy and practice from the State to the federal level. A representative national body has become the only effective means by which bodies such as MERGA can hope to influence what is happening in mathematics education. In September 1991, a National Education Forum was formed, also in recognition of the need for a national voice to address national agendas. The AMSC has been represented at Forum meetings. The meetings highlight the powerlessness of mathematics interests in broad educational forums where, in a large group, only a handful present have any real knowledge of mathematics education and some are openly hostile to the perceived position of mathematics on government agendas.

RESEARCH AND THE POLITICS OF MATHEMATICS EDUCATION

A number of researchers could be described as being involved in the politics of mathematics education through their involvement in bodies such as the AMSC, FASTS and the National Education Forum. At this stage there is little documented research in the current politics of mathematics education. The shifts in the political structures, which currently drive the environment in which mathematics education operates, are only a few years old and constantly changing. Researching the politics of mathematics education in the later part of this century should provide an interesting area of investigation.

Accordingly, the literature reviewed in the following sections has been selected on the basis that it informs the discussion above. There are numerous reports and documents emanating from State and federal governments that impinge on the politics of mathematics education. Kennedy (1988) noted that 21 major reports on education were produced between 1981 and 1987 and there have been many more since. Speedy (1992) listed over eighty publications from the National Board of Employment, Education and Training alone in the period between late 1988 and the beginning of 1992. It is also possible that some of the more significant documents that are guiding government policy are not even public. At the December 1991 meeting of the National Education Forum, a representative of the Australian Teachers' Union noted that there was an aural tradition developing in government departments. Papers were not tabled with the result that the influence of people on key committees was increasing (National Education Forum, 1991, December). The representative noted the difficulties this presented for researchers trying to respond to the many documents being circulated. The literature reviewed is therefore limited to some extent and grouped under four headings:

* historical perspectives,
* funding for mathematics education research,
* the politicisation of mathematics education, and
* recent reports and initiatives

The first two of these, and the issue of the social context of mathematics education that was so much part of the International Congress on Mathematical Education in 1988, have provided much of the stimulus for the politicisation of mathematics education. It is recent government initiatives, and the processes adopted for their implementation, which provide the current stimulus.

Historical Perspectives

One of the forces driving the politicisation of mathematics education in Australia is a clearer understanding of the past. Clements, Grimison and Ellerton (1989) and Ellerton and Clements (1988) documented the influence of a colonialist tendency to follow developments in school mathematics in England that persisted until quite recently. In part they attributed this to the assumption both here, and in England, that mathematics is culture-free. The development of political perspectives on mathematics education, tied as they are to social context, must be seen as part of this breaking away from the dominance of the ideas embedded in English mathematics education. Horwood (1992) noted that "the evolution of mathematics education in Australia took place alongside, if not before, its emergence overseas"(p. 33). The early strength of mathematics education in Australia has probably assisted in breaking down the dominance of overseas influence.

A number of writers developed this theme in describing developments in the States. Clements (1989) provided historical perspectives of school mathematics in Victoria within an Australian context. The final chapter (pp. 54-74) provided a timely account of recent, distinctly Australian developments which included background to the formation of groups such as MERGA and the conferences and structures that now exist to support mathematics education. Truran (1991) examined aspects of mathematics teaching in South Australia and concluded that the forces operating in the eastern States were not always those operating elsewhere. Horwood (1991) examined historical and political aspects of curriculum, particularly in Victoria, and with reference to the recent introduction of a new certificate for the final years of schooling. Major changes in the curriculum in the final years of schooling are a feature of developments in the current history of mathematics education (Grace, Horne, Stephens, Baxter, Jamieson, Karmelita, & Reeves, 1989).

Taken together these writings show an emerging independence in mathematics education in Australia that is influenced, but not tied to, overseas developments. They particularly support a resistance to mathematics education tied to economic rationalism that does not take account of equity and social justice issues in the teaching and learning of mathematics in Australia.

Funding for Mathematics Education Research

An appropriate mathematics curriculum for Australians cannot be developed without a strong research base and funding for research is also a major political issue. The major source of research funds, the Australian Research Council, funds educational research, including mathematics education research, through its allocation to the humanities and social sciences. Hence mathematics education research in Australia is very much tied to educational research and to research in the humanities and social sciences. This has been documented as part of a comprehensive review of educational research in Australia that was initiated in December 1990. An interim report was published in December 1991 (McGaw, 1991). While responses are currently being sought before the final report is prepared, the interim report is a rich source of information about current educational research funding in Australia. It would seem that mathematics education research has fared well in recent grants allocations through the traditional mechanisms when compared with other areas of education. This must be set however in the context of a very low level of funding for educational research overall. Mathematics education research appears to be getting a reasonable share of a very small cake.

Unlike many reviews in Australia, which have tended to rely solely on government funding (e.g., DEET, 1989), this one has a somewhat unique funding base. The list of financial sponsors includes professional associations, State governments and the Australian Research Council. It remains to be seen whether this affords it the same status as solely government funded reviews as it is not even clear to whom the review panel is reporting. It appears to be an example of a profession taking some control of review processes when it was suspected that there were shortcomings which needed to be documented.

MERGA's submission to this review was jointly prepared with MELA and endorsed by the AMSC (Clements & Sullivan, 1991). This comprehensive submission, which concluded with three pages of suggested areas of mathematics education research in Australia, was thus representative of the views of the mathematics education profession in Australia. While the importance of mathematics education research is acknowledged in the interim report, it is tied to fairly narrow views related to economic development (McGaw, 1991, p. 59). It would appear that the mathematics education profession must continue to try and educate the rest of the education profession about other reasons for the study of mathematics. The interim report would appear to hold little joy for mathematics education research while it remains tied solely to educational research, unless there is a considerable increase in resources, as the pool of funds is very small. The AMSC is presently investigating research funding for mathematics education in an attempt to achieve a firmer basis (Gaudry, 1991).

The Politicisation of Mathematics Education

While desires to break with the past, and to achieve adequate support for research, have been important factors in the politicisation of mathematics

education, a number of writers have also contributed to a growing awareness that mathematics educators can no longer afford not to be informed about government policies and actions which impinge on mathematics teaching and research at all levels from infant to tertiary.

Four chapters by Pitman, Kemmis, Thomas and Noss, with an introduction by Leder, in Ellerton and Clements (1989) provide some insight into the emerging issues of the politics of mathematics education. In her introduction, Leder (1989) quotes an example of the Chinese government's political agenda affecting curriculum content and continues in the following manner:

> It is perhaps easier to identify and be objective about such political interference with educational practices and curriculum content in cultures different from our own. It is tempting to reject the existence of parallel, though perhaps more subtle, pressures in our educational systems. That complacency is challenged by this section. A common theme runs through its four chapters: the extent that political and economic considerations dictate the content of school mathematics in Western societies. (p. 97)

Taken together these chapters provide compelling arguments for Australian mathematics educators to continue to argue for a voice in determining what happens in mathematics education in Australia. Pitman (1989) surveyed the political, social and economic factors influencing curriculum and views economic factors as driving more recent reforms. Kemmis (1989) dealt with theory, policy and practice and the way in which the authority of policy and practice has come to dominate theory. Thomas (1989a) continued the theme of economic imperatives driving curriculum which is not adequately informed by theory. Noss (1989) discussed recent developments in the United Kingdom in a chapter described as "pessimistic" by both Leder (1989) and Conroy (1991).

Concern about recent developments is also expressed by Thomas and Costello (1990) in their discussion of the changing context of the politics of mathematics education. They argue that mathematics educators should play a more active role in the politics of mathematics education and also note the emphasis on the economic value now placed on mathematics. Particular concern is expressed about the lack of support for teachers in terms of both resources and professional development, and the dangers of the trend towards a national curriculum without this support. Thomas (1989b) has documented how support for the professional development of teachers is as much dependent on political factors as on what a program has to offer.

Probably the most difficult task facing mathematics educators is educating other educators, and the wider community, about mathematics education. Thomas (1989c, 1990) has gone outside of traditional forums to argue that the general education community needs to engage in some radical re-thinking in its views of mathematics and mathematics education in addressing equity issues.

This however is not a common occurrence. McTaggart (1991), an educator at Deakin University who has clearly been conversing with the mathematics educators there, demonstrated that this is something that needs to be taken more seriously. His discussion of the possibility of coalitions between such bodies as the Australian Curriculum Studies Association and the AMSC in responding to government initiatives in mathematics was welcomed. Both Thomas (1990) and McTaggart (1991) have drawn attention to concerns about the way issues relating to mathematics education have been dealt with politically. They suggested that the broader education community needs to be aware of current issues in mathematics education because the difficulties have been encountered are likely to arise in other discipline areas.

Involving the wider community is often even more difficult and it is important to understand prevailing community attitudes to mathematics. The work of Galbraith and Chant (1990) in documenting public opinion is a valuable contribution to knowledge in this area. Knowledge about enrolment patterns in mathematics is also important, and the role of Dekkers, De Laeter and Malone (1991) in describing this is an important contribution to the politics of mathematics. Finally, assessment practices are always a major political issue beyond the scope of this chapter. The work of Galbraith (1991) however, that locates issues of mathematics and assessment in a wider educational and societal context, should be considered as a particularly important contribution.

Recent Reports and Initiatives

A number of national projects and reports currently drive the politics of education and mathematics education. Only those that directly impinge on mathematics education are discussed.

The Discipline Review. The Discipline Review of Teacher Education in Mathematics and Science was established in July 1988 and concluded in late 1989 (DEET, 1989). While MERGA had some concerns about the processes used in the initial stages, these were allayed by the extensive consultation which occurred during the review and the preparation of the final report. If the recommendations of the report were fully implemented, mathematics education would be in a very strong position in the tertiary sector. Unfortunately, while the recommendations were supported by government, and the position of mathematics education has undoubtably been strengthened, no additional financial support was forthcoming (Dawkins, 1990). The responsibility for implementation has rested principally with the Deans of Education, few of whom are mathematics educators, and has required the re-allocation of existing funds. FASTS conducted a symposium on the report in February 1990. Government spokespersons made it quite clear that the implementation of the recommendations was an institutional, not a government, responsibility (Thomas, 1990, April).

MERGA collected data on the implementation of the Review in 1991. It indicated that the Review was having some impact but only on the more readily achievable recommendations. Where capital expenditure was required to improve resources little was occurring. Recommendations in relation to research have not been adequately supported by government although a number of institutions - including Monash University, the University of Melbourne and Queensland University of Technology - have recently supported Chairs in Mathematics, or Science, Education. The real test of government commitment to the Review will come in 1993 as the final recommendation of the Review was that a report on the implementation of the recommendations be commissioned in that year (DEET, 1989).

Recent developments. There are several developments driving mathematics education in Australia at the present time. The most significant are The *National Statement on Mathematics for Australian Schools* (AEC, 1991), the mathematics assessment profiles project (New South Wales Department of Education, 1991) and a report on post-compulsory education and training commonly referred to as the Finn Review (AEC Review Committee, 1991). All of these projects are controlled by the AEC.

Mathematics is one of eight areas where it has been determined that there will be national statements and profiles. In the case of mathematics, the process began with a mapping of the curriculum offering by the various States (AEC, 1988). This document was then virtually ignored and a project team employed to write a national statement. It has been the view of many MERGA members for some time that a national framework, but not a national curriculum, would be a way of bringing some cohesion into mathematics education across Australia. MERGA however, believed that this should be strongly influenced by the researchers and teachers. In what has become a common feature of such AEC projects, the profession was not consulted, responsibility for overall management was given to a State Department of Education, a project team was assembled from personnel from within the State ministries rather than through consultation with the profession, the project was underfunded and it had quite unrealistic timelines. The federal ministry did fund a consultant from the tertiary sector but the position was not advertised. There was essentially no consultation with the mathematics education community until it was asked to comment on poorly prepared draft documents just before the Easter break. The deadline for reply was only a few weeks (AMSC, 1990).

The AMSC, and all constituent societies, raised numerous objections to the way in which the document was being prepared. MERGA was particularly opposed to the way in which it was being done and eventually a number of members were paid for providing comment on some sections. The AMS was invited to a meeting with the project team. The AEC's deadlines were extended and the comment provided greatly improved the document. The AMSC however, has not endorsed the final document and some members have serious reservations about it. The AAMT has been funded by the Federal government

to run professional development to introduce teachers to the Statement but even this is seen as providing a period of review rather than endorsement or complete support (AMSC, 1991, July). The AMSC is undertaking a detailed analysis of the Statement (H. Rubinstein, personal communication, February 1992).

Inadequate consultation on the National Statement culminated in a workshop in Melbourne in October 1990. One outcome of this was a decision that there needed to be further work done on assessment practices in mathematics to support the National Statement. The AMSC also believed that the AEC had agreed that the processes used in the consultative processes for the National Statement were unsatisfactory and that the Council would be consulted before any further national mathematics projects were undertaken (Thomas, 1991).

In April 1991, however, the AMSC was asked to provide comment on draft material emanating from a project preparing mathematics subject profiles. It was apparent that the project was already nearly six months old and there had been no consultation prior to this. Subject profiles have been described by the AEC as "assessment and reporting frameworks that can provide a means of monitoring student performance in order to inform students, teachers, schools, education systems and the community" (MERGA, 1991, July). The project is causing considerable concern in the AMSC and all member societies (Gaudry, 1991). Clements (AMSC, 1991, April) has been particularly concerned that they appear to be behavioural objectives, not linked to learning. Speedy (1992) has raised similar concerns in regard to the development of competency-based training in vocational and professional education.

Unlike its early involvement with the National Statement, to date the AMSC has not been prepared to be involved with the profile project and has continued to try and have the profiles project stopped until its aims, objectives and research base are clearly articulated in proper consultation with the profession. There is considerable concern about the ultimate uses of the profiles and whether they are the precursor to national testing. There is also a view that it cannot be done properly with the resources allocated.

At the beginning of 1992, representatives of AMSC, MERGA, MELA, AAMT and other groups were invited to a reference group meeting to be held in Sydney on February 13. The lack of commitment by the AEC to proper consultation with the most appropriate people, and the inadequate budget for the project, is exemplified in the letter of invitation which stated: "Unfortunately the budget for the Project does not include provision for travel or accommodation for reference group meetings. You may therefore wish to nominate a Sydney-based person to represent your organisation to minimize costs" (AMSC, 1992a, January). The reference group should have been formed, and funded to meet, before the project began. The question that confronted the AMSC, MERGA and others was what participation would imply. In the event representatives of all four groups participated. Subsequent to the meeting however, the President of the AMSC wrote to the convenor "to

place on record some of the questions to which adequate answers were not given". He noted that the Council had demonstrated that it was prepared to be involved to this and other projects but only subject to some conditions (AMSC, 1992, February).

The most recent national report which is likely to have serious impact on mathematics and mathematics education is the Finn Review, which has taken a very limited view of mathematics and identifies the key competencies as computation, measurement and understanding mathematical symbols (AEC Review Committee, 1991, p. 68). The report places considerable emphasis on nationally comparable assessment in terms of National Competence Profile Levels (p. 69). Subsequent to the release of this report, a committee was established to undertake further work on the development of key competencies (Mayer, 1992). Once again the mathematics profession has not been consulted although a sub-committee had been established in mathematics (AMSC, 1992b). The implementation of the competencies is planned to begin in 1993.

Taken together, these recent developments can only be viewed with concern. They confirm the fears of mathematics educators writing about the politics of mathematics and mathematics education in the last few years that the driving forces would be socio-political rather than educational. Speedy (1992) has noted that the principal force for this is economic.

CONCLUSIONS

There have been dramatic and rapid changes in the political climate in which mathematics education research in Australia operates. Recent developments outlined in this chapter demonstrate the particularly difficult context in which mathematics education research operates in Australia at the present time. The informal, but productive networks, which operated at the State level have collapsed. The shift to national agendas has seen the emergence of processes which ignore the voice of the professions and make token attempts at consultation that are used to legitimise the resulting products. It is regrettable that in the current political climate, the voices of Australian mathematics education researchers are more likely to be heard overseas than in the corridors of power in Canberra.

To redress this situation, MERGA must focus on national agendas and the formation of national bodies which can attempt to develop an effective voice at a national rather than state level. It is a difficult and confusing shift for all involved in education, not just those involved in mathematics education. The science community as a whole, through FASTS, has shown the way to effective political influence in this environment. The AMSC, under the FASTS umbrella, has shown that collaborative efforts between groups who have previously been in competition with one another is possible, productive and necessary if there is to be a professional voice at the national level. To be effective there must be openness and trust by all constituent societies in such

alliances. This can only be achieved by democratic and open operation of these bodies. It also requires that the mathematics education community as a whole recognises what has happened and that the old ways are no longer effective.

Providing appropriate mathematics education is a fundamental equity issue because it is concerned with access to power and privilege. In Australia, achieving equity is a complex task involving students in remote rural settings and in large cities and is made more challenging by the multicultural nature of Australian society. Only the mathematics education community itself has any comprehension of the task involved in achieving a mathematics curriculum worthy of a wealthy, democratic nation. The simplistic notions underpinning recent policy initiatives exemplified by the National Statement and the Profiles project must be addressed. This needs the support of all concerned with mathematics and the development of an effective national voice.

A final observation can be made: as the processes used by governments in Australia to determine what happens to mathematics education teaching and research become progressively less democratic and less concerned with equity issues, the profession becomes increasingly democratic and more concerned with these issues. In that lies the real political strength of the mathematics education community in Australia.

Acknowledgments. The valuable comments and advice provided by John Horwood (Victoria University of Technology) and David Widdup (Executive Director, FASTS) in the preparation of this chapter are gratefully acknowledged.

REFERENCES

Australian Education Council. (1988). *Mapping the Australian curriculum: The mathematics curriculum*. Canberra, Australian Capital Territory: Department of Employment, Education and Training.

Australian Education Council (1990). *A national statement on mathematics for Australian schools*. Melbourne, Victoria: Curriculum Corporation.

Australian Education Council Review Committee. (1991). *Young people's participation in post-compulsory education and training (Finn Review)*. Canberra, Australian Capital Territory: Australian Government Publishing Service.

Australian Mathematical Sciences Council. (1990). [Minutes of meetings Nos 3-7]. GPO Box 2181, Canberra ACT 2601.

Australian Mathematical Sciences Council. (1990-1991). [Minutes of meetings]. GPO Box 2181, Canberra ACT 2601.

Australian Mathematical Sciences Council. (1991, April 28). [Minutes of Meeting No. 11]. GPO Box 2181, Canberra ACT 2601.

Australian Mathematical Sciences Council. (1991, July 10). [Minutes of Meeting No. 14]. GPO Box 2181, Canberra ACT 2601.

Australian Mathematical Sciences Council. (1992a, January 21). [Letter to the President, G. Gaudry]. GPO Box 2181, Canberra ACT 2601.

Australian Mathematical Sciences Council. (1992b, January 31). [Correspondence from D. Widdup]. GPO Box 2181, Canberra ACT 2601.

Australian Mathematical Sciences Council. (1992c, February 25). [Letter from the President, G. Gaudry]. GPO Box 2181, Canberra ACT 2601.

Clements, M. A. (1989). *Mathematics for the minority: Some historical perspectives of school mathematics in Victoria*. Geelong, Victoria: Deakin University Press.

Clements, M. A., Grimison, L., & Ellerton, N. F. (1989). Colonialism and school mathematics in Australia 1788-1988. In N. F. Ellerton & M. A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 58-78). Geelong, Victoria: Deakin University Press.

Clements, M. A., & Sullivan, P. (1991). Mathematics education in Australia: Prospects and needs. In J. A. Malone & L. D. Miller (Eds.), *Collected papers from the ninth Biennial Conference of the Mathematics Education Lecturers Association* (pp. 8-20). Perth, Western Australia: Curtin University of Technology.

Conroy, J. (1991). Review of School mathematics: The challenge to change. *Mathematics Education Research Journal*, *3*(1), 45-50.

Dawkins, J. S. (1990, January 9). Media Release: Release of major report on teacher education. Canberra, Australian Capital Territory: Department of Employment, Education and Training.

Dekkers, J., De Laeter, J. R., & Malone, J. (1991). *Upper secondary school science and mathematics enrolment patterns in Australia, 1970-1989*. Perth, Western Australia: Curtin University of Technology.

Department of Employment, Education and Training (1989). *Discipline review of teacher education in mathematics and science*. Canberra, Australian Capital Territory: Australian Government Publishing Service.

Department of Employment, Education and Training (1988). *Higher education: A policy statement*. Canberra, Australian Capital Territory: Australian Government Publishing Service.

Ellerton, N. F., & Clements, M. A. (1988). Reshaping school mathematics in Australia 1788-1988. *Australian Journal of Education*, *32*(3), 387-405.

Ellerton, N. F., & Clements, M. A. (Eds.). (1989). *School mathematics: The challenge to change*. Geelong, Victoria: Deakin University Press.

Federation of Australian Scientific and Technological Societies. (1991, December). *Newsletter No. 42*, GPO Box 2181, Canberra ACT 2601.

Galbraith, P. L. (1991, July). *Paradigms, problems and assessment: Some ideological implications*. Paper presented at the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia. [Available from Author, Department of Education, The University of Queensland, QLD 4072].

Galbraith, P. L., & Chant, D. (1990). Factors shaping community attitudes to school mathematics: Implications for future curriculum change. *Educational Studies in Mathematics, 21*, 299-318.

Gaudry, G. I. (1990, October). *President's report to the Australian Mathematical Sciences Council.* Australian Mathematics Sciences Council, GPO Box 2181, Canberra ACT 2601.

Gaudry, G. I. (1991, November). *President's report to the Australian Mathematical Sciences Council.* Australian Mathematics Sciences Council, GPO Box 2181, Canberra ACT 2601.

Grace, N., Horne, M., Stephens, M., Baxter, J., Jamieson, R., Karmelita, W., & Reeves, H. (1989). Changes in mathematics curriculum: A view across the States. *The Australian Mathematics Teacher, 45*(2), 4-10.

Horwood, J. (1991). Mathematics curriculum in transition - Victoria. In J. O'Reilly & S. Wettenhall (Eds.), *Mathematical Ideas* (pp. 91-96).Melbourne, Victoria: Mathematical Society of Victoria.

Horwood, J. (1992). Birth of AAMT. *The Australian Mathematics Teacher, 48*(2), 32-33.

Kemmis, S. (1989). From curriculum innovation to educational reform: Changing schools. In N. F. Ellerton & M. A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 120-138). Geelong, Victoria: Deakin University Press.

Kennedy, K. J. (1988). The political context of curriculum reform in Australia in the 1980s. *Australian Journal of Education, 32*(3), 357-374.

Leder, G. C. (1989). Dimensions of power and mathematics education: Introduction. In N. F. Ellerton & M. A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 97-100). Geelong, Victoria: Deakin University Press.

Lovitt, C., & Clarke, D. (1988). *The mathematics curriculum and teaching program.* Canberra, Australian Capital Territory: Curriculum Development Centre.

Mayer, E. (1992). *Employment-related competencies for postcompulsory education and training: A discussion paper.* Melbourne, Victoria: Ministry of Education and Training.

McGaw, B. (1991). *Educational research in Australia: Interim report of the Review Panel: Strategic review of research in education.* Melbourne, Victoria: Australian Council for Educational Research.

McTaggart, R. (1991, July). *Community movements and school reform: A new coalition for action research.* Paper presented at the biennial conference of the Australian Curriculum Studies Association, Adelaide, South Australia. (Available from Author, Faculty of Education, Deakin University, Geelong VIC 3217).

Mathematics Education Research Group of Australasia. (1991, July). [Letter to the President, M. A. Clements]. Deakin University, Geelong VIC 3217.

National Education Forum. (1991). Minutes of meetings. PO Box 884, Belconnen NSW 2616.

National Education Forum. (1991, December). Unconfirmed Minutes, Meeting No. 2, PO Box 884, Belconnen NSW 2616.

Noss, R. (1989). Just testing: A critical view of recent change in the United Kingdom school mathematics curriculum. In N. F. Ellerton & M. A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 155-169). Geelong, Victoria: Deakin University Press.

New South Wales, Department of Education. (1991). *Draft mathematics profiles*. GPO Box 33, Sydney NSW 2001.

Organisation for Economic Cooperation and Development. (1990). *Education in OECD countries*. Paris: Organisation for Economic Cooperation and Development.

Pitman, A. (1989). Mathematics education reform in its social, political and economic contexts. In N. F. Ellerton & M. A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 101-119). Geelong, Victoria: Deakin University Press.

Speedy, G. (1992). *Curriculum initiatives*. Canberra, Australian Capital Territory: Australian Government Publishing Service.

Thomas, J. (1987, July). *The political agenda and mathematics education and research*. Paper presented at the tenth annual conference of the Mathematics Education Research Group of Australasia, James Cook University, Townsville, Queensland.

Thomas, J. (1989a). The politics of mathematics education: A personal construction. In N. F. Ellerton & M. A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 139-154). Geelong, Victoria: Deakin University Press.

Thomas, J. (1989b). A professional development model in support of teacher change. In J. Malone, H. Burkhardt, & C. Keitel (Eds.), *The Mathematics Curriculum: Towards the Year 2000. Proceedings of the Sixth International Congress on Mathematical Education* (pp. 445-451). Perth, West Australia: Curtin University of Technology.

Thomas, J. (1989c). Mathematics education, equity and national interests. In R. Woock (Ed.), *Education, Equity and National Interests: Proceedings of the Australian and New Zealand Comparative and International Education Society Conference* (pp. 259-267). Melbourne, Victoria: University of Melbourne.

Thomas, J. (1990 April) [Report to MERGA Executive]. P.O. Box 64, Footscray VIC 3011.

Thomas, J. (1990). Mathematics education: Crisis and response. *The Australian Teacher, 25*, 18-20.

Thomas, J. (1991, 6 October). Maths project draws fire. *The Age*, Melbourne, Victoria.

Thomas, J., & Costello, C. (1990). The changing context of politics of mathematics education in Australia. In K. Milton & H. McCann (Eds.), *Mathematical Turning Points* (Vol. 2, pp. 536-542). Hobart, Tasmania: Australian Association of Mathematics Teachers, 1990.

Truran, J. (1991, July). *Secondary school mathematics in South Australia 1870 to 1913*. Paper presented at the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia. (Available from Author, P O Box 157, Goodwood SA 5034).

Victorian Government. (1987). Victoria: The next decade. Melbourne, Victoria: *Government Printer*.

Widdup, D. (1988). FASTS reports to its education sector members. *The Australian Mathematics Teacher, 44*(2), 9-10.

Working Group on Education for Science and Technology (1987). Improving the science, mathematics and technology education base in Victoria. Melbourne, Victoria: *Government Printer*.

THE SOCIAL AND CULTURAL CONTEXT OF MATHEMATICS EDUCATION

Bill Atweh
The Centre for Mathematics and Science Education
Queensland University of Technology

Tom Cooper
The Centre for Mathematics and Science Education
Queensland University of Technology

Clive Kanes
The Institute for Learning in Mathematics and Language
Griffith University

Social context is a growing area of mathematics education research in Australia. In a similar manner to the rest of the mathematics education world, Australian researchers have seen the need to take the social context of instruction and the learner into account in attempting to understand teaching and learning.

The context of instruction is as varied as the learners who attempt learning. Australian researchers have become interested in most situations - the social interactions in the classroom, the background of the students, the role of the teacher and the politics of schooling. Researchers have taken a language, a cultural and a socially critical perspective to their work. Particular emphasis has been placed on mathematics learning of the indigenous Aboriginal population.

The problem for the reviewers has been to find a structure for this chapter that will encompass the wide variety of articles in this area. To maintain coherence, the articles have been divided into four foci: culture and language, students' social background, classroom interaction, and the teacher. The scope of the articles in each foci is such that at times it has been difficult to cleanly assign articles to each section. Where there has been contention, the reviewers have attempted to be true to the major thrust of the article. Further, issues that are normally addressed under the heading of social context overlap with topics discussed under other chapters in this review, in particular, the chapter on gender and mathematics and the chapter on politics of mathematics education.

Articles have been collected from Australian sources and from international sources where Australians have been authors. There have been

difficulties finding resources that have not been indexed. Sources such as technical reports and unpublished theses may have been missed.

FOCUS ON CULTURE AND LANGUAGE

This section covers studies of Aboriginal learning, bilingual students and other cultural and socio-linguistic perspectives.

Studies of Aboriginal Students

A Commonwealth Task force investigating needs for a policy statement on Aboriginal education (National Aboriginal and Torres Strait Islanders Education Policy, 1989) concluded that Aboriginal people remain the most educationally disadvantaged group in Australia. The task force found that participation of Aboriginal people in all forms of education was the lowest experienced by any group in Australia. This low participation has attracted mathematics-education researchers in Australia.

Aboriginal culture and mathematics education. A critique of mathematics education as it relates to the schools where Aboriginal students study was presented by Watson (1988). This critique was intended to be useful in developing appropriate mathematics curricula and pedagogy in these schools. Watson noted that the traditional policy and practice of educational systems as they related to Aboriginal students were based on assimilation and the prevention of the reproduction of the Aboriginal culture. Instead she called for an Aboriginal education policy that is based on the bi-cultural nature of Aboriginal society and acknowledges the differences between the Aboriginal and Australian cultures and assists Aboriginal children to be fluent and confident in functioning in both cultures.

Watson argued that the problem with previous attempts to develop mathematics-education programs for Aboriginal children was a failure to note that the needs and backgrounds of the different Aboriginal societies around the country lacked uniformity. Watson stated that Aboriginal societies ranged from traditional societies and homeland centres to urban dispersed communities, with languages ranging from high use of oral form to high use of printed material. She concluded that researchers, and indeed practitioners, in mathematics education should acknowledge the needs of each of these communities separately.

Watson discussed three main aspects of mathematics education that effect the teaching and learning of mathematics for Aboriginal students. Firstly, Watson pointed out that students appropriate a schema of meaning in learning mathematics, a schema which grows from the semantic structure of Indo-European languages. Hence, she argued that Aboriginal students who were developing two alternative schemata for coping with their traditional culture as well as the dominant European culture need to develop a higher level meta-

schema that bridges the two cultures. This, she suggested may mean that research into mathematics education of bilingual children may be used to better understand the development of mathematical concepts in other children. In an earlier study of bilingual children in Australia and Nigeria, Watson (1987) had found evidence that monolingual Yoruba children were slower in appropriating concepts associated with number than their bilingual compatriots and monolingual English speaking children and that the bilingual children's learning was enhanced by their bilingualism.

Secondly, Watson (1988) argued that mathematics-education lessons usually incorporate a style dominated by writing and sequential progression, while oral cultures learn by experience and example rather than by formal lessons. Further, she contended, their knowledge was more holistic, where meanings of abstract words were associated with metaphors which often invoked physical landscape images. Thirdly, Watson argued that mathematics meanings are value-laden and thus the learning of mathematics was emotionally arousing. She contended that Aboriginal learners have good reasons to feel suspicious about, and resist, learning which will induct them into a society for which they have mixed feelings, particularly when the learning involved mathematics, a subject which is highly valued in a society that devalues Aboriginal culture and knowledge.

In her book *Singing the Land, Singing the Land*, Watson (1989b) made a close study of the differences between Aboriginal and European cultural views on some very basic mathematical concepts such as patterns, relations, number and space. In another study, Watson (1990) compared the concept of number in Nigerian traditional society and English speaking communities.

Another critical analysis of mathematics education and the needs of the Aboriginal student was presented by Crawford (1990). The main topic of this analysis was the role of technology in mathematics teaching. While students from English speaking backgrounds have difficulties with using English registers to discuss mathematical ideas, these difficulties were exacerbated with bilingual students from non-technological societies. The author noted that Aboriginal dialects constituted a restricted code in mathematical discourse. Further, Aboriginal students were often exposed to mathematical content and skills without having a chance to understand the cultural and social context of the mathematics they were exposed to.

Crawford argued that new educational software provided several potential benefits for the education of Aboriginal students. Collaborative learning environments provided Aboriginal students with a non-threatening context in which the cooperative and non-competitive values of their societies could be practiced. Similarly, they provided students with opportunities to reflect and hence learn about their learning. Crawford contended that this enabled Aboriginal students to acquire "secret" knowledge traditionally not available to them in normal mathematics classes. Lastly, she argued that the use of computers in mathematics classes had the potential of changing the social organisation of the classroom to allow for the discussion of the social and

cultural context of mathematics knowledge, an innovation that would assist Aboriginal learners.

A wide ranging review of the relevant literature on mathematics education and Aboriginal children was presented by Graham (1988). The review was carried out with the intention of developing more effective educational procedures for these children. Graham agreed with Watson (1988) that aboriginal students grow in a bi-cultural society. As Graham (1988) stated: "Where there is conflict between ways of perceiving, talking and thinking about reality they must be presented as alternatives ..." and children should be "... able to act appropriately in the situation in which they find themselves" (p. 131). Graham contended that Aboriginal students need more time, particularly in the early years, to develop the foundation concepts of mathematics. She argued that while number concepts of Aboriginal students may not be as developed as equivalent non-Aboriginal students, more use should be made of the visual/spatial abilities that these students bring with them to school. She concluded that the ethnomathematics of the Aboriginal culture should be used whenever possible and that students should have the opportunity to be involved in real-life mathematical experiences beyond those provided by the use of concrete materials. She warned, however, that care should be taken that this does not lead into a watered down version of mathematics that would dis-empower Aboriginal students in their future lives in Australian society.

J. Harris (1987) rejected the long held claim by earlier ethnographers and sociologists studying the Aboriginal societies that Aborigines could not count beyond three. Harris presented a large amount of evidence that showed that different Aboriginal languages had many more words for numbers than had been widely accepted. He presented evidence of use of compound names for numbers as well as body counting that he contended showed that Aboriginal societies did count with some precision. He argued that since counting in Aboriginal societies was a meaningful activity and not an abstract science, school examples such as the task of counting sticks would have appeared a very useless and meaningless activity.

In another paper P. Harris (1989) illustrated the complexity of the context factors that need to be taken into account for bringing about change in the design of mathematics curricula for bi-cultural students. Her chapter "Contexts for change in cross-cultural classrooms" was based on an extensive review of the literature as well as her own experiences as a teacher in the Warlipiri community in the Northern Territory. Bringing about curriculum change should take into account the historical context of such communities, "if we are not to risk perpetuating the errors of the past in planning new educational programs for Aboriginal people..." (p. 82). Similarly, the socio-political context is an essential factor to be studied since Harris asserts that "in Aboriginal schools ... the effects of political decisions are a more constantly felt influence than they are in schools in mainstream Australia" (p. 83). In support of the different studies that considered the differences in between Aboriginal languages and English, Harris asserted that the distance between

these languages and English is far greater than the distance between most migrant languages and English. In her discussion of the cultural context of mathematics in the Aboriginal tradition, Harris used three categories of Mathematical thinking identified by Bishop (1988), namely, number, space, and measurement. In each case, she pointed out significant differences between the concepts and use of such constructs between the two cultures. Harris concluded her study by presenting some pointers for practitioners involved in teaching mathematics to Aboriginal students.

P. Harris's (1991) book *Mathematics in a Cultural Context: Aboriginal Perspectives on Space, Time and Money* is the culmination of twenty years of involvement of the author with studying Aboriginal education and culture, researching Aboriginal mathematical thinking and teaching Aboriginal children in remote areas. Ideas in the book evolved from the Mathematics in Aboriginal Schools project supported by the Curriculum Development Centre and the Northern Territory Department of Education. The aim of the book was to increase the awareness of teachers of Aboriginal students of the differences in conceptions between the Aboriginal and Western cultures on concepts such as time, space and money. Of noticeable absence from the book was the discussion of the concept of number, which usually plays a central role in Western programs in primary mathematics. Harris pointed out that the concept of number "was given the lowest priority because it is the area of mathematical thinking which is probably most alien to an Aboriginal world view; and the first aim of the project was to highlight what *is* important to Aboriginal people" (p. 2). This book is valuable resource for understanding the Aboriginal context for mathematics teaching, both for researchers and practitioners.

Aboriginal children and cognitive processing. Very few researchers have investigated the cognitive ability and processing of information of Aboriginal children. Boulton-Lewis, Neill and Halford (1987a, 1987b) studied Aboriginal children's capacity to process information. Tests were designed to minimise the influence of background knowledge, style or strategy on performance. This was achieved in two ways. The context of the questions was selected to be familiar to Aboriginal communities. Secondly, two parallel forms of items were used that differ in the demand on cognitive processing needed. The logic was as follows. If students passed the less demanding form and failed the more demanding form, then the test provided evidence that the difficulty lay in the information processing factor rather than content or student background.

Seventy five children from 4 years 1 month to 8 years 11 months were chosen from a traditional rural community about 290 kilometres from Brisbane. The study concluded that Aboriginal children in the sample possessed information processing capacity that was similar to, and comparable with, non-Aboriginal children of same ages. Similarly, the performance of these children on length and number tasks were comparable to non-Aboriginal children. The authors concluded that "... lower levels of achievement by Aboriginal children

in school mathematics are probably a function of motivational, educational and environmental factors" (1987a, p. 65).

A follow up study into two samples of urban children both of Aboriginal and European background showed similar results (Boulton-Lewis, Neill & Halford 1988). The authors concluded that "... mathematics should be taught across the primary grades (and probably later) so that it was related, as far as possible, to important issues and activities in everyday life" (p. 35).

Aboriginal students at universities. Aboriginal students at the tertiary level of education are under-represented in Australian Universities. While most universities in Australia have special programs for Aboriginal and Islanders students in higher education funded by the Australian Federal government, very few publications were identified that evaluate the effect of these programs or their students. Mathematics and science are often crucial subjects for students aspiring tertiary studies. It is surprising that not more research has been carried out on programs that target Aboriginal students. Treagust, Malone, Fraser and Taylor (1987) reported on the initial stages of a program at the Curtin University in Western Australia. The evaluation of this program was largely formative. Data as to the effect of this program on students was not available.

Studies in Socio-Linguistic Contexts

Watson (1989a) departed from the recognised paradigms of cognitive psychology, and focused instead on language and social context as provided for in the later philosophy of Ludwig Wittgenstein, 1889-1951. Watson argued that, in terms of Wittgenstein, mathematics was seen as a family of broad ranging social activities. She contended that this contrasted with other established views of mathematics, for example, Platonism (the view that mathematical entities, although abstract, exist in reality, that mathematical truth is fixed and timeless); Empiricism (that mathematical concepts grow unproblematically out of the physical world); Psychologism (that mental states and internal constructions provide the basis of mathematics, external manifestations of mathematics such as the linguistic or graphical accompaniments are irrelevant); and, Formalism (the view that mathematics is a game consisting of the manipulation of meaningless symbols).

Watson argued that the way in which ordinary language is used is of central importance in Wittgenstein's view: mathematics is a language-game based on the rules of ordinary language use and ordinary material practice, it is therefore an extension of the specific social context or "form of life" within which mathematical activities are practiced. She deduced that understanding the nature of mathematics therefore involves an ethnography on the making of mathematical meanings. Watson noted that this language-centred view of mathematical knowledge has important implications for teachers of mathematics. It could be argued, for instance, that mathematical concepts have no essential meaning. She provided the example that there was not *a* concept

of seven. Instead, she argued, there was a myriad of words and sentences (language-games) with crisscrossing uses and meanings. She contended that the existence of the concept of "seven", say, depends on the strength of the "family resemblance" established between the language-games involving the word seven and its cognates in written, spoken and drawn genres. She argued that meanings of single words could only be grasped by understanding the purpose of the language-game in which they are placed; i.e., understanding was reached when the student was able to use words meaningfully whilst doing what needs to be done. Thus, she concluded, mathematical concepts could only be appropriated by the learner if their meanings were negotiated within a wide variety (oral, written, graphic, material) of textual forms; texts were not passively consumed by the learner, for learners were actively engaged in a process of reconstructing meanings; and moreover, learners must also be active in producing a wide range of texts.

In summary, Watson construed mathematics learning as: (a) learning to use written, spoken and drawn texts; (b) learning to recognise the types of text which constitute mathematical discourse; and (c) learning to construct texts. This view of learning mathematics treated learners first and foremost as social agents and mathematical meaning as intrinsically social in character.

Kanes (1991a, 1991b) also used the Wittgensteinian concepts of "language-game", "form of life" and "family resemblance" to explore learning and teaching interactions in school classrooms. Kanes (1991a) suggested that these formulations offered hope for the development of a socially interactive model for learning in mathematics. He contended that such a model would automatically incorporate the context, social and cultural, of the learner into what mathematics is learnt by the learner and how it is learnt. The argument was that mathematics should be thought of as something the learner *does* rather than something merely thought about and that language rather than thinking should be held to mediate mathematical experience. Examples of how mathematical concepts arise as language-games were provided. These were drawn from pre-school (conservation), primary school (arithmetic) and secondary school (algebra). Textual devices such as metaphor and metonymy were found to play crucial roles in the structure of the language-games explicated in these examples. Another feature was the important role accorded to affect. This derived from the *gaming* aspect of language-games. Kanes concluded that a more complete analysis may involve poststructural thought - the work of Lacan and Derrida in particular. The final part of the paper developed this approach further by speculating on how game playing and, in particular, the game of learning the (language) games of mathematics may be structured.

In another paper, Kanes (1991b) attempted to extend coverage of these themes to include both the teacher and the learner. He observed that constructivism, defined as the belief that learners learn by actively reorganising existing schemata for conceptual and procedural knowledge, had not been effectively operationalised as a system for teaching mathematics. He argued

that attempts to do so usually fail because they implicitly reassert the traditional absorption theory of learning. He contrasted such a theory with constructivism, in that the learner is thought to receive knowledge rather than construct knowledge. He supported the contentions of Bereiter and others that constructivism seems to imply that the learners learn by constructing, paradoxically, more complex structures from less complex structures. Kanes suggested that this paradox, the so-called learning paradox, underlies the difficulty researchers have in developing instructional practices fully consistent with constructivist theory.

Taking the example of an actual classroom episode, Kanes contended that a Wittgensteinian conception of the learning and teaching interaction can, in contrast, provide a coherent account of mathematics education. His argument was as follows. Classroom language does not merely point out or facilitate mathematical relations to be encoded and schematised. Instead, language falls into a myriad of language-games which are activities whose forms provide direct instances of mathematical experience itself. Elements of an epistemological theory which involves meaning and context from the outset emerge from this account. In contrast, other epistemologies often assume, explicitly or implicitly, that mathematical knowledge exists separate from mathematical contexts. Indeed, one portion of the pedagogical problem to which they give rise is to bring these distinct aspects of mathematical experience into accordance. No such problem faces the theory outlined in Kanes (1991b).

Crawford (1990) used theoretical constructs from Vygotsky, Bernstein and Halliday which related understanding of speech to motivation. She concluded that classroom language reflected hierarchical relationships not qualitative aspects of the contexts in which tasks are performed outside of school, leading to low achievement in students in non-technological cultures, cultures where knowledge of the social and physical world is not commonly shared.

Studies on Bilingual Students

A longitudinal study in bilingual students was reported in Butler and Marsh (1986). English and non-English speaking students from seven primary schools were followed for six years. In Year 6, a mathematics test was administered. Butler and Marsh found that English speaking students performed better in reading in all grades, but that there was no difference in performance on the Year 6 test between the two groups. Butler and Marsh argued that because one might have expected that verbal ability in Year 6 was essential for performance in mathematics, then it seemed reasonable to assume that there may be other factors that made bilingual students perform in Year 6 mathematics at the same level as English speaking students. The authors posited motivation as a factor.

A second study on bilingual mathematics education was conducted in New Zealand by Ohia, Molone and Knight (1990). This study reported on the first stage of a project that investigated bilingual mathematics classes held in New

Zealand secondary schools. This first stage of the project involved a questionnaire sent to different schools that were thought to operate bilingual units. Over the period of two years 52 such schools were identified. This sample contained a wide cross-section of secondary schools, both urban and rural, large and small, single sex and co-educational and containing proportions of Maori students above and below the national average. The questionnaire sought information about student and teacher characteristics, teaching methods and resources employed, nature of the school and community and effectiveness of the programs as seen by the teachers and administrators.

Ohia et al. observed that in almost all cases mathematics teaching in the responding schools was conducted in English with Maori being used for organisation and control. The mathematics content taught did not differ from the mainstream schools except that some Maori examples were included. The authors identified the major problem in such schools as being the lack of knowledge of non-Maori teachers of the Maori culture and the lack of mathematics qualifications of the participating Maori teachers.

The authors concluded that the effect of such programs of achievement is still not known. However, there was a obvious effect in raising students attitudes to Maori culture and language. It was postulated that the promotion of the Maori language and culture would increase the student's self esteem, which in turn, would lead to improve achievement. The direction of effect of these three variables would work in the other direction as well.

Ethnomathematics

A single research project attempted to identify an aspect of the Australian culture that may be regarded as a basis for ethnomathematics. Peard (1991) noted that gambling in Australia is a noticeable phenomenon that constitutes an integral part of the social life of many Australians. Australia, as a nation, spends more money, per capita, on gambling than any other nation in the world. The study reported on the early stages of a project to investigate differences in misconceptions and cognitive processing on probability tasks between children from gambling and non-gambling backgrounds. Twenty students were selected from a low socioeconomic school, and classified as gamblers or non gamblers depending on their participation in chance games, interest in horse racing or family interest in horse racing. This initial research did not yield consistent patterns of errors that students from different backgrounds showed. Even without formal instruction in probability, however, students from gambling backgrounds did show some evidence of having developed a concept of "expected value" not very different from the standard mathematical concept.

FOCUS ON STUDENTS' SOCIAL BACKGROUND

This section covers studies which focus on the effects of student background on learning and teaching mathematics. Because of other chapters in this book, e.g., covering gender differences and affective traits, this section is mostly restricted to issues related to family background, community influences and socio-economic status.

Family Influences

Prediction of students' success and/or selection of subjects has been the subject of few research studies in Australia. One of these is that of Marjoribanks (1987a). In this paper, Marjoribanks differentiated between psychological studies which focused on relationships between ability, attitudes and achievement and sociological studies which looked at student background factors such as socio-economic status and parent socialisation. The paper described a study which aimed to investigate the interaction between socio-economic status and parent socialisation variables. In particular, it tested the hypothesis that the interaction between attitudes and achievement varies between students from different family background. The study used a sample of 928 students from four family backgrounds: middle status-getting by, middle status-getting ahead, lower status-getting by, and lower status-getting ahead. Student performance was tested by standardised mathematics and word comprehension tests and their attitudes to school were measured by a previously published Likert scale instrument. Multiple regression analysis was used to investigate the relationship between attitudes and achievement for each of the family types.

The study showed moderate family-grouping differences in children's word performance and more modest differences in ability, attitudes toward school and mathematics achievement. Further, the study showed that the relationship between attitude and achievement was different in the four family types.

In another study, Marjoribanks (1987b) used an interactional model to investigate the relationship between children's individual characteristics, family influences and mathematics achievement. A sample of 883 Australian families was examined. A family interview measured the degree of involvement of the family with the mathematics studies of their children and the degree to which the family allowed for the development of independence of the child from the family. Marjoribanks found that mathematics achievement had a strong association with intellectual ability, a moderate association with the family measure, and negligible to modest association with school related attitudes. He found some gender influences in the results. For boy's, some family measures interacted positively with intellectual ability to be related to achievement in mathematics. For girls, on the other hand, family status only had a moderate association with achievement. The author hypothesised that such interactions

may explain the often reported sex differences in mathematics in the adolescent years.

Community Attitudes

Galbraith and Chant (1990) investigated community attitudes to school mathematics and discussed the implications for intended curriculum change. They gathered data on attitudes towards mathematical content, values, attitudes and beliefs with a questionnaire to 660 subjects (326 male and 334 female) equally distributed by age, educational background and occupational categories. A factor analysis of the questionnaire items provided five factors as predictors of society's view of mathematics and five factors which delineate mathematics stereotypes. Analysis of the influence of predictor variables and demographic variables (e.g., age, sex, education level, sex bias, and ability bias) on these stereotypes was conducted. Galbraith and Chant highlighted the following results: a substantial body of opinion supported "back to basics"; mathematics was not viewed as a male domain; mathematics was seen as a proxy for general ability; and mathematics was regarded as important though also seen as distasteful. They concluded that there was overlap between the values held by the community concerning mathematics education and views held within the mathematics-education community. They argued that this confluence could be a basis for forging reform.

Social Class and Gender

A series of investigations of the role of social class and gender in mathematics teaching has been undertaken by Atweh and Cooper (1989a, 1989b, 1991, in press). In these investigations, mathematics teaching in Year 9 classes in schools with different social class and gender students was compared. The first study compared two classes, a professional class boys private high school with a working class girls private high school. The second study compared four classes, a professional class boys private high school, a professional class girls private high school, working class boys private high school and working class girls private high school. The third study also compared four classes, a professional class state high school, a middle-class state high school, a working-class state high school and a welfare state high school. In each study, the schools being compared were chosen because there were similarities in the mathematics curricula and teaching in the schools. They all followed the same official syllabus, their teachers followed the same traditional "chalk and talk" approach to teaching and their learning activities were dominated by textbooks and by the teacher.

A single Year 9 class (the second year in the secondary schools in Queensland) was selected from each school. As far as possible the Year 9 classes chosen followed the same text book and were observed doing the same mathematical topic. A multiple instrument technique was employed to collect information about the mathematics teaching in these classes. Each class was observed for the duration of one topic (between one and two weeks). A

classroom observation instrument, modified from Good and Brophy (1987) by the authors in the first stage of the project, was used to record and classify teacher-student interactions. Each lesson was audio-taped and the tapes transcribed. In addition, interviews were conducted with the principal and mathematics subject master of each school, each class teacher and selected students from each class, to provide a comprehensive view of the different aspects of the mathematics curriculum in the schools and to allow for triangulation with the observations. Teachers' and students' perceptions of mathematics, their own and each others' abilities and attitudes, and each others' perceptions, were the primary focus of the interviews with the students and the class teachers. Data were gathered on student-teacher interactions, teacher modification of textbook topics, student classroom behaviour, aspirations of teachers and students and background, and socio-economic status of students and community.

The theoretical basis of the investigations is critical sociology as described in Cooper (1988). Cooper argued that, since the mid sixties, the mounting evidence of the failure of many school compensatory programs to raise the conditions of minorities and the disadvantaged has put under question the role that schools play in reproducing social and cultural inequalities. He argued that schools play a primary role in preparing students from different social backgrounds to meet the needs of an unequal society. He contended that correspondence theories between the power relations in the school and the workforce have been too deterministic with regard to the role of schools in reproducing inequalities in society and that hegemony is now seen as the process by which the dominant classes of society use ideology, not brute force, to impose their views and values on the needs and interests of the subordinate groups.

Cooper presented arguments to support mathematics having a formidable role in reproducing crucial ideologies, a role made more powerful because of the notion of objectivity that surrounds mathematics itself and masks its hegemonic function. These arguments supported Stake and Easley (1978) in seeing mathematics as neither culture-free nor value-free, as the crucial subject for reproducing existing social values and as functioning as a "behavioural badge for eligibility for the various privileges of society," and in seeing that teachers modify their curriculum accordingly.

The conclusions with regard to these data deduced by Atweh and Cooper (1991) were in six general areas. The first of these was that mathematics does carry into its teaching beliefs and values that constrain that teaching and provide the basis of the differential effect that teaching has on different students. This, argued Atweh and Cooper, can be seen in the second conclusion in that there is a belief that the best mathematics is rigorous and formal and, consequently, the best teaching is to students that excel in that style of mathematics. The result of this, contended Atweh and Cooper, is that the best teachers of mathematics tend to congregate into the better schools. Because of the shortage of mathematics teachers, this means that the lower

socio-economic class schools end up with an inordinate proportion of poorly trained mathematics teachers. This then, has a consequence related to the third conclusion about mathematical beliefs presented by Atweh and Cooper: that mathematics is a mysterious subject known only to a few strange people, above the subjective nature of other subjects and not to be tampered with by novices. Atweh and Cooper argued that the teachers in the schools with the most problems with the rigour of mathematics are, due to their background, the least likely to innovate and do anything about that rigour.

The fourth conclusion was that because of its public and seemingly objective nature and because it appears to be able to be marked right/wrong, mathematics strongly reinforces the myth that people are totally responsible for their own success or failure. The institutional consequence of this for Queensland is that funding for schools is primarily related to size of the school and not its need. The schools have to rely on the support of parents and this is in proportion to the socio-economic level of the community surrounding the school. The lower socio-economic class schools are left with little support and low self esteem.

The fifth conclusion of Atweh and Cooper was to support Stake and Easley's (1978) view that mathematics appears to be taught as if it were value laden and should be the basis for stratifying society for employment. The consequence of this is that teachers tend to give children the mathematics they feel the children's future life chances require. This means that there is a tendency for mathematics to be seen as more important for boys than girls, more important for "smart" boys than those who do not show aptitude, and more important for students whose socio-economic background means that they will likely attain a professional job than for those from the lower socio-economic classes who are likely to attain only unskilled work. The expectations for learning are that boys with a high socio-economic background will do (and need to do) well and others (girls and low class boys) will not do as well, need less rigour and more help.

The last conclusion in these papers was to offer some support, as documented in Anyon (1981) and Willis (1977), for the belief that the primary hegemonic consequence of mathematics teaching is the myth of separation of labour and management, and hence that mathematics is a subject for which knowledge of how to control it should be given only to those students with a high probability of being managers in their future careers. Atweh and Cooper concluded that this is singly the major difference between low and high socio-economic class schools: teachers of mathematics in the lower socio-economic schools tend to want to control every activity of the students, while those in the high socio-economic schools encourage students to take control of their own learning, to look up answers for themselves and to work out how to do problems for themselves - to act as mathematicians. The focus in the lower schools is on setting out and neatness, while the focus in higher schools is on understanding, creating and controlling.

Participation in School Mathematics

School mathematics plays a crucial role in entry to different university courses and many careers (Bishop, 1988, Stake & Easley, 1978, Willis, 1989). Concerns about the questions of equity in society must involve research into the patterns of participation of the different social groups in school mathematics. This issue was addressed by a few Australian researchers such as Ainley, Jones and Navaratnam (1990); Dekkers, DeLaetor & Malone (1991); Jones (1988); and Tesse (1989).

These studies differed in the scope of school subjects considered, the scope of students studied, the year for which data were analysed and the range of background variables investigated. The following factors were shown to be related to the choices that students make to study mathematics at the high school level: the gender of the student; the school type, ie., state *versus* private and single sex *versus* coeducational; and the location of the school, ie., urban *versus* country. Some of these factors were important by their own while others were effective in interaction with other variables.

FOCUS ON CLASSROOM INTERACTIONS

This section covers research into the social interactions of mathematics classrooms, the quality and quantity of interactions between students and between teachers and students.

Classroom Activity and Environment

The classroom offers a rich environment for the study of teaching and learning in mathematics. Researchers were able to focus on the instructional activities and the learners to provide rich detail on the learning process and insight into actual practices. O'Brien and Richardson (1987) reported on an ethnographic study in which twenty-one fourth grade classrooms in eight schools were observed using a partially structured classroom observation schedule. The aim of the research was to study the way mathematics was taught in primary school classrooms. O'Brien and Richardson made observations using categories and subcategories constructed around content (e.g., mentals, tables, and concepts), teaching and management style (e.g., attitudes and teaching style) and child activity (e.g., attention and time on task). Their results indicated that what was taught departed seriously from the set curriculum, while teaching methodologies fell well short of ideals espoused by the system syllabus. They reported that some teaching was not competent in terms of subject matter and little effective use was made of concrete materials, real life situations, developing language and differential rates and styles of learning. They concluded that although the teachers observed appeared to be well motivated and hard working, "... their current practices in mathematics teaching are quite inadequate" (p. 29). O'Brien and Richardson criticised the quality of the some teacher education programs.

Tobin and Malone (1989) investigated the participation in mathematics classrooms of "target" students, students who had been found to dominate interactions with the teacher in whole class situations by asking and responding to questions. They aimed to understand the role these students play in the classroom. They observed five Year 8 classes (a total of forty-two lessons per author), maintained records of lessons and interviewed teachers and students during and after lessons to support and refute hypotheses being developed. They identified and coded the interactions for cognitive level.

Tobin and Malone argued that the teachers appeared to monitor understanding through the target students. They suggested that "... the pace of instruction was probably geared to the understanding of target students" (p. 325). They found that the target students were most often male (although the evidence of this was thought inconclusive by the authors) and that their prominence was sometimes due to accepting responsibility for their own learning and/or being competitively engaged with other target students in the same class. The authors also deduced from their observations that classroom management problems were conducive to targeting of students. They concluded that target students did not necessarily constitute a problem for teaching, but that such students "... might exist for a variety of reasons and fulfil different niches in the classroom ecosystem" (p. 330).

Researchers in West Australia have been involved in research on the classroom environment. Fraser and his colleagues (Fraser, 1986) developed inventories to measure students perceptions of and preference for different classroom environments. Fraser, Malone and Neale (1988) claimed that evidence has consistently shown that students perceptions are good predictors of variability in educational outcomes beyond that attributable to students ability and past achievement. In this paper they reported a case study of one primary school teacher who used a classroom environment instrument to gauge students perceptions of her class. The teacher then introduced a series of activities to change the classroom environment. On the posttest, students perceptions on the Difficulty scale showed significant improvement. The authors were careful not to over-generalise from this single case study, yet this result gives rise to questions for further research such as what are the most appropriate intervention activities to change the classroom environment and whether such changes do improve students' performance and attitudes.

The question of class sizes in mathematics teaching was investigated by Bourke (1986). Besides studying the relationship between class size and achievement, the author investigated the extent to which the postulated relationship is mediated through teaching practices. A sample of 63 teachers from 33 schools at Year 5 were observed during mathematics classes. Information about students' achievement, attitudes, and background factors were collected. The class sizes in the observed classes varied from 22 to 33. Bourke noted a positive, but not significant, relationship between class size and achievement for the 63 classes. However, when controlling for student ability, the partial correlations between class size and achievement was negative and

significant. Bourke observed that in small classes teachers tended to use whole class activities that yielded higher achievement. Teachers in larger classes tended to use more small group work and this resulted in lower achievement. Grouping may lead to fragmentation of lessons.

Considering variations in student-teacher interactions in large classes, Bourke noted that teachers required more management and this did not enhance student learning. In smaller classes, teachers asked more probing questions and waited longer for answers. Further, smaller classes had more homework and more time was spent on correction of homework. Noisy classes were, in general, less productive. Bourke concluded that the varying teaching practices observed in larger and smaller classes provided an explanation of the causal relationship between class size and achievement.

Classroom Communication

Of particular interest to many mathematics education researchers were the interactions between teachers and students. Leder (1987,1988) undertook such a study, focusing on the patterns of teacher-student interactions to answer the question of how these patterns vary between genders, subject areas and high and low achieving students. Nine lessons were video-taped in a year six classroom (17 girls and 13 boys) in mathematics, science and language over three weeks, using a radio-microphone to record teacher-student dialogue. Wait time and subsequent engagement time recorded student-teacher interactions coded under -Good and Brophy (1987) conventions. Measures of student achievement were obtained and these were used to divide students into categories of above average, average and below average.

Leder found some inconsistencies in the way teachers communicate with boys and girls. She found that teachers asked proportionately more process and product questions to those students they rated best; however this was not statistically significant. She found that wait and attention times were also in favour of top students. Her conclusions covered the type of questioning, the form of the interactions, gender differences and wait time. She argued that teacher variability in interactions reflected the extent to which teacher behaviour adapted to requirements, that girls were given more product than process questions, that extra wait time for low-achieving students did not extend to constructive teachers time, and that the best students benefited most. She contended that teacher behaviour was modified to suit perceived needs reinforcing learning styles that may maximise differences in achievement. In the author's own words:

> The data presented reveal an interesting paradox. On one hand they provide evidence of teacher behaviour being appropriately modified to suit the perceived needs and capabilities of individual students and the requirements of the material being taught, yet such adaptation may ... reinforce different learning styles and students' self

> perceptions and ultimately maximise differences in student achievement. (Leder, 1987, pp. 268-269)

The question of wait time was also the subject of an extensive review of literature conducted by Tobin (1987). The author defined wait time as a measure of the pauses between two sentences in a verbal interaction. The studies reviewed covered a range of subjects and levels of schooling. Experimental studies that manipulated the length of wait time indicated an increase in achievement in science and, at least in middle school mathematics. The author posited the hypothesis that longer wait time provided students, and teachers, with time to think and hence appeared to have facilitated higher cognitive level learning. This level of thinking is a necessary, but not sufficient, condition for learning.

FOCUS ON THE TEACHER

This section covers research which focuses on the values, beliefs, background and role of the teacher as a major player in teaching and learning mathematics.

Teacher Practice

In his study of the role of the teacher, Bourke (1988) used questionnaires to investigate the quality of school life experienced by students across the interface between primary and secondary school, the classroom context established by the teacher and the teaching priorities used in lessons. He found that there was a marked decline in the quality of school life from primary to secondary. He identified eight factor clusters that related to mathematics achievement and enjoyment: teachers' intentions and emphases for lesson content; teachers' perceptions of students' ability; teachers' workload and conditions; class grouping types; individual teacher-student interactions; autonomy and professionalism of teacher; and teacher perceptions of importance of curriculum topics. He also identified three factor clusters that related to mathematics outcomes: quality of instruction; classroom management; and instructional practices. He concluded that there was a need to research the relationship between the quality of school life and classroom contexts and teaching practices.

Annesley and Clark (1988) described on a study of senior mathematics teachers to determine the effect that affective traits had in the implementation of the school based assessment scheme in Queensland schools, ROSBA. Fifty-four teachers in eleven schools completed a questionnaire and participated in follow-up interviews. Data were also gathered from a sample of twenty percent of their students. Eighty-five percent of teachers said that they attempted to foster affective domain goals in their students and the same percentage ranked mathematics as the hardest subject in which to do this. Annesley and Clark reported that teachers showed little confidence in their own abilities to promote affective traits, evaluated these traits no more deeply than

low/high achieving or hard/lazy worker, agreed that enhanced affective traits promoted cognition and, by a slender margin, thought that assessment should be concerned with a broader range of criteria than just achievement. Annesley and Clark argued that there was little support for affective objectives in senior mathematics syllabi and that teachers felt a need for training in this area. Annesley and Clark concluded that although teachers accepted that schools had a responsibility for affect, they were basically unaware of the syllabus' affective objectives and needed training to deal with them.

Sullivan (1989) examined factors inhibiting mathematics curriculum change in primary school classrooms. Reported in this paper was a case study of one teacher covering periods within two phases of teacher development: initial education and induction into regular classroom teaching. Sullivan's subject was found to be a highly conscientious teacher exhibiting a strong degree of professional commitment. She was found to have overcome concerns with self and class management, characteristics usually found in experienced teachers. Her instructional procedures and explanations were clear, and she evidenced a capacity to reflect on both planning and teaching. Moreover, she demonstrated an intelligent understanding of the recommendations of her training for teaching. For instance, she maintained a teaching goal for student learning which focused on relational over instrumental understanding and she displayed an awareness of the efficacy of concrete aids in order to achieve these ends.

Nevertheless, Sullivan found that his subject's reflection and planning were much more related to actual lessons than stated beliefs and opinions. Much of the subject's thinking appeared to be attributable to personal factors, rather than formal training. Sullivan also noted that a major dichotomy existed between his subject's espoused belief in the efficacy of concrete aids and their actual deployment in the classroom. He identified teacher reliance on short term planning, rather than teacher laziness as posited by some, as a critical variable in the determination of these phenomena.

Sullivan hypothesised that the divergence observed between the subject's classroom behaviours and her espoused beliefs could in part be explained by the use made by his subject of the mathematics text set for her teaching. The text was found to govern, and therefore unduly restrict, a major portion of her planning; it appeared to make teaching easier but it did not improve the teacher's self-confidence; its mathematical intentions were not always understood by the subject; reliance of the text appeared to stimulate an emphasis an short term goals and ultimately to promote instrumental understanding in students.

Sullivan concluded by proffering two considerations for pre- and in-service teacher education. Firstly, it must be recognised that the classroom teacher's paramount need is to maintain a stable and harmonious classroom environment. Principle reliance on the provision of adequate knowledge of teaching and learning mathematics cannot therefore, by itself, sustain desired innovations or reforms of curriculum or teaching methodology. Instead, when

proposing change, reformers need to be more cognisant of actual classroom contexts and the impact of these on teacher practice. Secondly, in primary teacher education at least, in-service programs may need to seek to broaden the general understanding or scope of mathematics content.

Exemplary Teaching

Studies looking at exemplary mathematics teaching have been undertaken by Tobin and Fraser (1988a, 1988b). These papers summarised the findings of five case studies (involving seven teachers) relating to the nature of exemplary teaching in mathematics. These teachers were observed teaching and they were interviewed. The materials these teachers used were also examined. In addition, data were gathered from their students using Fraser's classroom environment instrument (Fraser, 1986) which measures the psycho-social learning environment of students. The results reported by Tobin and Fraser related to the context of learning within a classroom. They found that exemplary factors included teacher management strategies such as monitoring student behaviour, independent and cooperative group work. The exemplary teachers, argued Tobin and Fraser, seemed to be able to set up a classroom in which "students knew what to do and appeared to enjoy working in the classroom" (1988a, p. 6).

Tobin and Fraser contended that exemplary teachers encourage participation by setting up "safety nets" for their students and emphasising private interactions, saving loss of face as much as possible. They argued that such teachers tended to base their lesson organisation around student understanding, using their excellent management skills to monitor student progress and to provide rich experiential contexts. The student data showed, according to Tobin and Fraser, that exemplary teachers achieved significantly greater teacher control, personalisation, participation, differentiation and organisation.

CONCLUSIONS

It is safe to say that the early mathematics education research in Australia and the South-west Pacific region was primarily based on paradigms and concerns from the behavioural sciences, in particular, psychology of learning. These concerns were reflected in the areas of curriculum design, teaching methods, and diagnosis and remediation of learning difficulties. Even research on special needs, such as girls in mathematics, to a certain degree, followed the psychological models in looking at differences in mathematical ability and achievement between girls and boys. More recently, however, theories and questions arising from the social sciences, sociology in particular, have contributed new challenges and questions for investigation as well as new paradigms and methodologies for research. Arguably, the question as to which approach, the psychological or the sociological, can be expected to yield the

truth about the nature of difficulties in the learning and teaching of mathematics is a mis-constructed question. The paradigms may be complementary in yielding a better understanding of the complex nature of mathematics learning.

It is difficult to demonstrate quantitatively that research in the area of context has increased in quantity and quality during the last four years since earlier reviews of mathematics education research in this region did not include context as a separate category. Suffice to note that the inclusion of a separate chapter in this current review is an indication of the increasing importance of this area of research in Australasia. Obviously this increase parallels similar trends in Europe and the Americas.

Research into the context of mathematics education may be conveniently categorised into two main areas: The social context and the cultural context. Research in both areas was reviewed in this chapter. From the number of studies reviewed here, it is obvious that the issues of concern to Australasian researchers in the context of mathematics education are very wide ranging. They cover concerns about Aboriginal and other indigenous people, bilingual education, family and background factors, participation and access, communication, language and the role of the teacher in the classroom. This spread of issues and factors studied has a negative side effect. In a situation where research output is limited, due to number of researchers involved, areas of interest tend to be somewhat fragmented. Many of the studies reported above are single examples of studies on a particular topic. This implies that results of studies can only be compared with studies from overseas. Very little confirmatory research is reported in Australian publications.

The studies reviewed in this chapter also reveal a variety of methodologies used. Some studies were ethnographic while others employed complex quantitative techniques. Once again, arguments may be developed that different methodologies are useful in answering different types of questions.

This chapter concludes by raising some considerations, indeed challenges, that researchers in the social context may need to address in the future. Firstly, based on the observations made above about the lack of long term research projects in this area, i.e., in contrast to individual studies, there seems to be a need for establishing lines of research or long term projects. These lines of research would allow for replication studies and for theories to be developed and tested.

Secondly, studies into the context of mathematics education should become part of all types of research in mathematics education rather than remain as a separate category. The studies carried out by Boulton-Lewis, Neill and Halford illustrate how psychological studies can successfully combine social context concerns with other concerns. Just as concern for gender issues has infiltrated research into learning and teaching methods, all researchers in mathematics education should be aware of the social and cultural considerations in their conduct of research. Further, this new research interest has opened up possibilities for research cooperation between mathematics educators and other

social scientists and sociologists. Great benefits can be obtained from increasing this cooperation.

Lastly, isolated research findings may be much more useful if they contribute to theories. Some of the studies reported above stem from specific theories from sociology or socio-linguistics. Theoretical considerations may not be as obvious in other studies. Arguably, research in mathematics education that leads to theory building and verification is much more effective in initiating and sustaining change.

Considering the relatively recent increase in interest in this area of mathematics education research, one can be reasonably optimistic that this research will lead to a greater understanding of the development of mathematical thinking and contribute to the solution of many of the difficulties that teachers and students face in school mathematics.

REFERENCES

Ainley, J, Jones, W. & Naravatnam, K. (1990). *Subject choice in senior secondary school*. Canberra: Australian Government Publishing Service.

Annesley, F. & Clark, J. (1988). Issues in the assessment and reporting of affective objectives in Queensland senior mathematics syllabus. In N. Baumgart (Ed.), *Reports and records of achievement for school leavers* (pp. 211-223). Deakin, Australian Capital Territory: Australian College of Education.

Anyon, J. (1981). Social class and school knowledge. *Curriculum Inquiry, 11*(1), 3-42.

Atweh, W. & Cooper, T. J. (1989, July). *A tale of two schools*. Paper presented at the Annual Conference of the Mathematics Education Research Group in Australasia, Bathurst, New South Wales:

Atweh, W. & Cooper, T. J. (1989, November). *Gender, social class and social reproductive nature of mathematics*. Paper presented at the Annual Conference of the Australian Association of Research in Education, Adelaide, South Australia.

Atweh, W. & Cooper, T. (1991). Hegemony in mathematics curricula: The effect of gender and social class on the organisation of mathematics teaching for 9 students. In F. Furinghetti (Ed.). *Proceedings of the Fifteenth International Psychology of Mathematics Education Conference* (pp. 88-95), Assisi, Italy.

Atweh, W. & Cooper T. J. (in press). *An exploratory Investigation of the relationship between gender, social class, school mathematics and student teacher interactions: The final report*. Queensland University of Technology: The Centre for Mathematics and Science Education.

Bishop, A. (1988). *Mathematical enculturation: A cultural perspective on mathematics education*. Dordrecht: Kluwar Academic Publishers.

Boulton-Lewis, G., Neill, H. & Halford, G. (1987a). Information processing and mathematical knowledge in Aboriginal Australian children in South-east Queensland. *Australian Aboriginal Studies, 2*, 63-65

Boulton-Lewis, G., Neill, H. & Halford, G. (1987b). *Research into educational potential of Aboriginal children in South-east Queensland.* (Report to the Australian Institute of Aboriginal Studies), Canberra, Australia.

Boulton-Lewis, G., Neill, H. & Halford, G. (1988). *Educational potential and mathematical achievement in urban Aboriginal children in South-east Queensland.* (Report to the Australian Institute of Aboriginal Studies), Canberra, Australia.

Bourke, S. (1986). How smaller is better: Some relationships between class size, teaching practices and student achievement. *American Educational Research Journal, 23*, 558-571.

Bourke, S. (1988). The role of the teacher. In J. Pegg (Ed.), *Mathematical Interfaces*, (pp. 58-65). Armidale, New South Wales: Australian Association of Mathematics Teachers.

Butler, S. & March, H. (1986). Reading and arithmetic achievement in primary years for students from non-English speaking families: A seven year longitudinal comparison. *Australian Journal of Education, 30*, 23-44.

Cooper, T. (1988). Negative power and negative processing. *Research in Mathematics in Australia*, December, 29-41.

Crawford, K. (1990). Language and technology in classroom settings for students form non-technological cultures. *For the Learning of Mathematics, 10*(1), 2-6.

Dekkers, J., DeLaetor, J. & Malone, J. (1991). *Upper secondary school science and mathematics enrolment patterns in Australia 1970-1988.* Perth: Curtin University of Technology.

Fraser, B. (1986). *Classroom environment.* London: Croom Helm.

Fraser, B., Malone, J., & Neale, J. (1988). Observing and improving the mathematics classroom environment. *Research in Mathematics Education in Australia*, June, 8-15.

Galbraith, P. & Chant, D. (1990). Factors shaping community attitudes to school mathematics: Implications for future curriculum change. *Educational studies in mathematics, 21*, 299-318.

Good, T. & Brophy, J. (1987). *Looking into the classroom* (4th ed.). NY: Harper and Row Publishers.

Graham, B. (1988). Mathematics education and Aboriginal children. *Educational Studies in Mathematics, 19*, 119-135.

Harris, J. (1987). Australian Aboriginal and Islander mathematics. *Australian Aboriginal Studies, 2*, 29-37.

Harris, P. (1989). Contexts for change in cross-cultural classrooms. In N. Ellerton and M.A. Clements (Ed.). *School Mathematics: The challenge to change* (pp. 79-95), Geelong, Victoria: Deakin University.

Harris, P. (1991). *Mathematics in a cultural context: Aboriginal perspectives on time, space and money*. Geelong, Victoria: Deakin University Press.

Jones, W. (1988). Secondary school mathematics and technological careers. (ACER monograph # 32). Hawthorn, Victoria: Australian Council for Educational Research.

Kanes, C. (1991a). Games and meta-games: Towards a socially interactive theory of mathematics learning. In F. Furinghetti (Ed.), *Proceedings of the Fifteenth International Psychology of Mathematics Education Conference* (pp. 229-236), Assisi, Italy.

Kanes, C. (1991b, November). *Language and text in mathematics education: Towards a Post-structuralist account of mathematics education.* Paper presented at the Annual Conference of the Australian Association of Research in Education, Gold Coast, Queensland.

Leder, G. C. (1987). Teacher student interaction: A case study. *Educational Studies in Mathematics, 18*, 255-71.

Leder, G. C. (1988). Teacher-student interactions: The mathematics classroom. *Unicorn*, 14, (2), 107-110 and (3), 165-166.

Marjoribanks, K. (1987a). Ability and attitude correlates of academic achievement: Family group differences. *Journal of Educational Psychology, 79*, 171-78.

Marjoribanks, K. (1987b). Individual environment correlates of children's mathematical achievement. *Educational Studies, 13*, 115-123.

National Aboriginal and Torres Strait Islander Education Policy. (1989). Canberra: Department of Employment, Education and Training.

O'Brien, P. & Richardson, A. (1987). *"That's the only maths today." "Yahoo!" "You beauty.": Qualitative observations of grade 4 mathematics classes, Mt Druitt longitudinal study*. Macquarie University, North Ryde, Sydney.(ERIC Document Reproduction Service No ED299110)

Ohia, M., Malone, M. & Knight, G. (1990). A survey of mathematics teaching in New Zealand secondary schools bilingual units. In A. Begg, B. Bell, F. Biddulph, M. Carr, M. Carr, J. McChesney, & J. Loveridge (Eds.), *Science and mathematics education papers* (pp. 133-158). Hamilton, New Zealand: University of Waikato.

Peard, R. F. (1991). *Misconceptions in Probability: A comparison of gamblers and non-gamblers within the Australian social context.*(Technical Report 3) Brisbane, Australia: The Centre for Mathematics and Science Education.

Sullivan, P. (1989). Factors inhibiting change - a case study of a beginning primary teacher. *Mathematics Education Research Journal, 1* (2), 1-16.

Stake, R. and Easely, J. (1978). *Case studies in science education*. Urbana: University of Illinios.

Teese, R. (1989). Australian Private schools, specialisation and curriculum conservation. *British Journal of Educational Studies, 37*, 235-252.

Tobin, K. (1987). The role of wait time in higher cognitive level learning. *Review of Educational Research, 57*, 69-95.

Tobin, K. & Fraser, B. (1988a). Investigations of exemplary teaching in Australian mathematics classes. *The Australian Mathematics Teacher, 44* (2), 5-8.

Tobin, K. & Fraser, B. (1988b). Investigations of exemplary practice in high school science and mathematics. *Australian Journal of Education, 32*, 75-94.

Tobin, K. & Malone, J. (1989). Differential participation in whole class activities. *Australian Journal of Education, 33*, 320-331.

Treagust, D., Malone, J. Fraser, B. & Taylor, P. (1987). *Bridging program for Aborigines wishing to study science and mathematic in higher education*. Paper presented as part of Symposium on Participation and Equity at Annual Meeting of the American Educational Research Association, Washington DC.

Watson, H. (1987). Learning to apply numbers to nature. *Educational Studies in Mathematics, 18*, 339-357.

Watson, H. (1988). Language and mathematics education for Aboriginal-Australian children. *Language and Education, 2*, 255-273.

Watson, H. (1989a). A Wittgensteinian view of mathematics: Implications for teachers of matheamtics. In N. Ellerton and M. A. Clements (Eds.), *School mathematics: The challenge to change*. Geelong, Victoria: Deakin University.

Watson, H. (1989b). Finding patterns in the world. In H. Watson & D. Chambers (Eds.), *Singing the land, signing the land* (pp. 31-42), Geelong, Victoria: Deakin University.

Watson, H. (1990). Investigating the social foundation of mathematics: Natural number in culturally diverse forms of life. *Social Studies of Science*, 20, 283-312

Willis, S. (1989). *"Real girls do not do maths" :Gender and the construction of Privilege*. Geelong, Victoria: Deakin University.

Willis, P. (1977). *Learning to labour*. Lexington, MA.: D.C. Heath.

GENDER: A CRITICAL VARIABLE IN MATHEMATICS EDUCATION

Gilah Leder
School of Graduate Studies
Faculty of Education
Monash University

Helen Forgasz
School of Graduate Studies
Faculty of Education
Monash University

Australian research concerned with gender differences in mathematics learning and published in the years 1988-1991 is reviewed in this chapter. While participation and performance trends are discussed, particular emphasis is placed on explanatory and intervention studies. The substantial number of government policy documents and reports published during the period surveyed have lead to a variety of professional development and curriculum initiatives. A representative set of these is described in the chapter. Despite the special problems created by the current difficult economic climate this chapter concludes that there are encouraging trends which augur well for the future.

This chapter reviews the progress made towards achieving equity in the area of mathematics during the past four years. As a prelude to this, it is useful to summarise very briefly the major findings of the earlier Mathematics Education Research Group of Australasia (MERGA) sponsored reviews of Australian research in mathematics education (Leder, 1984a; Barnes, 1988a).

THE EARLIER REVIEWS OF RESEARCH

In the first of MERGA reviews of Australian research in mathematics education published in 1984 to mark Australia's hosting of the fifth International Congress on Mathematical Education (ICME-5), Leder (1984a) summarised 100 articles that dealt with gender differences in mathematics learning under the headings of "literature review" (15 articles), "performance data" (10), "participation data" (21), "possible explanations" (33), and "intervention studies" (21). The review covered work published in refereed journals as well as material reported in subject association journals, education departments publications, and the proceedings of national conferences, i.e., on publications in the conventional research literature as well as those with a wide circulation among practitioners in the educational community.

In the second review, Barnes (1988a) summarised a further 74 articles. Twelve (16%) were categorised as "literature review", 7% as concerned with "performance data", 12% with "participation data", while the "possible explanations" and "intervention studies" groupings both consisted of 32% of the articles reviewed. Thus by 1988 the bulk of the work concerned with gender and mathematics had moved away from documenting broad inequities which had already been recognised and reported in formal documents. Far more energy was being spent on explaining and exploring the factors that appeared to contribute to the differences identified and on documenting and evaluating intervention studies aimed at redressing disadvantages apparently experienced by females in mathematics.

Work carried out in the Australian context or using Australian samples in the past four years is summarised next. Particular attention is focused on articles that attempt to explain gender differences and work that describes intervention strategies. Because of recent increased interest in the topic, work dealing with single-sex and coeducational schooling issues is also singled out.

GOVERNMENT INITIATIVES

The large number of government sponsored publications, specifically concerned with the education of females, that have appeared in recent years testify to a widespread concern about the enrolment, career, and employment discrepancies identified. The Commonwealth Schools Commission's *Girls, School and Society*, published in 1975, was the first major government report to do so. Its conclusions included:

> Girls achieve at least equally well with boys in school, but the subject choices they make are more limiting than those made by boys in the opportunities they open up for further education and employment. .... The greatest disparity between the sexes exists in industrial and technical training which strongly attracts boys leaving school before completing a full secondary course. (p. 154)

Girls and Tomorrow, published by the Commonwealth Schools Commission Working Party on the Education of Girls some ten years later in 1984, noted that

> Girls continue to be afforded far less opportunity than boys to realise their potential. The quality of girls' education does not match that of boys in terms of developing confidence and self-esteem or marketable skills.... Teachers continue to allocate less of their time to girls than to boys. Teacher interaction with girls does not encourage creativity and inquiry to the same extent as with boys.... School hierarchies are providing even fewer role models for girls than in 1975. (p. 2)

Publication of the *National Policy for the Education of Girls in Australian Schools* (Commonwealth School Commission, 1987), represented another important formal recognition of the special difficulties faced by many females in the Australian educational system. Its set of objectives fell into four broad areas: raising awareness of the educational needs of girls; achieving equal access to and participation in all curriculum areas; establishing a supportive school environment, and ensuring equitable resource allocation.

Various national and state reports have been published during the period reviewed in this chapter. A representative selection of these documents is summarised in Table 1.

December, 1990 saw the publication of *A National Statement on Mathematics for Australian Schools* from the Australian Education Council. Amongst the underlying principles of the statement is the recognition that "participation in mathematics in Australia has been dependent on being a member of particular groups in society" (p.8) and that:

> We are now beginning to understand some of our past curriculum practices in mathematics which have disadvantaged groups of students. For example, many of the contexts in which mathematical concepts were developed, applied and assessed were more likely to be central in the lives of boys than in the lives of girls. In some subtle and not-so-subtle ways, the message was communicated to girls and boys that mathematics was of more relevance in the lives of adult men than adult women. (p. 9)

The Australian Association of Mathematics Teachers Inc. (AAMT) also published *A National Statement on Girls and Mathematics* during 1990. The statement provides a short background to the issues concerning girls and mathematics and recommends that "to achieve gender equity will involve concurrent changes in teacher development and training, curriculum, society's attitudes, schools and systems" (p.6). A list of requirements to meet these recommendations is presented. Teachers were kept informed of the development, relevant issues and recommendations of the national statement (see "Girls and mathematics: the development", 1990; "Girls and mathematics: Some recommendations", 1990).

Table 2 summarises a representative selection of government sponsored curriculum and professional development initiatives which relate to the education of females at the various levels of schooling. Many have special relevance to mathematics education.

From the range of reports and curriculum and professional development initiatives published over the past few years, it is evident that governments, at both federal and state levels, have begun to address the issues highlighted in the *National Policy for the Education of Girls in Australian Schools*.

Table 1
Government Policy Documents and Reports

SOURCE	BRIEF DESCRIPTION
Department of Employment, Education and Training (DEET) (1988). *Girls in schools: Report on the National Policy for the education of girls in Australian schools*. Canberra: Commonwealth of Australia. DEET, (1989; 1990). *Girls in schools 2; 3: Report on the National Policy for the education of girls in Australian schools*. Canberra: Commonwealth of Australia	Following the endorsement by the Australian Education Council (AEC) in 1987 of the national policy for the education of girls in Australian schools, these documents form the annual reports of policy implementation across the nation. Included are reports from the Commonwealth, each state government's education department/ministry and from the non-government sectors. Each report outlines the directions taken to implement the policy and indications of continuing needs and future intentions are presented. Several references are made to mathematics education provisions for girls.
Department of Employment, Education and Training [DEET]. (1989). *Discipline review of teacher education in mathematics and science. Volumes 1 and 2*. Canberra: Australian Government Publishing Service.	A detailed report on the state of teacher education in mathematics and science across Australia. Amongst its findings, the Review panel noted its disappointment in finding so little concern in higher education with respect to females' participation in mathematics and science. Many teacher education courses included some work on gender; usually these were restricted to single sessions.
Studies Directorate, NSW Department of Education. (1989). *Girls and mathematics: Report on a project of national significance*. Sydney: Author.	With implications for teachers' professional development in relation to mathematics education for girls, this report outlines action research conducted to investigate girls' participation in primary school mathematics. The project was conducted in conjunction with the Basic Learning in Primary Schools (BLIPS) numeracy program.

NSW Girls Technology Strategy Working Party (1988). *Report of the Working Party on the Girls Technology Strategy. Part 1. Overview and recommendations.* Sydney: NSW Department of Education.	In response to the NSW government's girls technology strategy (GTS) aimed at increasing girls' confidence and competence in mathematics and other science and technical areas, this Working Party report identifies factors which serve as impediments to girls' participation and achievement in these areas. Strategies to be implemented over a five year period (1988-1992) and recommendations for girls' education in these areas are outlined.
Ministry of Education Western Australia (1991). *Social Justice in Education: Policy guidelines for gender equity.* Author.	Gender equity is a central issue in the social justice policy. The National Policy for the Education of Girls in Australian Schools is endorsed and four main objectives are set out to achieve gender equity in education.
Ministerial Advisory Committee on Women and Girls [MACWAG] (1991). *Working for gender justice in schools.* Ministry of Education and Training, Victoria.	The work of equal opportunity workers in Victorian government schools is documented. Outcomes of intervention and the process of educational change related to gender issues are discussed.
State Board of Education (1990). *Social Justice in Schools: Some recent findings. Research in Victorian schools.* Victoria: Author.	Includes a chapter which outlines research on female students in Victorian schools. Comparisons are given for male and female school experiences, achievements, school curriculum participation and retention rates, and post-school participation. Resources and strategies are suggested.

* References in this table are not included in the reference list at the conclusion of the chapter

* The Appendix lists additional references for which we were unable to obtain original copies

As yet the long term impact that these policies and initiatives will have on schools and teachers, on the teaching and learning opportunities of females, on their participation rates in those subject areas still dominated by males, and on their career and employment prospects, is largely unknown. Other intervention strategies, to be discussed later in this chapter, have also been implemented during the period of review.

In *Dismantling the Divide* Sampson (1991) reaffirmed that, despite undeniable progress, many of the gender inequities reported in the earlier documents remain. Sampson's careful review of many of the major projects funded by Governments and private sources is particularly valuable. "One off

or singly focused activities" appeared to be least successful in terms of increasing females' participation in traditional male studies, activities or occupations. Her reminder that "using the job classification system in use until 1985, the proportion of women employed in disproportionately female occupations actually increased from 78% in 1947 to 82%" (p. 1) that year is particularly sobering.

Table 2
Curriculum and Professional Development Activities

SOURCE	BRIEF DESCRIPTION
Ministry of Education, Victoria and the Ministerial Advisory Committee on Women and Girls (1988). *Primary strategies: Continuing the change for equality*. Melbourne: Office of Schools Administration: Author.	Contains strategies for raising awareness about sexism in primary schools and a program for developing self-esteem. Suggestions for developing gender-inclusive curriculum are offered. Work units and teaching strategies in a range of subject areas, including mathematics and science, are included.
Ministry of Education (1988). *The mathematics framework P-10*. Victoria: Author	A non-prescriptive curriculum planning guide for P-10 mathematics education in Victoria. Emphasis is placed on changes in teaching and learning within the classroom. Issues and directions in mathematics education are presented with a chapter devoted to 'Girls in mathematics'.
Lewis S. & Davies, A. (1988). *Gender equity in mathematics and science*. ACT: Curriculum Development Centre.	The principles underlying this professional development manual include the need to address the particular issues relating to girls when teaching maths and science. To assist groups of teachers to explore issues and plan strategies, activity based ideas and worksheets are provided, many of which are adaptable as classroom teaching activities.

Davies, B. (1988). *Gender, equity and early childhood*. Curriculum Development Centre.	One in a series of publications concerned with education and gender issues. Examines how thinking on gender has influenced teachers in primary schools. Identifies ways in which traditional gender patterns are produced by teachers and students. Strategies for change are recommended.
Clark, M. (1989). *The great divide: The construction of gender in the primary school*. Canberra: Curriculum Development Centre.	The world of the primary school is described and some research on primary children illustrating how gender is constructed is presented. A framework for action to bring about gender inclusive curricula in primary schools is provided.
Department of Employment, Education and Training (1990). *Gender Equity Network (GEN) Teacher's Kit*. Canberra: Commonwealth of Australia.	A three volume kit for teachers at all levels: junior primary, senior primary, and secondary. Based on the objectives of the National Policy for the Education of Girls in Australian Schools, each volume provides practical strategies for implementation in the classroom across all curriculum areas and for physical activities.
Ministerial Advisory Committee on Women and Girls (1991). *That's a good idea!: Practical strategies for a gender-inclusive education in secondary schools*. Victoria: Ministry of Education and Training.	A collection of practical ideas is presented for secondary school teachers and for those concerned with equal educational opportunity. The objectives of the National Policy for the Education of Girls in Australian Schools are addressed and strategies are offered and resource lists provided across all curriculum areas.
Schwarz, V., Allard, A. & Mathews, B. (1990). *A fair go for all: Guidelines for a gender-inclusive curriculum*. Education Shop, Ministry of Education, Victoria.	Provides a rationale for the necessary changes across the school curriculum to bring about gender-inclusiveness. Although not specific to mathematics education, the areas addressed: language, learning and teaching, assessment and reporting, and school organisation are areas generic across disciplines. The final sections deal with the roles and responsibilities of the school community and the support services available.

* References in this table are not included in the reference list at the conclusion of the chapter
* The Appendix lists additional references for which we were unable to obtain original copies

PARTICIPATION

When the first female graduate received her degree from an Australian university more than one hundred years ago it seemed that all formal barriers preventing females from participating fully in education had finally been removed. Indeed, according to Grimshaw (1982) "the Australian colonies were among the earliest to emancipate women politically [and] to offer girls equal educational opportunities" (p. 3).

Current enrolment statistics reveal that more females than males now remain in full time education till the end of secondary schooling. The overall student retentivity rate grew from 30.0% in 1970 to 60.3% in 1989 (Dekkers, De Laeter, & Malone, 1991). For males, the retentivity rates changed over that period from 33.8% to 55.4%. For females the changes were even more dramatic: from 26.1% to 65.4%. However, a more careful perusal of these statistics indicates that males and females do not participate equally in all subject areas.

Dekkers et al. (1991) collected detailed enrolment data for the period 1970 to 1989 for students studying mathematics and science courses in the upper secondary school years. The number of students electing to study type I mathematics courses in Year 12 (i.e., courses designed for students not intending to proceed to tertiary studies) increased fourfold over the period monitored. Typically, more females than males seemed to select this mathematics course option (male and female enrolments are not available for each of the years of interest, however). Type 2 subjects, or those mathematics courses which serve as an adequate pre-requisite for tertiary courses "in which the mathematical content is minimal -for example, in architecture, pharmacy or economics" (p. 23) also increased steadily in the two decades since 1970: by a factor of 9 in 1990. In recent years proportionately equal numbers of females and males have been attracted to this mathematics course option. Major discrepancies in females' and males' participation continue, however, in type 3 mathematics subjects, i.e., those which "involve `specialised' or `advanced' mathematics leading to tertiary studies in which mathematics is an integral part of the discipline, as in mathematics, physical science or engineering" (p. 23). In absolute terms, more males and females enrolled in this type of subjects each year, as can be seen from the data in Table 3

The entries also show that the proportion of Year 12 students taking these subjects appears to be dropping steadily. Males outnumber females approximately two to one (taking into account the different numbers of students staying on till Year 12). This ratio appears to be remarkably constant despite the increasing number of students staying on at school till the end of Year 12 and the growing number of students continuing with some form of mathematics till this level.

Table 3
Enrolment in Type 3 Mathematics Subjects

Year	Year 12 population	Enrolment in type 3 mathematics subjects	Proportion of year 12 students taking this subject
1985	M 55713	28393	.51
	F 60613	15620	.26
1986	M 61038	26681	.44
	F 67074	14803	.22
1987	M 67585	27938	.41
	F 74522	16022	.21
1988	M 76322	30646	.40
	F 85147	17553	.21
1989	M 78830	30617	.39
	F 89015	18867	.21

(Adapted from Dekkers et al., 1991)

More localised evidence has been presented by Gardner (1990) and Jones (1990). The former, which summarised student participation in mathematics subjects in 208 schools involved in the Maths and Science Incentive Grant schemes in Victoria in 1988 and 1989, confirmed the patterns of enrolment for girls and boys already described, and also revealed that "participation rates for the (sic) almost all the selected maths and science subjects in years 11 and 12 for girls and boys, fell from 1986 to 1988, despite large increases in the numbers of those still at school" (p. 9). In her survey of a smaller number of schools in New South Wales, Jones (1990) found that both gender and type of school attended (State v. Catholic) affected enrolment patterns in mathematics. Considerably more boys than girls in both school systems were enrolled in three unit mathematics at Year 12. Statewide, proportionately more boys than girls elected to take both three and four unit mathematics.

That the participation rates vary somewhat from state to state can be seen from the National Database on the Education of Girls in Australian Schools (Department of Education, Employment and Training [DEET], 1991) data summarised in Table 4. The headings in this table have been changed to make them consistent with the groupings adopted by Dekkers et al. (1991).
Throughout Australia, the data reveal, females and males participate at least equally in the "softer" mathematics subject options. However, considerably more males than females study intensive mathematics courses at grade 12. The ACT provides the only exception.

Evidence of the lower participation rate of females in the physical sciences more generally is reinforced by recent statistics, published by the Department of Employment, Education and Training, which indicate that among those with

post-school qualifications in science, computing, mathematics, agricultural and veterinary science males outnumber females more than 2:1 (Sampson, 1991).

Table 4
Participation Rates in Year 12 Mathematics Courses, by State/Territory, 1987 and 1988

		Maths 3		Maths 2		Maths 1	
		Total	Fem %	Total	Fem %	Total	Fem %
NSW	1987	9 487	37.4	16 358	52.0	3 719	58.5
	1988	11 167	39.0	18 457	52.3	15 663	59.0
VIC	1987	5 273	30.7	13 978	45.4	n.a.	n.a.
	1988	5 894	31.4	15 546	46.2	n.a.	n.a.
QLD	1987	4 824	29.0	13 031	44.0	12 200	60.2
	1988	5 372	27.7	13 977	43.4	13 163	59.9
WA	1987	1 964	29.9	7 089	50.1	n.a.	n.a.
	1988	2 005	27.9	8 083	49.9	n.a.	n.a.
SA	1987	2 374	32.1	3 456	53.2	n.a.	n.a.
	1988	2 435	31.7	3 944	50.9	n.a.	n.a.
NT	1987	166	34.3	215	57.2	n.a.	n.a.
	1988	125	31.2	280	53.9	n.a.	n.a.
ACT	1987	1 243	57.6	1 438	46.5	n.a.	n.a.
	1988	1 639	53.5	1 510	46.2	n.a.	n.a.

(Adapted from Department of Employment, Education and Training, 1991, p. 38)

PERFORMANCE

Performance in mathematics at school level attracted limited research attention over the past few years. Johnston and Rennie (1990), in their analysis of the effects of the introduction of the Unit Curriculum in mathematics and science curricula on gender equity in enrolment and performance in Western Australia provided some data on mathematics performance at Years 9 and 10 over the two year period 1987-1988. 1987 was a pilot year for the new curriculum and the sample size was much smaller than for the first full year of implementation in 1988. The data suggested that there was no consistent trend for females to

take less science and mathematics than males; in 1988 there was no mathematics unit where the enrolment difference between males and females exceeded 5%. Females were found to have performed as well as males in mathematics; any differences found tended to favour females.

Performance in mathematics competitions and at tertiary level attracted the attention of some researchers. Performance data from the Australian Mathematics Competition (AMC) were presented by Leder and Atkins (1990). These data, analysed by gender, revealed that at each of the Years 7, 8, 9 and 10, boys as a group, on average, outperformed girls as a group. The trend was less evident amongst the top 1% of students. When items on the AMC which were common at each grade level were analysed, boys did better than girls at some grade levels and girls did better at other grade levels. Leder and Atkins (1990) carefully pointed out that within gender group performance variations were considerably greater than between gender group differences found on these items. Carty (1990) reported that, on average, boys gave correct responses to 1.5 more questions than girls whilst girls omitted more questions than boys. Penalties for incorrect answers apply in the AMC and Carty (1990) speculated whether "girls may be more cautious than boys in their approach to guessing" (p. 302).

At tertiary level Watson (1988, 1989) analysed data over two separate years from first year University of Tasmania mathematics students enrolled in a particular mathematics course where the conventional lecture/tutorial model of teaching was applied. Females comprised twenty four percent of the cohort. Twenty two variables were used and the characteristics explored involved those often associated with the learning of mathematics. On only three variables were significant gender differences found. Females were found to suffer more from negative test anxiety than males. Females were also found to be more conscientious than males in that they completed a greater number of assignments and attended more tutorials. Many correlations amongst the variables proved significant and predictive multiple regression equations were developed. A self-selection process producing fewer females in mathematics courses who are highly motivated towards success was hypothesised to explain the lack of significant gender differences on measured variables and for the better predictability of results for males than for females - findings at variance with other studies. The model was found to have an 80% success rate for predicting passes and fails. The multiple regression models suggested that "a measure of previous work in the subject, a general ability test in the area, and a pretest on the specific subject matter are the most useful covariates overall in predicting course results and long-term retention" (Watson, 1988, p. 211).

Feather (1988) found that the enrolment decisions of a large sample of Flinders University students enrolled in humanities, social sciences and science in an introductory psychology course were related to students' self-concept of ability in mathematics and English and to the subjective value assigned to these subjects. On average, students rated themselves as better at English than mathematics and assigned higher subjective value to English, particularly the

females. No statistically reliable gender differences were found for self-concepts of ability for mathematics or English. Male students assigned higher value to mathematics than did females and females assigned higher value to English. Females were also found to assign more importance to values involving concern for people.

POSSIBLE EXPLANATIONS

Comprehensive overviews of previous literature in the field were presented by Leder and Fennema (1990) and Willis (1989). Also relevant here is the work of Carss (1988), Harding (1990), Leder (1990a) and Stewart (1991).

Patterns of classroom interactions in school mathematics lessons and the attitudes of students towards mathematics and their beliefs about themselves as learners of mathematics have been the focus of several reported research studies .

Leder (1990b) reported on a study involving students in Years 3, 6, 7 and 10 in 14 metropolitan schools in Melbourne, Victoria. No significant gender differences in mathematics achievement were evident for students in Years 3, 6, or 7; at Year 10 the boys scored significantly higher than the girls. Teachers' perceptions of students' ability generally coincided with achievement test scores. Students in three Year 7 classes who had performed comparably on the achievement test were asked to indicate their perceptions of their mathematics ability. A larger proportion of the boys (65%) compared with girls (18%) rated themselves as above-average compared with their classmates. The males also indicated greater confidence in coping with mathematics. Gender differences were also apparent in attributions for success and failure. Males more than females attributed success to ability; females more often than males attributed failure to ability and to task difficulty. Leder and Forgasz (1991) in another study with Year 7 mathematics students, and Leder (1989c) following interviews with almost 100 Year 3 and 6 students, also reported similar gender differences in beliefs where no achievement differences were found.

Leder's (1990b) study also involved investigating patterns of classroom interaction. Males were found to interact more frequently with the teacher than did females. Although no clear pattern was evident for high and low cognitive questions, males attracted more high-cognitive level questions at Year 10. At all grade levels, males dominated public interactions and received more disciplinary exchanges. When the data were correlated with teacher ratings for Year 7 students, below average females had the greatest number of interactions. Excellent Year 10 females had more interactions than other female groups at that level but significantly fewer than excellent males. Analyses of wait times for interactions revealed that longer wait times were accorded females at Year 3 and for low-cognitive questions at Year 10. Males received longer wait times for high-cognitive level exchanges at Year 6 and 7. Lowest-

rated males in Year 3 received three times the attention time of any other group. Other aspects of Leder's (1990b) study have been reported elsewhere (see Leder, 1988; 1989a; 1989b).

According to Leder (1990b), "schools do not work in a vacuum. They reflect the dominant cultural values and expectations of the society they serve" (p. 166). The influence of the societal context in which mathematics learning takes place and gender-stereotyped behaviours, values and expectations have been put forward as explanations for females' differential attitudes, achievement and participation in mathematics.

Gender differences in the early years of education have also been examined. Leder (1989c) found that pre-school children's beliefs about how their parents use mathematics were gender-stereotyped despite the fact that the mothers of the two pre-school children interviewed were tertiary-qualified in science-related fields. With particular emphasis on the early childhood years, Willis (1990) briefly sketched some of the beliefs and attitudes people still hold about girls and mathematics and their detrimental impact on girls.

Crawford (1988) cited Australian evidence that indicates that more males than females perform extremely well in mathematics, e.g., in the Australian Mathematics Competition, and in selected topic areas such as measurement and problem solving. Females, on the other hand, appeared to do better than males on arithmetic procedures. She argued that females thus appear to be advantaged in settings where the correct application of arithmetic procedures was emphasised (such as the primary and lower secondary classrooms) while males were advantaged in higher profile events such as mathematics competitions and "in tests of mathematical giftedness" (p. 156). The emphasis in many mathematics classes on following instructions and memorising procedures was, she contends, consistent with the role typically assigned to females in the broader society - a role in which they are given less autonomy and less opportunity for decision making than males. Thus cultural experiences and expectations contributed to the observed gender differences in participation and performance in mathematics. These differential experiences were further reinforced by "gender differences in the social interaction conditions of learning in the classroom" (p. 157), e.g., interactions with the teacher. Crawford concluded that there were two important areas that require further attention: males' and females' outside school experiences and a redefinition of gender equity in terms of conditions of learning as well as opportunity for participation.

Numerous policy statements emphasise that increasing participation rates is an aim to be achieved in order to redress inequities in girls' education and in their career opportunities. An interesting, more `radical' perspective has recently emerged. In reflective discourses, Hayes (1989), Shelley (1989) and Willis (1991) questioned the worth of encouraging girls into mathematics and science in a society which they perceive as based on a male value system. In order to succeed, Shelley (1990) contended girls must necessarily accept "the male world view" (p. 12). Cynical about the Australian government's rhetoric

in promoting greater female participation in mathematics and science, Hayes (1989) questioned whether this is a genuine effort to address gender equity issues or simply a means by which national productivity goals might be met. According to Willis (1990): "Over recent years, governments, science and industry have `discovered' that women `are a vital asset'" (p. 37). Hayes (1989) contended that mathematics and science will receive increased federal funding "regardless of whether women participate in them or not" (p. 18). According to Shelley (1990), the male value system impinges on the teaching and learning of mathematics, the nature of the discipline itself, and on the society served by mathematics. She contended that when mathematics is regarded as a static discipline to be absorbed, creative thinking and development based in personal experiences are denied. Promoting access, she believed, depends on the choice made by educators and society on the world view for the future. Willis (1991) stated that young women should not have to become like young men and are unlikely to enter the fields of science and technology unless there is a preparedness to make changes to the rules of the game.

Bell (1989) reported the findings of research conducted by the Working Party on the Girls and Technology Strategy (GTS) for the New South Wales Department of Education. Within the small sample of schools involved, both structural and pedagogical barriers to girls' participation in mathematics, science, industrial arts and computer studies were identified. The former included unequal allocation of senior and junior teaching responsibilities to male and female teachers, and timetabling inflexibility. Amongst the pedagogical issues highlighted were: differential interaction patterns and expectations for male and female students, male students' monopolising of equipment, alienating learning environments, and the predominance of competitive whole class teaching practices indicative of teachers' lack of awareness of gender differences in preferred learning styles which "unwittingly continue to disadvantage some of their students" (Bell, 1989, p. 32). Highlighting examples from science syllabi, Bell indicated that, as yet, syllabi and curriculum documents in New South Wales cannot confidently be said to be gender-inclusive. Bell (1989) also described some important intervention initiatives in NSW aimed at raising both girls' and parents' awareness of the importance of studying mathematics in order to broaden career options. The aims of the NSW Girls' Education Strategy policy was also outlined.

COEDUCATION AND SINGLE-SEX

Since 1988, coeducation and single-sex settings have been discussed in the literature from two perspectives: teaching and learning of mathematics within single-sex schools and, single-sex mathematics classes within coeducational schools.

From the perspectives of single-sex schools and the optimal education for

girls, Gill (1988) provided a comprehensive overview of the issues involved, summarised and critiqued the cases presented by both sides of the debate, and provided an extensive annotated bibliography of the literature from Australia, Britain, the United States and other international sources. With contrary research findings, Gill (1988) concluded that "the situation is much too complex to reduce to a straight comparison between school types" (p. 14). While there seemed to be "indications of significant differences between the environments of mixed and single sex schools" (p. 13), it also appeared that "there may be important differences between schools" (p. 13). Mathematics and science enrolment and achievements were found to be a focus of many studies. Gill (1988) suggested that investigating learning environments which are enabling and enhance girls' academic achievements, comparing mathematics and science classroom processes with those in which girls have traditionally excelled (English and history, for example) to understand better girls' preferred learning strategies, are directions for future research. She also advocated continued research into affective outcomes: self-esteem, confidence, enjoyment of school and career aspirations, as forming "an important contribution to the issue of girls and educational outcomes" (p. 14).

That students' participation and performance in mathematics (and other school subjects) is affected not only by school setting (single-sex v. coeducational) but also by socio-economic status, associated parental attitudes, as well as the attitudes and practices of teachers and administrators, was argued most cogently by Jones (1990). "Being female", she concluded, "is a greater disadvantage the lower one's socioeconomic status" (p. 166).

School type was a critical variable in the study reported by Foon (1988). A large sample of year 10 students from sixteen independent schools in Melbourne (5 girls' schools, six boys' schools and 5 coeducational schools) completed an eighteen page questionnaire to explore whether school type attended affected self-esteem, locus-of-control and affiliation needs. Pertinent to the mathematics dimension of the study, students reported their grade in mathematics from the previous year, their rank order preference for mathematics, science and English, and their ratings of academic performance in these three subjects. It was found that males rated their achievements in mathematics and science higher than did females and females preferred and rated their achievements in English higher than did males. Females from single-sex schools were more likely to prefer science and rate their achievements in mathematics and science higher than their coeducational counterparts. Students from single-sex schools seemed to be less traditional about the appropriateness of subject areas by gender. The self-esteem of females was lower than for males yet no significant differences were found for the self-esteem of females from single-sex and coeducational schools. Whilst females were found to attach greater importance to the value their parents held for school success, males felt friends and teachers attached greater importance to their success at school. Since the attitudes of parents, teachers and family appeared important, Foon suggested that future research might usefully be

directed to their relationship with school outcomes.

A unique situation in which two single-sex high schools in the same area were re-organised to form two coeducational high schools provided the base for the study reported by Marsh, Smith, Marsh and Owens (1988) (see also Marsh, Owens, Marsh & Smith, 1988). Multidimensional self-concept measures showed an increase over the four years of the transition for both males and females; gender differences in these measures were unaffected by the transition. Gender differences in achievement, with males continuing to achieve higher in mathematics and females in English, were also unaffected during the period of the transition. The authors suggested that the benefits in self-concept derived from the transition from single-sex to coeducation were not at the expense of academic achievement.

Late in 1991, several independent boys schools in Victoria announced that they were moving to coeducation (Slattery, 1991). These announcements rekindled the single sex versus coeducation debate in the popular media with the main focus on girls', not boys', education; an earlier media debate was summarised by Leder (1984b). The recessed economy has prompted sceptical observers to imply that the commitment to coeducation is driven by economics rather than educational principle (Munro, 1991). Greater opportunity for girls in mathematics and science and higher self-esteem have been amongst the arguments put forward by the single-sex supporters (Crawford, 1991). Advertisements for coeducational schools proclaim that coeducation is a more "natural" setting for all students, will prepare students more adequately for the world, and that single-sex schooling is anachronistic (Crawford, 1989).

Battersby (1989) argued that a coeducational setting viewed as `natural' may disadvantage girls. Through an historic case study, Battersby (1989) demonstrated how an ideology of naturalism resulted in gender differentiation, and suggested that the current ideology of neutrality (coeducation) may have the same effects. Battersby contended that single-sex provisions, particularly in subjects like mathematics, may significantly influence the social construction of gender.

As will be discussed in the section on intervention strategies, research on single-sex mathematical classes within coeducational school environments was also undertaken. Gill (1988) recommended that more attention is required in monitoring schooling processes by ethnographic studies. She claimed that the ethnographic style of research "would have a lot to offer in terms of generating an understanding of curriculum processes and girls' education in both the single sex and coeducational setting" (p. 14).

INTERVENTION STUDIES AND PROJECTS

The period under review has witnessed a range of intervention strategies at all levels of the educational spectrum from pre-schoolers to mature adults to meet the identified needs of females in mathematics. The reports and findings of

studies associated with some of these initiatives will now be described.

Cross (1988) described a 12 week intervention program aimed at making "careers which involve the use or prerequisite demands of mathematics and physical science in technology more attractive and worthwhile" (p. 400). The study was carried out in a high school situated in a northern suburb of Melbourne. There were considerably more girls than boys at the school (girls : boys = 1.6 : 1). Data about participation in Year 10 elective subjects revealed a number of differences. Boys dominated in subjects such as technical drawing, woodwork, electronics, horticulture, graphics and computer studies; girls in home economics, decorative fibre, typing, and shorthand. While the proportions of males and females in the top mathematics group were approximately as expected at Year 10 and in the general mathematics course at Year 12, only 1 girl, compared with 12 boys, was enrolled in pure and applied mathematics at Year 12. In science, participation in Year 12 biology was gender neutral. Chemistry and physics, however, attracted few girls. The author noted a number of problems encountered. Because the program was initiated outside the school not all staff supported it. The intervention sessions were more successful in an all-girls setting. Substantial adjustments to the school program were thus required. Some girls saw the program's emphasis on non-traditional career options as down-grading their intended career choice.

Vasey (1989) presented an outline of the background to the Family Maths Project (Australia) (FAMPA) in the Australian Capital Territory (ACT). Included among the project's objectives were: improving attitudes towards mathematics and demonstrating that boys and girls can achieve equally in mathematics. Drawing on research on the influence of parental attitudes on the attitudes of their children, FAMPA believed in encouraging the family to be a base for the learning of mathematics. Covering the years from pre-school through to year 12, FAMPA ran in-service activities for parents and teachers at which the participants carry out calculation or problem-solving examples. Schools then developed their own in-school programs and FAMPA facilitated the sharing of ideas between schools. FAMPA also provided complementary services to other ACT maths education programs and projects. Evaluation was used to refine programs to meet local community needs. Initial research indicated that girls with poor attitudes to maths were being reached and that their attitudes were more positive. Parents were found to hold gender-related attitudes about mathematics which FAMPA plans to pursue.

In considering the causes and explanations put forward for girls' lower participation in and preparation for tertiary level studies of mathematics, Barnes (1989a) mounted a case for the intervention program, the Mathematics Learning Centre, provided at the University of Sydney. Increasing the women's access to mathematics and to the courses which depend on it were underlying aims of the program. Assistance with the mathematical content in a wide range of courses was provided to students on a free and voluntary attendance basis. A range of teaching methods was adopted and additional courses in conjunction with other services or departments within the university

were run. From the students, the Centre identified and addressed a range of obstacles to learning including: lack of prior knowledge, dependence on rote learning, lack of problem-solving skills, lack of confidence, difficulties with understanding mathematical symbols, perceptions of mathematics as a male domain. Barnes (1989a) also presented several case studies of individual women which illustrate the positive impact that the program had on them.

During 1989, a well-conceived and novel intervention project was carried out across the state of Victoria. Using the power of the media, in particular television, a successful three-month advertising campaign entitled `Maths Multiplies Your Choices' was conducted (Women's Employment Branch, c.1989; Baker, 1989). The aim was to raise the awareness of parents and girls to the breadth of career options for which mathematics and science are needed. Baker (1989) indicated that comparisons of pre- and post-campaign survey data revealed a marked change in perceptions and attitudes in favour of mathematics and science by both parents and Year 10 girls. For example, there was an increase in the number of girls indicating they would pursue mathematics to Years 11 and 12 and greater awareness that a background in mathematics and science provided a wider range of career opportunities. Other states in Australia appear to have followed Victoria's lead and adopted similar initiatives involving the media.

Within the one high school, Rowe (1988) (see also Schwarz & Rowe, 1990) studied the effects on students' confidence and achievement in mathematics when allocated to single-sex or coeducational classes. Years 7 and 8 students were randomly allocated to single-sex or mixed classes. Measures of confidence and achievement were taken on three occasions: at the beginning and end of the school year and at the end of the following school year. In the second year, due to school constraints, some students changed classroom settings. It was found that over the three test occasions, gender differences in achievement and confidence were not significant. For students remaining in single-sex classes or in mixed classes, the most notable gains were amongst girls in single-sex classes followed by boys in single-sex classes. Students in single-sex classes indicated significantly higher gains in confidence over time than those in mixed classes. For students who changed class type, there was a significant class-type effect on their confidence in learning and using mathematics: negative for girls moving from single-sex to mixed classes, positive for boys moving to single-sex classes. For the grade 9 students, being in single-sex classes was associated with high confidence levels which increased the likelihood of their subsequent participation in Year 10 mainstream classes. Rowe (1990) cautioned that "the long-term effectiveness of single-sex class grouping as an appropriate intervention strategy to increase the participation of students in coeducational postprimary schools [for any area of the curriculum] has yet to be established" (p. 197).

Table 5 provides a short synopsis of a selection of projects which have been reported since 1988. Several other of these interventions involved investigating (Girls project/Paralowie R-12 school, 1990), evaluating (Process

mathematics/ Holden Hill primary school, 1988) and/or encouraging (Dunstan, 1990) the provision of single-sex mathematics classes or single-sex small group work in coeducational settings.

Papers presented at conferences for mathematics teachers provide opportunities for researchers and practitioners to share their findings and experiences. Many creative ideas, in particular numerous pragmatic suggestions for gender-inclusive mathematics curriculum design and classroom implementation are presented. Reviews and reflections on the successes of a range of curricular and intervention initiatives over recent years aimed at improving the mathematics education of girls were presented by Sampson (1989) and Lustig-Selzer (1988) . The papers described below were presented at the conferences of the Mathematical Association of Victoria since 1988 and would be representative of similar conferences in other states of Australia.

Having targeted girls talented in mathematics as a group at risk, Taylor (1990) described a case study of a multi-faceted program for Year 4 students in one independent girls' school in Victoria. A second enrichment program for girls in Years K-6 was presented by Wirth (1991). In this program, promoting positive attitudes in girls and the provision of both female-oriented mathematics themes and practical mathematical experiences were regarded as vital. Fitzsimmons (1991) worked successfully with a group of women of diverse ages and mathematical backgrounds. The classroom environment created, the learning strategies adopted and the curriculum presented were described. Zevenbergen (1991) argues a case for the constructivist mathematics classroom and the role it can play in addressing social justice issues, including gender. With its potential for gender-inclusive curriculum and group work, and its emphasis on communication, she contended that the learning environment engendered meets the needs of girls.

Spence (1990) suggested that including the experiences of women mathematicians will make the teaching of mathematics more interesting and relevant to females. Gender-inclusive approaches to the teaching of calculus were discussed by Barnes (1988b) and Lowe (1990) who described a new text book suggesting gender-inclusive approaches to the teaching and learning of algebra. Barnes (1989b) reflected on what constitutes girl-friendly mathematics teaching while Hildebrand (1989) provided a range of general gender-inclusive mathematics teaching strategies. Groves and Prelaz (1989) listed practical strategies and relevant resources for raising career awareness in mathematics classes. Costello, Long and Marr (1989) suggested a number of ways in which parents and teachers can assist in the development of the spatial abilities of children, particularly girls. Leder and Penna (1990) described the background to and the content of two videos which have been produced to link mathematics to the workplace. One of the purposes behind the production of the videos was to redress the gender stereotypes of women in the workplace by showing "women at work in positions of responsibility requiring mathematical skills" (p. 267).

Table 5
Intervention Initiatives

SOURCE	BRIEF DESCRIPTION
Department of Employment, Education and Training (1988). *Teaching Girls: Bulletin, 1*. Canberra: Commonwealth of Australia.	Examples of the ways in which education systems have addressed the implementation of the National Policy for the Education of Girls in Australian Schools. Case studies, programs and projects, conferences and seminars, and available resources are included.
Preston, S (1989). New centre to boost science education in Victoria. *Education Victoria, 4*(4), 2.	Profiles the newly established John Gardiner Centre in Hawthorn, Victoria. The focus of the Centre is to stimulate greater participation, particularly of girls, in maths and science studies. The Centre will also provide in-service education programs for teachers.
Pilley, T. (1988). Maths science technology: Increasing student interest and participation. *Education Victoria, 3*(3), 7-10.	Describes initiatives undertaken to increase student participation in maths and science (eg. Girls and Maths and Science Teaching, GAMAST), strategies for increasing numbers and enhancing teachers' skills in maths and science, and other projects.
Mathematics/Salisbury junior primary school. (1990). Project in progress. South Australia	With special reference to disadvantaged groups, including girls, the project assessed the impact of mathematics programs on the development of 11 targeted children.
Maths/Salisbury North primary school. (1990). Project in progress. South Australia	Developed from a booklet aimed at increasing parents' involvement in their children's mathematics learning. The program includes mathematics electives for children, parents and teachers demonstrating how activities may assist children in understanding mathematical concepts - an accompanying booklet complements the activities. The program has the potential to examine issues including girls and mathematics.

Dunstan, J. F. (1990). *Special staffing needs of all girls classes in maths, science/or technology (Executive memorandum no. 436)*. Victoria: Ministry of Education.	Ministry indicates willingness to assist schools having a preference for female staff in running all girl classes in mathematics, science and technology. School principals asked to indicate requirements in writing.
Girls project/Paralowie R-12 school. (1990). Project in progress. South Australia.	Program with dual purpose: to gather information about girls learning in single-sex classes, with a focus on mathematics and science, and in exploring sexual harassment issues. The school's commitment to girls' education issues has been extended.
Girls' education strategy: Pilot schools. (1989). NSW.	Project supporting the Girls' education strategy (GES) policy. Selected pilot schools are to encourage girls into technology by developing action plans to meet girls' needs and interests in focus subject areas including mathematics.
Girls in schools. (1989). [Videorecording]. SBS television.	Video outlining the National Policy for the education of girls in Australian schools. Areas covered include gender inclusive curricula and single sex classes. Includes interviews with students and others and scenes showing girls in mathematics classes.
Process mathematics/ Holden Hill primary school. (1988). Project in progress. South Australia.	Mathematics classroom based project based in the practice of open education. Resources were provided for children to engage in open tasks in mathematics and to plan and record progress and success. Children working in gender groups resulted in girls viewing themselves as independently successful in maths.
Equal opportunity projects in Victorian schools. (1988). Melbourne: Equal Opportunity Unit, Ministry of Education, Victoria.	Under four broad subject headings, including mathematics, an inventory, with a brief description and other additional information, of equal opportunity projects in Victorian primary, post primary and TAFE colleges is presented.

* References in this table are not included in the reference list at the conclusion of the chapter
* The Appendix lists additional references for which we were unable to obtain original copies

CONCLUSIONS

The impact of the National Policy for the Education of Girls in Australian Schools (Commonwealth Schools Commission, 1987) emerges as a significant feature of the period 1988 to 1991. Regular publications including the monthly newsletter, The GEN (Gender Equity Network) and the professional development magazine GEMS (Gender Equity in Maths and Science) form a means of disseminating information to practitioners and other interested parties. The projects of national significance and the range of curriculum and research publications from the Curriculum Development Centre attest to efforts aimed at addressing targeted audiences. At the state level, government documents, reports, intervention strategies and curriculum initiatives have also addressed the four broad objective areas of the policy with respect to girls and the provision of appropriate mathematics education. Participation data for the same period are unable to reflect the extent of the effects on girls of the range of initiates undertaken. The need for continued monitoring is clear.

The range and scope of the reported intervention initiatives form a second significant feature of the period under review. Recognising the contributions of research findings, both local and international, practitioners have implemented recommendations and are reporting on the initiatives, often school-based, which they have undertaken. Short term findings are encouraging. Supporting and maintaining the enthusiasm and efforts of those involved in this work must be endorsed for the longer term impacts to be studied and evaluated; established communication networks must also be maintained.

A comprehensive range of gender-inclusive curriculum and professional development materials are now available. It would appear, however, that little of the work with teachers in professional development has been reported. The extent of the incorporation of such materials in mathematics classrooms across the various levels of schooling is generally unknown. How teachers are changing their beliefs about the teaching and learning of mathematics, how they are adapting materials in their classrooms, the effect of changing classroom learning environments on student outcomes, both cognitive and affective, seem fruitful areas for future research.

Valuable findings have emerged from the basic research over the period. Affective variables continue to be of significance and the importance of continued research in this field has been suggested by many. Gender differences in achievement in the Australian Mathematics Competition invite closer investigation. The shift towards more ethnographic methodologies has provided richer and more personalised data for close and critical analysis. Advocacy for this form of research has been noted. With respect to studies in a range of areas including classroom interaction, single-sex and coeducational settings (whole-school, whole class within coeducational schools and within class group settings), the qualitative data obtained may help in understanding better how to address the gender inequities still evident within mathematics

education.

The recessed economic climate in Australia over recent times has impinged on education in a number of ways. Funding for new initiatives, and continued support for ongoing projects have been curtailed. Programs that allow for individual differences and that focus on the process and not merely the product of learning are severely challenged by politicians' increasingly vocal advocacy of national curricula and testing programs. In this period of high youth unemployment and increased competition for tertiary places, arguments that sound mathematical qualifications lead to open educational and career opportunities are developing a hollow ring. Concerns have begun to be expressed about the continuing emphasis on encouraging females to aim for the still male-dominated spheres of science and mathematics. Economic concerns, rather than educational considerations, appear to be driving some single-sex independent schools to become coeducational. Yet Ellerton and Clements' (1988) historical perspective on traditions within school mathematics in Australia makes sobering reading. It is, after all, only in the last 25 years that these traditions have been seriously questioned and challenged, that efforts have been made to increase access to mathematics to groups, including females, previously prevented from participating. We are making headway. The educational community is more aware of the issues; curriculum direction and policy statements are supportive. Ingrained social attitudes and expectations are slow to change; some success, however, has been demonstrated.

Since gender equity has not yet been attained in the sphere of mathematics education in Australia, keeping the issue alive and on the national educational agenda is a vitally important priority. Despite the shadows of doom cast by the economic times, the work reported in this review and the encouraging trends they convey ultimately augur well for the future.

REFERENCES

Australian Association of Mathematics Teachers. (1991). *A national statement on girls and mathematics*. Melbourne, Author.

Australian Education Council. (1990). *A national statement on mathematics for Australian schools*. Melbourne: Curriculum Corporation.

Baker, M. (1989). *The Maths multiplies your choices information line: Report to the Ministerial Advisory Committee on Women's Employment*. Victoria: Department of Labour.

Barnes, M. (1988a). Research in gender and mathematics: An annotated bibliography of Australian research 1984-1987. In D. C. Blane and G. C. Leder (Eds.), *Mathematics education in Australia: A selection of recent research* (pp. 22-48). Melbourne: Mathematics Education Research Group of Australasia.and Monash University

Barnes, M. (1988b). Can we make calculus gender-inclusive? In D. Firth

(Ed.), *Maths counts - who cares?* (pp. 63-67). Parkville: Mathematical Association of Victoria.

Barnes, M. (1989a). A rescue operation. In G. C. Leder & S. N. Sampson (Eds.), *Educating girls: Practice and research*(pp. 105-117). Sydney: Allen & Unwin.

Barnes, M. (1989b). Girl friendly mathematics. In B. Doig (Ed.), *Everyone counts* (pp. 290-291). Parkville: Mathematical Association of Victoria.

Battersby, D. (1989). Gender construction, mathematics and single-sex schools. *Independent Education, 19*(4), 36-38.

Bell, L. (1989). Girls can be calculating and inventive - but why are there so few women scientists, mathematicians and engineer? *Independent Education, 19*(4), 29-35.

Carss, M. (1988). What would a nice girl like you want with a calculator? In S. Taylor & M. Henry (Eds.), *Battlers and bluestockings: Women's place in Australian education* (pp. 95-100). Canberra, ACT: Australian College of Education.

Carty, J. (1990). A teacher's experience on the Australian Mathematics Competition problems committee. In K. Clements (Ed.), *Whither mathematics?* (pp. 300-304). Parkville: Mathematical Association of Victoria.

Commonwealth Schools Commission. (1975). *Girls, schools and society*. Canberra, ACT: Author.

Commonwealth Schools Commission. (1984). *Girls and tomorrow: The challenge for schools* (Report of the working party on the education of girls). Canberra, ACT: Author.

Commonwealth Schools Commission. (1987). *A national statement for the education of girls in Australian schools*. Canberra: Author.

Costello, P., Long, K., & Marr, B. (1989). `Girls need space too!'. In B. Doig (Ed.), *Everyone counts* (pp.151-152). Parkville: Mathematical Association of Victoria.

Crawford, A. (1991, December 15). School lessons that can last a lifetime. *The Sunday Age*, p.9.

Crawford, K. (1988). Equality of opportunity and equality of conditions in mathematics learning for girls. *Unicorn, 14*, 55-160

Cross, R.T. (1988) Task value intervention: Increasing girls' awareness of the importance of mathematics and physical science for career choice. *School Science and Mathematics, 88*(5), 397-412.

Dekkers, J., De Laeter, J. R., & Malone, J. A. (1991). *Upper secondary school science and mathematics enrolment patterns in Australia, 1970-1989*. Perth: Curtin University of Technology.

Department of Employment, Education and Training. (1991). *National database on the education of girls in Australian schools*. Canberra: Australian Government Publishing Service.

Ellerton, N. F., & Clements, M. A. (1988). Reshaping school mathematics in Australia 1788-1988. *Australian Journal of Education, 32*(3), 387-405.

Feather, N. T. (1988). Values, valences, and course enrollment: Testing the role of personal values within an expectancy-valence framework. *Journal of Educational Psychology, 80*(3), 381-391.

Fitzsimmons, G. (1991). Teaching and learning mathematics can be a joyful experience. In J. O'Reilly & S. Wettenhall (Eds.), *Mathematics: Ideas* (pp. 10-14). Parkville: Mathematical Association of Victoria.

Foon, A. E. (1988). The relationship between school type and adolescent self-esteem, attribution styles, and affiliation needs: Implications for educational outcome. *British Journal of Educational Psychology, 58*, 44-54.

Gardner, K. (1990). *Gender participation in maths and science in Victorian secondary schools: A report prepared from the maths and science incentive grant data.* Melbourne: Women's Employment Branch, Department of Labour.

Gill, J. (1988). *Which way to school?: A review of the evidence on the single sex versus coeducation debate and an annotated bibliography of the research.* Canberra: Curriculum Development Centre.

Girls and mathematics (the development of a national statement). (1990). *The Australian Mathematics Teacher, 46*(1), 6.

Girls and mathematics: Is there a problem. (1990). *The Australian Mathematics Teacher, 46*(3), 10-11.

Girls and mathematics: Some recommendations. (1990). *The Australian Mathematics Teacher, 46*(4), 16-17.

Grimshaw, P. (1982). Introduction. In P. Grimshaw and L. Strachan (Eds.), *The half open door* (pp. 1-9). Sydney: Marle and Iremonger.

Groves, U., & Prelaz, S. (1989). Girls, maths and careers. In B. Doig (Ed.), *Everyone counts (*pp. 340-342). Parkville: Mathematical Association of Victoria.

Harding, J. (1990). What counts in mathematics education? In K. Clements (Ed.), *Whither mathematics?*, (pp. 345-351). Parkville: Mathematical Association of Victoria.

Hayes, F. (1989). On equitable cake-cutting, or: Caring more about caring. *Educational Philosophy and Theory, 21*(2), 12-22.

Hildebrand, G. (1989). Gender inclusive mathematics education girls and mathematics: Year 7-12. In B. Doig (Ed.), *Everyone counts* (pp.24-26). Parkville: Mathematical Association of Victoria.

Johnston, J. E., & Rennie, L. J. (1990 April). *Structural change in curriculum: The implications for gender equity in science and mathematics.* Paper presented at the annual meeting of the American Educational Research Association Boston, MA. (ERIC Document Reproduction Service No. ED 317 404)

Jones, J. (1990). Outcomes of girls' schooling: Unravelling some social differences. *Australian Journal of Education, 34*, 153-167.

Leder, G. C. (1984a). Girls and mathematics: An annotated bibliography of Australian Research. In J. Briggs (Ed.), *Summary of research in*

mathematics education in Australia (pp. 63-81). Brisbane: Mathematics Education Research Group of Australasia.

Leder, G. C. (1984b). Girls and mathematics: Single sex or mixed schooling? In P. Costello, S. Ferguson, K. Slinn, M. Stephens, D. Trembath, & D. Williams (Eds.), *Facets of Australian Mathematics Education, Fifth International Congress on Mathematical Education, Adelaide, Aug. 24-30* (pp. 169-172). Blackburn, Melbourne: Australian Association of Mathematics Teachers.

Leder, G.C. (1988). Teacher-student interactions: The mathematics classroom. *Unicorn, 14,* 161 -166.

Leder, G. C. (1989a). Inside the mathematics classroom: A question of equity. *The Australian Mathematics Teacher, 45* (3), 30-31.

Leder, G. C. (1989b). Do girls count in mathematics? In G. C. Leder & S. N. Sampson (Eds.), *Educating girls: Practice and research,* (pp. 84-97). Sydney: Allen & Unwin.

Leder, G. C. (1989c). Girls and mathematics: A question of equity. In B. Doig (Ed.), *Everyone counts,* (pp.18-23). Parkville: Mathematical Association of Victoria.

Leder, G. C. (1990a). Gender differences in mathematics: An overview. In E. Fennema & G. C. Leder (Eds.), *Mathematics and gender.* (pp. 10-26). New York: Teachers College Press.

Leder, G. C. (1990b). Teacher/student interactions in the mathematics classroom: A different perspective. In E. Fennema & G. C. Leder (Eds.), *Mathematics and gender,* pp.149-168. New York: Teachers College Press.

Leder, G. C., & Atkins, W. (1990). The Australian Mathematics Competition: Results and impications. *Australian Senior Mathematics Journal, 4*(2), 65-73.

Leder, G. C., & Fennema, E. (1990). Gender differences in mathematics: A synthesis. In E. Fennema & G. C. Leder (Eds.), *Mathematics and gender* (pp.188-199). New York: Teachers College Press.

Leder, G. C., & Forgasz, H. J. (1991). Learning mathematics: Some student perspectives. In J. O'Reilly & S. Wettenhall (Eds.), *Mathematics: Ideas* (pp. 3-9). Parkville: Mathematical Association of Victoria.

Leder, G. C., & Penna, C. (1990). Mathematics in the workplace: Video perspectives. In K. Clements (Ed.), *Whither mathematics? (*pp. 266-270). Parkville: Mathematical Association of Victoria.

Lowe, I. (1990). Algebra for all Australians. In K. Clements (Ed.), *Whither mathematics?* (pp. 296-299). Parkville: Mathematical Association of Victoria.

Lustig-Selzer, A. (1988). Curricula and gender: where to now? *Educational Research and Perspectives, 15*(2), 97-111.

Marsh, H.W., Owens, L., Marsh, M., & Smith, I.D. (1988). From single-sex to coed schools. In G. C. Leder & S. N. Sampson (Eds.), *Educating girls: Practice and research,* (pp. 144-157). Sydney: Allen & Unwin.

Marsh, H.W., Smith, I.D., Marsh, M., & Owens, L. (1988). The transition from single-sex to coeducational high schools: effects on multiple dimensions of self-concept and on academic achievement. *American Educational Research Journal 25*, 237-269.

Munro, J. (1991, December 8). Boys into girls' schools won't go. *The Sunday Age*.

Rowe, K. J. (1988). Single-sex and mixed-sex classes: The effects of class type on student achievement, confidence and participation in mathematics. *Australian Journal of Education, 32*(2), 180-202.

Sampson, S. N. (1989). It's a long haul but I think you're winning - girls and mathematics and the classroom teacher. *Vinculum, 26*(2), 20-21.

Sampson, S. N. (1991, June). *Dismantling the divide*. Background paper prepared for NBEET seminar. Available Women's Bureau, DEET, Canberra.

Schwarz, V., & Rowe, K. J. (1990). *Count us in: Girls, boys and mathematics*. Melbourne: Ministry of Education, Victoria.

Shelley, N. (1989). A perspective on mathematics. *GEMS, 1*(2), 12-13.

Slattery, L. (1991, December 10). Reviving the coeducation debate. *The Age*, p. 13.

Spence, P. (1990). Women and mathematics and girls. *The Australian Mathematics Teacher, 46*(2), 26-28.

Stewart, J. A. (1991). Why don't girls study mathematics and physical science? *The Australian Science Teachers Journal, 37*(3), 18-23.

Taylor, M. A. (1990). Catering for mathematically able girls in the primary classroom. In K. Clements (Ed.), *Whither mathematics?* (pp.407-413). Parkville: Mathematical Association of Victoria.

The Australian Association of Mathematics Teachers Inc. (1990). *A national statement on girls and mathematics*. [GEMS, July 1990]. Melbourne: Author.

Vasey, J. (1989). The family maths project. In G. C. Leder & S. N. Sampson (Eds.), *Educating girls: Practice and research*, pp.98-104. Sydney: Allen & Unwin.

Watson, J. M. (1988). Student characteristics and prediction of success in a conventional university mathematics course. *Journal of Experimental Education, 56*(4), 203-212.

Watson, J.M. (1989). An exploration of gender differences in tertiary mathematics. *Mathematics Education Research Journal, 1*(1), 1-10.

Willis, S. (1989). *"Real girls don't do maths". Gender and the construction of privilege*. Geelong, Victoria: Deakin University Press.

Willis, S. (1990). Some beliefs about girls, mathematics and the early childhood years. *Australian Journal of Early Childhood, 15*(1), 47-48.

Willis, S. (1991). Science and technology participation by women: Wherein lies the problem? In W. A. Pullman (Ed.), *Women in science and technology: A vital asset!* Proceedings of a seminar, 27 Sept., Perth. Perth: Industry Education Forum.

Wirth, E. (1991). Putting magic into maths and keeping it there. In J. O'Reilly & S. Wettenhall (Eds.), *Mathematics: Ideas*, pp.23-29. Parkville: Mathematical Association of Victoria.

Women's Employment Branch. (1989). *Maths multiplies your choices.* Melbourne: Department of Labour.

Zevenbergen, R. (1991). *Constructivism in the classroom.* In J. O'Reilly & S. Wettenhall (Eds.), *Mathematics: Ideas*, pp.15-18. Parkville: Mathematical Association of Victoria.

APPENDIX

Byne, J. S., & Zeplin, D. (1990). *A problem-solving approach: Brunswick Maths Task Centre* (The social justice curriculum collection). Victoria.

Department of Employment, Education and Training, Economic Policy Analysis Division. (1991). *Retention and participation in Australian schools 1967-1990.* Canberra: Australian Government Publishing Service.

Dow, D. (1989). *Mathematics and girls.* Bathurst, NSW: CSU-Mitchell.

Drechsler, B. (1988). *Myths that matter in equality of outcome in education for girls in low socio-economic situations.* Adelaide: Education of Girls Unit, South Australian Education Department.

Education of Girls in Primary Schools Project (South Australia) Equal opportunities: Northern area: Education of Girls in Primary Schools Project, year one 1988. (1988). Elizabeth, South Australia: Education Department of South Australia, Northern Area Education Office.

Girls and achievement. (1990) Darwin: Northern Territory Department of Education.

Girls learning: From theory to practice: Port Adelaide, Mitcham and Gepps Cross Girls High Schools. Adelaide: Education of Girls Unit, Education Department: PEP.

It's just the thing you do: A study of year 8 girls and boys making decisions about subjects and careers (1988). Hobart: Education Department, Tasmania.

Johnston, S. (1990). *Retention rates, more than just counting heads.* Brisbane: Department of Education, Queensland.

Maths matters: An introduction to Victoria's numeracy strategy. (1990). Victoria.

Ministerial Advisory Committee on Women and Girls. (1988). *Primary practices: A practical guide to assist the promotion of equal opportunity practices in Victorian primary school communities - No. 1.* Victoria.

Ministry of Education and Training (1991). *Girls and maths, science and technology: school-based social justice initiatives.* Melbourne: Ministry of Education and Training, Victoria.

New South Wales Department of Education, Inservice Education, Library. (1988). *Non-sexist resource collection: Audio-visual catalogue 1988.*

North Sydney: Non-Sexist Resource Unit, Services Directorate, NSW Department of Education.

NSW Ministry of Education and Youth Affairs. (1988). *National database on the education of girls in Australian schools*. Sydney: Author.

Office of Schools Administration. (1989). *Equal opportunity action plan for girls in education 1988-1990*. Victoria.

Owen, J. M., & Hurworth, R. E. (1988). *Roles for evaluators as critical friend: An evaluation of the Girls and Mathematics and Science Project [GAMAST]*. Carlton, Victoria: Centre for Program Evaluation, Melbourne College of Advanced Education.

Rennie, L. J. (1989). *Monitoring of unit curriculum enrolment patterns*. Perth, Western Australia: Secondary Education Authority, Western Australia.

The book of chalkface. Volume two, maths activities. (1989). Victoria.

Unit Curriculum: A survey of student selection patterns. (1989, May). East Perth: Ministry of Education, Western Australia.

Waite, H. (1988). *National database on girls and education: Discussion paper*. Sydney: Social Policy Unit, NSW Ministry of Education.

RESEARCH IN PRACTICE: TEACHERS AS RESEARCHERS

Judith A. Mousley
Faculty of Education
Deakin University

This chapter describes some of the participatory research which has been undertaken in Australasia over the past four years within mathematics classrooms. Some extensive programs which commenced before 1988 have been included if they are still having a significant impact on mathematics education developments and have continued to involve teachers in researching classroom practice.

It is difficult to decide when reflective or innovative practice becomes "research". In this chapter, the notion of participatory research includes informal development, testing and publication of ideas by teachers. Emery (1986), in the introduction to a set of papers on qualitative research wrote, "Asking questions, working together to see what happens after different kinds of actions are performed, clarifying and understanding circumstances; all are fundamental research functions" (p. i). Common aspects of participatory research are collegial reflection on practice and the planning, introduction and evaluation of pedagogical innovations in staged and systematic ways.

Recently, there have been many published studies which fall into the category of participatory research, representative samples of which were selected for inclusion in this article. Preference was given to projects where small groups of teachers initiated and carried out the research, and where the primary focus was on changing pedagogical paradigms and classroom discourse, as opposed to experimenting with new content: but there was no shortage of publications. Chartres (1988) noted amongst Australian teachers a recent shift in focus from content and resource innovations to the longer-term exploration of educational philosophy when stating: "The past two years have enabled me to work with an ever-increasing range of teachers who are changing their mathematics programs from two major perspectives, firstly from their views about children's learning, and secondly from broadening and clarifying their thoughts about the nature of mathematics" (p. 8).

Also included in this chapter are summaries of extended research projects undertaken by teachers, as well as the work of academics in initiating, facilitating and supporting teacher-researchers. Reference is made to those funded school-based improvement programs which focused specifically on the development of field knowledge about mathematics education processes.

TIMES OF CHANGE

During the 1980s, boundaries between the professional and political aspects of Australian teachers' work were broken down to some extent. Increasing devolution of responsibility for curricular and administrative planning required that teachers be better prepared for planning, developing and evaluating innovations within individual schools and communities. A confusion of expectations from many directions involved teachers in making new curricular choices, in developing some non-traditional practices and topics, and in responding to the challenges and constraints of new assessment systems. Also, an increasing recognition of the diversity of Australian socio-cultural scenes demanded that teachers and administrators become involved in the determination and creation of optimal conditions for teaching and learning within individual community and institutional contexts.

While moves towards curriculum control at state and national levels in recent years resulted in less autonomy for teachers (Bessat, 1988; Kemmis, 1989), many continued to be innovative and professional in their attitude to the improvement of teaching. This was encouraged by in-service programs, such as those reviewed by Kennedy and Hodgens (1989), Robinson (1989) and Witford (1988). Both teachers and their employers increasingly recognised the fact that traditionally passive roles in in-service courses do little to develop teachers' interpersonal skills and attitudes appropriate to the complex and shifting relationships within schools. Thus the habit of offering short, one-off in-service courses, remote from classrooms, declined. To balance this, there was a growing recognition of the importance of small-scale school-based experiences as forms of research into, and sources of data about, the learning of mathematics. The majority of professional development now occurs at less formal levels.

During the period of this review, many teachers challenged the previously dichotomous nature of the teacher *or* researcher tradition by researching their own beliefs and classroom practices. Pateman (1989) characterised the teacher-researcher as not simply a consumer but a producer of knowledge:

> Such a classroom teacher is guided in the practice of teaching by the results of his or her own classroom research and does not rely exclusively on the results of others. Research and practice might then be viewed as two sides of the same coin. (p. 36)

THIS WORKS FOR US

Mathematics teachers often unconsciously engage in a dialectical process of adapting comfortable ways of working to the unique aspects of specific

educational settings, or of trying to find a structured path through the tangled web of conflicting and competing expectations that characterise the current world of mathematics education. These processes generate forms of knowledge with the common characteristic of "what works for us". When such investigation is brought to the level of consciousness and shared with a professional audience, teachers engaged in a form of experimentation that not only allows them to resolve issues of the immediate domain but also to develop a repertoire of tested knowledge in the wider mathematics education community. Because it is seen as immediately applicable to classroom use, such research can have a powerful impact on mathematical pedagogy.

The avenues for discursive construction of knowledge between mathematics teachers in Australia over the past four years have been widely varied but have demonstrated sustained growth. The period of 1988-1991 saw a strengthening of channels for the dissemination of findings of classroom research through teachers' journals and presentations by practitioners at state and national mathematics education and generalist conferences. The conferences of the Mathematical Association of Victoria, for instance, attracted over 1700 teachers yearly, and many of these contributed the results of their classroom research, presenting a picture of multiple and shifting images of curriculum development and classroom interaction studies.

Typical of the research reports with a "this works for us" flavour are those emanating from South Australia, where teams of teachers were involved in researching early childhood and primary mathematics education and in the dissemination of their findings throughout the state. The sharing of reflective writing, research reports and case studies was facilitated by the Education Department (see Charters & Nancarrow, 1988), but research foci and directions were under the control of small groups of practitioners.

In Kidric and Galvin's (1989) journal article on "story book maths" and Swan's (1988) research, reflections on the results of using literature in mathematics classes were recorded, together with issues regarding implementation and a discussion of likely problems. Similarly, Francesconi, Willis and Money (1990) investigated the nature of changes which were involved in the introduction of regular, extended problem-solving activities throughout their school; and Richards (1987) raised issues regarding the problems experienced when working with mixed ability classes. Burrows (1991) recorded students' responses and detailed the process used to introduce similar interactive adventure games and "magic".

Libertino's (1989) report on co-operative learning strategies took research a stage further in that it raised as problematic the roles of teachers, and this theme was also taken up by Nulty and Williams (1988, p. 27) in their investigation of a "conference approach" to teaching mathematics. The development of thematic approaches (e.g., Hodgson & Richards, 1989; Stewart, 1989; Oliver & Byers, 1991) has involved integrating disciplines as well as different social interaction in classrooms. However, the most common topic researched by teachers over the past few years has been the introduction

of a cooperative problem-solving approach to teaching (e.g., Currie, 1991; Float & Harvey, 1991; James, 1991).

Goal-based learning and criterion-based assessment formed the focus of a number of research projects. Omar and De Sensi (1988), for instance, reported on their school's development of a Year 7-10 program based on these organisational factors. Maltzahn (1990) outlined how teachers at her primary school developed strategies for evaluating children's progress without testing. The introduction of a radically different assessment system (the Victorian Certificate of Education), using extended mathematical investigations and problem-solving activities, involved every upper-secondary school in changing practices. Many teachers wrote articles about new ways of working with senior students (e.g., Daly & Cody, 1990; Johnston, Money, Nixon & Pearson,1990; Leigh-Lancaster, 1990; Williams, 1990), with a common emphasis on how teachers' adapted expectations, interactions, student support and assessment procedures.

A rigorous approach to classroom improvement was made by Waters and Montgomery (1989, 1990, in press). They reported on their development of a classroom climate where children and teachers were seen as the creators of mathematical ideas and problems. Creating a more "risk-free" and supportive environment, where mathematical strategies were modelled by all classroom participants was the key element of their action research, aimed at producing young mathematicians able to take responsibility for their own learning. During their three years of research into the teaching of mathematics, Waters and Montgomery aimed to produce a "community of mathematicians" with respect for learners and respect for mathematics (1990, p. 562).

Another teacher who took on an extended, three-year investigation into the development of her own teaching, and then published her findings was Skinner (1990). In working with one group of children from midway through kindergarten to Year 2, Skinner developed a problem-centred approach to teaching, with particular emphasis on constructing a teaching style which encouraged and supported children in posing, discussing, solving and recording their own mathematical problems. In reporting on her work, Skinner kept a focus on teaching actions, tackling difficult questions like how to assess learning and when to intervene.

Individual teachers also undertook extensive research projects for postgraduate theses, covering a wide range of classroom situations and techniques. Both Messinbird (1991) and Burnett (1991), for example, researched their own classrooms to investigate teachers' roles in improving children's fraction concepts. They kept professional journals and had their students keep diaries. Emphasis in both projects was on the development of social activities and contexts which encouraged students to discuss, question and probe mathematical ideas. Wilson (1989) claimed that success in long-term research projects such as these, aimed at the development of more effective teaching strategies, depends upon three critical variables: the extent of self-investment by the teacher, the level of support offered by the school and/or

an external facilitator, and an on-going commitment to nurture, evaluate and revise.

While individual teachers generally shared the results of classroom research with their colleagues willingly, the task of formal reporting on progress and linking the work with other current research and theories was often that of an academic. Stacey (1991), for instance, outlined some of the positive effects on students' problem-solving behaviour of long-term teaching through a problem solving approach in one Year 9 classroom. The teacher had worked over a number of years to develop his own problem-solving approach to the teaching of mathematics, with an emphasis on developing questioning attitudes and resourcefulness in students. Similarly, Dixon (1990) researched the possibilities of developing independent thinkers through teacher-written individual programs which account for different children's interests and methods of operation. These freed the teacher for one-to-one teaching and allowed for fluid learning according to children's needs.

In Penang, Malaysia, a group of teachers in a secondary school worked together with academics on a collaborative classroom-based professional development program. Focus on a number of discrete topics and how traditional teaching methods may be improved led to more effective teaching (Tan, Noh & Sim, 1991). Klein (1991) reported on a project where teachers, student teachers and school students are involved in the re-negotiation of familiar roles, patterns of discourse and pedagogical practices. This gave rise, claimed Klein, to contestation as well as reflection and the development of further understandings about teaching, the social contexts of teaching and possibilities for change.

From 1984, teachers working in post-graduate programs with Deakin University in Victoria undertook school-based action research projects. While their findings were not published individually, Mousley and Rice (1990) researched the longer-term effects of this requirement for eighty course participants. Not only did they find some ongoing restructuring of pedagogy, but also a growth in teachers' understandings of the nature of mathematics, the processes of teaching and learning mathematics, the power of institutional contexts and the processes of pedagogical change. Generally, the research by teachers underwent a shift in focus from educational product to the actual processes through which knowledge is gained, then to the origins of schooling practices in the context of wider social structures. Some participants moved on to investigate and confront the traditional social practices, both within schools and in the wider context of societal sub-cultures, which constrain innovation.

Another group of lecturers who undertook action research on their own teaching at tertiary level, Cooper & Smith (1987) studied the effects of, and processes of, their own decision-making about pedagogy. By involving student teachers in school-based field studies and reflections on these, the Brisbane lecturers attempted to integrate theory, practice, content and methods closely. They recognised that their own course content and teaching actions helped shape for student teachers not only beliefs about teaching but also perceptions

of mathematics itself.

PARTNERSHIPS IN CLASSROOM RESEARCH

During the last four years, teachers' research was facilitated by academics and mathematics education consultants. Some studies focused on recording the effects of particular teaching strategies. Groups of teachers in Tasmania in Australia and Exeter in the U.K., for instance, adapted their mathematics teaching methods and assessment for young students to more closely match their teaching of emergent writing (Edmunds & Stoessiger, 1987; Stoessiger & Edmunds, 1987; Stoessiger & Wilkinson, 1991). Wait time, or the duration of pauses in classroom discourse, was a research focus for a group of teachers working with Tobin (1987), with the change in practice appearing to lead to higher cognitive levels by providing both teachers and students with more time to think. Clarke, Stephens & Waywood (1989) recorded a group of nine teachers' reflections as they worked together to encourage students to write about their mathematics classes in daily diaries. Middleton (1991) worked with groups of teachers in attempting to improve teacher-student interaction and task explanations through the use of visual imagery, finding that change in the nature of classroom discourse necessitated a change in teachers' beliefs about what it is to teach and to learn.

It is not uncommon for participatory research to enter the realm of social critique. For example, Clark (1989) outlined how one teacher's introduction of calculators as an integral part of her teaching and raised issues regarding the purposes of traditional curriculum content. Issacs (1987) prompted groups of student teachers over a period of four years to undertake classroom research into their teaching behaviours during the introduction and control of problem-solving lessons. He reported that most teachers felt constrained originally by the set syllabus and were tentative about developing innovative practices, but that many continued to use problem solving after their participation in the research projects.

Other projects aimed at basing teachers' work on students' understandings. Teachers working with Hunt (1987), for example, focused on improving their skills in close observation of children at work, using the students' language, actions and understandings for further curriculum development. A team of academics and teachers (see Fraser, Malone & Neil, 1988, 1989) worked together to obtain feedback on students' perceptions of actual and preferred classroom environments, then used these as a basis for reflection upon, discussion of and systematic attempts to improve classroom environments. Jeffrey (1990) collected data on how children become actively involved in a problem with reflective abstraction, in order to inform her progress towards a more constructivist teaching style. Miller (e.g., Miller & England, 1989; Miller, 1990a, 1990b, 1991a, 1991b) reported that asking students to respond to written prompts about algebra proved another avenue of

teachers' obtaining a clearer picture of students' understandings and misconceptions, enabling appropriate teaching and remedial strategies to be developed.

In projects such as those above, when academics are involved in either initiating or facilitating action research, teachers were generally reluctant to accept control over the project. When the initial idea and the organisational factors (e.g., meetings of colleagues and keeping of records) remain in the locus of control of academics, teachers can certainly benefit from being involved. But the changes made to classroom practices are inclined to be linked with individual settings: there is little sense of collegiate energies and decision-making. Pateman (1989) claimed that the idea of teachers setting their own research agenda and making further decisions about direction, data collection and the use of data takes a while to take root; and that this process needs to be facilitated by the university staff. Henry (1989) reported on how four groups of teachers worked on developing techniques of process/inquiry teaching. Henry provided an extensive critique of the action research process and of an external facilitator's role as well as a detailed record of classroom interactions and teachers' reflections. He found that group meetings throughout collaborative action research projects became vital discursive moments where common meanings and intentions were established and accounts of classroom organisation, classroom activities and teacher behaviours were evoked.

Taylor and Fleming (1987) and Fleming (1988) described a Year 12 mathematics teacher's attempts to improve classroom effectiveness by modifying his traditional style towards a more student-centred pedagogy with the controlled introduction of aspects of constructivist practice. The major focus of this study was the teacher's actions as a reflection of his beliefs about the transmission of mathematical knowledge and skills. The teacher involved, Fleming, asked the question how the acquisition of mathematical knowledge by students can be enhanced by specific teaching strategies, then kept daily records of classroom interactions and his reflections. This action research led to a gradual reconstruction of the teacher's central concepts of teaching and a greater understanding of those strategies which can assist the development of a student-centred style of teaching. The reports of Taylor and Fleming (1987) used the action research approach to examining the conceptual dynamics of the processes of classroom-based curriculum change. Several cycles of development, trialing and feedback were undertaken by the teacher and academic. Taylor (1989) presented an account of his role in the development of a collaborative partnership of co-researchers.

Johnston (1988) supported groups of teachers in three schools in Western Australia while they examined their beliefs and practices, comparing these with the philosophical underpinnings of the State curriculum. She also concluded that teachers need the space and time to develop professionally and personally as they take control over this type of research, and need to feel that they are at the centre of decision-making processes. Skehan (1988) reported that as

teachers take control, they generally appreciate the professional growth which results from formulating and negotiating their own strategies for change.

Watson (1990) demonstrated strategies for relinquishing control while working with a group of teacher-researchers and community members in the development of the notion of Aboriginal education for the Yolngu community. Yolngu schools straddled a cultural interface, preparing students for productive lives in two social orders. Negotiating meanings of what it was to educate and to be educated as well as the relevance of different aspects of mathematical knowledge was quite a different process from, although inextricably linked with, negotiation of content and teaching methods. This involved Watson and her co-researchers in a dynamic process of coming to understand some components of each others' traditions as well as subtleties of language and meaning; and in the creation of a newly understood living knowledge. It also made problematic the roles of all researchers in decision-making about the "action" aspects of action research.

Similarly, in New Zealand, groups of Maori and non-Maori teachers, as well as interested community members worked together to develop mathematics curricula for Maori children. At first, emphasis was on adding a Maori dimension to the European curriculum, but the last few years saw progress towards mathematics being taught in the Maori language as well as the use of vocabulary, resources and teaching approaches within the Maori context (Barton, 1990).

TEACHERS AS CURRICULUM DEVELOPERS

The introduction of new teaching materials can act as a stimulus to classroom research. The *Reality in Mathematics Education* materials (Lowe & Lovitt, 1984; Ministry of Education, Victoria,1988-90), are examples of materials based on an alternative pedagogical philosophy. Research on the use of such materials (e.g., Caughey & Cribb, 1990) involved teachers in examining and changing their own perceptions of mathematics as well as in challenging traditional teaching strategies and content. Research projects undertaken by small groups of teachers helped overcome a problem identified by Pegg (1989) - an automatic assumption that exemplary lessons will fit the understandings, skills and professional contexts of all teachers.

Communicating calculus ideas clearly to all students, but particularly girls, was the focus of the joint project of Barnes and Johnston (1992), teachers and students. While the calculus text was initially written by Barnes, teachers and students became part of the research team as they worked together on further development and implementation strategies. Such research on classroom interaction is generally linked with critical theory in that it critiques and aims to change previously unquestioned practices, such as the use of unknown terms and symbols or gender-biased examples and applications.

An extensive project involving academics as initiators, but placing

teachers at the centre of the research process, was undertaken from 1985 to 1989 (Quinlan, 1986; Quinlan, Low & Llewellyn, 1987; Quinlan, Low, Sawyer & White, 1989). Year 7 and 8 mathematics teachers focused their attention on developing materials and strategies appropriate to the use of concrete manipulatives, problem solving and group discussions in the teaching of algebra. The aim was to improve teachers' roles in drawing closer links between verbal and symbolic statements and physical manipulatives; the use of student understandings was a key focus in this action research project. Their work resulted in the publication of class materials and edited videotapes, together with extensive papers on the research process.

Another project which facilitated research by teachers was entitled "Supporting teachers in change: The introduction of calculators as an agent for change in the teaching of primary mathematics" (see Beeby & Cheesman, 1991; Groves, 1991; Groves, Cheeseman, Clarke, & Hawkes, 1991; Groves, Ferres, Bergfeld & Salter, 1990). This ongoing project investigated the extent to which the ready availability of technology, coupled with systematic professional support, resulted in changes in teachers' practices, attitudes and beliefs about mathematics pedagogy. The research questions which formed the initial focus of the project were the extent and purpose of calculator use, changes of teachers' expectations of children's mathematical performance and consequent changes in the curriculum, and changes in teachers' beliefs and teaching practices.

Groups of teachers within schools have published sets materials and teaching strategies, making available the activities they have found successful. Space limitations and accessibility of material preclude further discussion of all of these in this chapter. They included *Activity Maths* (Eddy & Reiffel, 1989) and *LAPS Mathematics* (Hammond, Hastings, Martin & White, 1991). Gargiuolo (1990) reported that a group of teachers working in Auckland, New Zealand pooled ideas and set about establishing a collection of modules. The first stage in this process, though, had to be articulation and clarification of a common philosophy about mathematics as well as about mathematics learning.

GROUP RESEARCH IN SCHOOL IMPROVEMENT PROGRAMS

Over the past four years, the major impetus to classroom research throughout Australia and New Zealand was provided by school improvement programs which introduced teachers to current theories but allowed periods of time for trialing the ideas and practices in classroom settings before teachers met again to discuss the results.

For instance, the *Exploring Mathematics in Classrooms (EMIC)* (Ministry of Education, 1989) program involved teachers in Victoria working in small groups with tutors to examine and change their teaching of mathematics. Cranston, Dungan and Grieve (1989) reported on a Queensland program, the

1-10 Mathematics In-service Project (MIP) with thirty participant teachers. Both programs provided opportunities to consider, reflect on and discuss key issues before teachers carried out research in their own classrooms. Both programs were aimed at broadening teaching approaches to include more activity-based and problem-based teaching as well as discussions, so there was an element of central control over research foci.

The *Participation and Equity Program (PEP, 1987)* and *Girls and Maths and Science Teaching* project *(GAMAST)* (see Lewis, 1987) led to publication of the results of action research into gender-inclusive teaching (e.g., Shuttleworth, 1987). Findings of individual teachers (e.g., Pilley, 1988) or groups of schools (e.g., Boomer, Davison, Hoy, Mayer & Roe, 1987; Ministry of Education, Victoria, 1989) added to the body of knowledge and pool of resources which encourage greater access to mathematical ideas for all. Some of the groups of professionals shared their ideas during in-service sessions which they organised for teachers and student-teachers, while others saw the need to establish more extensive networks (see, for example, De Mestre & Richards, 1990).

A major initiative which placed teachers at the centre of educational decision-making (with mathematics education only one component) was the federally-funded professional development project entitled *Basic Learning in Primary Schools (BLIPS)* (see, Clements, 1989, p. 60). Different Australian states developed similar professional development programs, such as *Teachers Investigating Mathematics Education (TIME)* in New South Wales, with the common feature of running over a period of weeks and involving small groups of teachers in organising classroom research.

Another example, *Key Group*, (see Granet, de Vries & Shuttleworth, 1987; Grealy, Farina & Roffey, 1990; Pateman, 1989; Pegg & Redden, 1991; Robinson, 1989; Robinson & Alexopoulos, 1987; van der Spek, 1989) was also a school-based professional development program which supported groups of colleagues to improve mathematics teaching in whichever way they choose. The use of the model varied, but it generally involved the formation of small groups made up of three teachers from one school, an outside consultant and, in many cases, a parent. At the commencement of a project, a number of "key groups" participated in a three day live-in conference where they were introduced to current ideas about mathematics teaching. They also received some coaching about ways to undertake school-based change and the roles they might play in re-shaping institutional practice. After identifying their own strengths and weaknesses and deciding on a focus and plan of change for their school, each group returned to its school to implement the plans and continually evaluate progress for approximately five months. Support was provided by consultants, and a small seeding grant was available to cover some costs and some teacher-release time. A second conference allowed participants to report on progress, share experiences, demonstrate activities and materials that had been developed and to discuss the process of classroom research.

Out of Key Group came a number of reports prepared by teachers (e.g.,

Whyte, 1988) which demonstrate a range of foci. Rice (1990) investigated the Key Group process as an empowerment paradigm for teachers. She found that this way of working impacted on power relations within the school and prompted an awareness of, and access to, political processes in which there had been little previous involvement.

Another project which recognised the importance of integrating practical and theoretical components within the school and community environment was the *Remote Area Teacher Education Program*, available to Aboriginal people from Queensland and the Torres Strait Islands (see e.g., Putt, 1991). While space does not allow the description of all the classroom research situations involved, there was a general emphasis on negotiating curricula, for both trainee teachers and students, and the development of classroom materials which were compatible with and supportive of Aboriginal cultures.

A major project which involved community members in Australia and Papua New Guinea in organised workshops and on-going activities was the *Family Maths Project, Australia (FAMPA)* (Costello, 1986, 1988, 1990a, 1990b, Costello & Weal, 1989; Vasey, 1989). The focus for teachers here was the development of meaningful partnerships with parents - through better communication, involving parents in mathematics teaching at home (and frequently in classrooms) and through the provision of family workshops and challenging activities. The involvement of parents was taken a step further in the *Parents-as-Tutors Programme (PTP)* and *Individual Assistance Program (IAP)*, researched by the University of Canberra (see Hawkins, 1991), *Families Count* (Fowler & Robinson, 1990; Robinson & Fowler, 1989) and similar programs (e.g., Brick, 1988). These projects involved teachers and parents in the development of working partnerships to improve school programs.

It is evident that there is a growing trend amongst teachers, administrators, curriculum developers and academics towards the acceptance of the proposal that teachers have a substantive role to play in the development of our understanding of the teaching process. This has been reflected in the number of extensive, funded programs using school-based research as a model of professional development.

CONCLUSIONS

The initiatives reported above were all based on the assumption that teachers *can* conduct useful and rigorous research, thus not only effecting their own professional development but also contributing to the knowledge and expertise of others. The collaborative nature of participatory research was a strong feature of all of the projects, as were their extended time frames and opportunities for reflection.

As this chapter demonstrates, many Australian teachers have recently been involved in mathematics education research, for the benefit of themselves and their students as well as the wider profession. It is possible, though, that

Australasian curriculum developers and established researchers could play a greater part in facilitating the movement of teachers into the realm of recognised classroom research, with potential benefits for all. More teachers could then experience what Pateman (1989) described when he summarised two projects involving teacher-researchers:

> Research was viewed by the teachers ... as something familiar and useful rather than as the private province of a group of specialists whose main function is to translate the results of their research into other people's practice. (p. 38)

REFERENCES

Barnes, M., & Johnston, J. (1992, April). *Humanising curriculum: The development and evaluation of a gender-inclusive course and text.* Paper presented to the annual conference of the National Council for Teachers of Mathematics, Nashville, TN.

Barton, W. (1990). He matauranga tau ahua reorua: He kitenga o tetahi kaiako [Bilingual mathematics education: A practitioners point of view] *SAME papers, 1990.* Hamilton, New Zealand: Centre for Science and Mathematics Education Research.

Baxter, J. (Project director) (1988). *Teacher education mathematics project: A report.* Sturt: Australian Association of Mathematics Teachers.

Beeby, T., & Cheesman, J. (1991, July). *Calculators as an agent for change: The Victoria College calculator project.* Paper presented to the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia.

Boomer, G., Davison, C., Hoy, K., Mayer, B., & Roe, R. (1987). *But this isn't science, Miss: A language across the curriculum project.* Melbourne, Victoria: Ministry of Education (Schools Division).

Brick, L. (1988). Organising a school-based family math session. In D. Firth (Ed.), *Maths counts: Who cares?* (pp. 80-82). Melbourne, Victoria: Mathematical Association of Victoria.

Burrows, D. (1991). How to plan interactive adventure games. In J. O'Reilly, & S. Wettenhall (Eds.), *Mathematics: Inclusive, dynamic, exciting, active stimulating* (pp. 30-34). Melbourne, Victoria: Mathematical Association of Victoria.

Burnett, L. (1991). *The implementation of a school-based program using manipulatives, hands-on activities and instructional games to develop early numeracy skills.* Unpublished honours thesis, Griffith University, Brisbane, Qld.

Caughey, W., & Cribb, P. (1990). *Primary RIME.* Melbourne, Victoria: Victorian Ministry of Education (School Programs Division).

Chartres, M. (1988). What is mathematics: A personal perspective. *The*

Australian Mathematics Teacher, 44(3), 7-9.

Charters, M., & Nancarrow, R. (1988). *Classroom experiences: Facilitating change in mathematics curriculum in South Australia.* Paper presented to the eleventh annual conference of the Mathematics Education Research Association of Australasia, Geelong, Victoria.

Clarke, D., Stephens, M., & Waywood, A. (1989). Improving students' learning in mathematics through diaries, journals and other communications. In B. Doig (Ed.), *Everyone counts* (pp. 454-460). Melbourne, Victoria: Mathematical Association of Victoria.

Clark, O. (1989).Calulators in the primary school. *Prime Number 4*(1), 6-11.

Cooper, T., & Smith, R. (1987). Towards a complete primary mathematics education program: A summary of major issues. *Mathematics Curricula: Change and Challenge.* Papers and Reports from the seventh Biennial Conference of the Mathematics Education Lecturers Association, Townsville, Queensland.

Costello, P. (1986). FAMPA - The beautiful monster. In N. F. Ellerton (Ed.), *Mathematics: Who needs what?* (pp. 136-144). Melbourne, Victoria: Mathematical Association of Victoria.

Costello, P. (1988). The Family Maths Project Australia (FAMPA). In J. Baxter (Project director), *Teacher education mathematics project: A report.* (pp. 75-79). Sturt: Australian Association of Mathematics Teachers.

Costello, P. (1990a). Family maths. *The Australian Mathematics Teacher, 46(4),* 13-14.

Costello, P. (1990b). Family Maths. In K. Milton & H. McCann (Eds.), *Mathematical turning points: Strategies for the 1990s* (Vol. 1)(pp. 190-193). Hobart, Tasmania: Australian Association of Mathematics Teachers.

Costello, P., & Weal, S. (1989). The equation: Children + parents + teachers + maths = fun + interest + confidence. In B. Doig (Ed.), *Everyone counts* (pp. 65-73). Melbourne, Victoria: Mathematical Association of Victoria.

Cranston, N., Dungan, J., & Grieve, C. (1989). *A review of the 1-10 mathematics inservice project: An interim report on an innovation in teacher in-service education.* Brisbane, Queensland: Department of Education, Queensland. (Division of Curriculum Services).

Currie, T. (1991). Winning the maths game: Teamwork scores goals. In J. O'Reilly & S. Wettenhall (Eds.), *Mathematics: Inclusive, dynamic, exciting, active stimulating* (pp. 210-215). Melbourne, Victoria: Mathematical Association of Victoria.

Daly, T., & Cody, M. (1990). The Lilydale Secondary College VCE Mathematics trial experience 1989-1991. In M. A. Clements (Ed.), *Whither mathematics?* (pp. 89-97). Melbourne, Victoria: Mathematical Association of Victoria.

De Mestre, N., & Richards, M. (1990). Problem-solving task centres. *The Australian Mathematics Teacher, 46*(2), 8-9.

Dixon, M. (1990). Individual programming in the mathematics classroom. In M. A. Clements (Ed.), *Whither mathematics?* (pp. 420-423). Melbourne, Victoria: Mathematical Association of Victoria.

Eddy, C., & Reiffel, S. (1989). Activity-maths. In B. Doig (Ed.), *Everyone counts* (p. 36-37). Melbourne, Victoria: Mathematical Association of Victoria.

Edmunds, J., & Stoessiger, R. (1987). Using natural learning processes in mathematics. *Mathematics in School, 19,* 30-37.

Emery, M. (1986). Introduction. In M. Emery (Ed.), *Qualitative research: Papers from a symposium*, Canberra, Australian Capital Territory: Australian Association for Adult Education.

Fleming, B. (1988). Project report: *Enhancing the teaching-learning process in mathematics through the development of a constructivist teaching style*. Perth, Western Australia: Curtin University of Technology.

Float, M., & Harvey, D. (1991). Learning and problem solving in mathematics. In J. O'Reilly & S. Wettenhall (Eds.), *Mathematics: Inclusive, dynamic, exciting, active stimulating* (pp. 216-220). Melbourne, Victoria: Mathematical Association of Victoria.

Fowler, H., & Robinson, I. (1990). Families counting on. In M. A. Clements (Ed.), *Whither mathematics?* (pp. 285-286). Melbourne, Victoria: Mathematical Association of Victoria.

Francesconi, J., Willis, J., & Money, R. (1990). Problem solving at Catholic Ladies College: An interview with Year 10 students. *Vinculum, 27*(1), 4-6.

Fraser, B. J., Malone, J. A., & Neil, J. M. (1988). Observing and improving the mathematics classroom environment. *Research in Mathematics Education in Australia*, 8-15.

Fraser, B. J., Malone, J. A., & Neil, J. M. (1989). Assessing and improving the psychosocial environment of mathematics classrooms. *Journal for Research in Mathematics Education, 20*(2), 191-201.

Gargiuolo, S. (1990). Teaching of mathematics to Year 11 students: A New Zealand experience. In M. A. Clements (Ed.), *Whither mathematics?* (pp. 438-440). Melbourne, Victoria: Mathematical Association of Victoria.

Granet, A., de Vries, L., & Shuttleworth, V. (1987). Blips counts. *Education Victoria, 2*(3), 6.

Grealy, R., Farina, L., & Roffey, C. (1990). Take-home maths cards. *Prime Number, 5*(1), 11-15.

Groves, S., Cheeseman, J., Clarke, C., & Hawkes, J. (1991). Using calculators with young children. In J. O'Reilly & S. Wettenhall (Eds.), *Mathematics: Inclusive, dynamic, exciting, active stimulating* (pp. 325-333). Melbourne, Victoria: Mathematical Association of Victoria.

Groves, S., Ferres, G., Bergfeld, S., & Salter, S. (1990). Calculators in the infant school: The Victoria College calculator project. In M. A. Clements (Ed.), *Whither mathematics?* (pp. 244-250). Melbourne, Victoria: Mathematical Association of Victoria.

Groves, S. (1991, November). *Calculators as an agent for change in the teaching of primary mathematics*. Paper presented to the annual meeting of the Australian Association for Educational Research, Gold Coast, Queensland.

Hammond,P., Hastings, M., Martin, B., & White, H. (1991). LAPS mathematics: The overlapping of number, measurement and space _ Its development and implementation. In J. O'Reilly & S. Wettenhall (Eds.), *Mathematics: Inclusive, dynamic, exciting, active stimulating* (pp. 119-124). Melbourne, Victoria: Mathematical Association of Victoria.

Hawkins, W. (1991, July). *Parents as tutors of mathematics*. Paper presented to the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Aurtralia.

Henry, J. A. (1989). *A critical analysis of action research-based inservice teacher education*. Unpublished doctoral dissertation, Deakin University, Geelong, Victoria.

Hodgson, B., & Richards, M. (1989). A thematic approach to VCE mathematics. In B. Doig (Ed.), *Everyone counts* (pp. 346-349). Melbourne, Victoria: Mathematical Association of Victoria.

Hunt, P. F. (1987). *Teacher perceptions about the use of observation in mathematics*. Unpublished master's thesis, Deakin University, Geelong, Victoria.

Issacs, I. (1987). Preparing mathematics teacher to teach problem solving using action research. *Mathematics curricula: Change and challenge*. Papers and Reports from the seventh Biennial Conference of the Mathematics Education Lecturers Association, Townsville, Queensland.

James, R. (1991). Problem solving: From puzzles to CATS. In J. O'Reilly & S. Wettenhall (Eds.), *Mathematics: Inclusive, dynamic, exciting, active stimulating* (pp. 189-197). Melbourne Victoria: Mathematical Association of Victoria.

Jeffrey, P. (1990). Constructivism, autonomy and mathematics education. In K. Milton & H. McCann (Eds.), *Mathematical Turning Points: Strategies for the 1990s* (Vol. 2, pp. 344-347). Hobart, Tasmania: Australian Association of Mathematics Teachers.

Johnston, L., Money, R., Nixon, B., & Pearson, J. (1990). Reasoning and Data, Units 3 and 4. In M. A. Clements (Ed.), *Whither mathematics?* (pp. 71-76). Melbourne, Victoria: Mathematical Association of Victoria.

Johnston, J. (1988). *Development and evaluation of an in-service program for secondary mathematics teachers: An exploratory study*. Unpublished master's thesis, University of Western Australia, Perth, Western Australia.

Kidric, M., & Galvin, G. (1989). *Prime Number 4*(1), 3-5.

Klein, M. (1991, July). *An investigation of constructivism and pre-service teacher education through action research*. Paper presented to the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia.

Leigh-Lancaster, D. (1990). Change (VCE) and Approximation (What we do). In M. A. Clements (Ed.), *Whither mathematics?* (pp. 98-103). Melbourne, Victoria: Mathematical Association of Victoria.

Libertino, J. (1989). *Prime Number, 3*(3), 13-16.

Lovitt, C., & Clarke, D. (1986). *MCTP: First year report*. Canberra, Australian Capital Territory: Curriculum Development Centre.

Lowe, I., & Lovitt, C. (1984). *RIME lesson pack*. Melbourne, Victoria: Curriculum Branch, Ministry of Education.

Maltzahn, K. (1990). Evaluation without testing. In M. A. Clements (Ed.), *Whither mathematics?* (pp. 418-419). Melbourne, Victoria: Mathematical Association of Victoria.

Messinbird, B. L. (1991). *An investigation into the use of structured games and heuristic activities to develop fraction concepts at the Year 4 level*. Unpublished master's thesis, Griffith University, Brisbane, Queensland.

Middleton, H. (1991, November). *The role of visual thinking and visual mediation in three-dimensional problem solving: Some theoretical issues*. Paper presented to the annual conference of the Australian Association for Research in Education, Gold Coast, Queensland.

Miller, L. D. (1990a). Writing to learn: Suggestions for classroom implementation. In M. A. Clements (Ed.), *Whither mathematics?* (pp. 364-369). Melbourne, Victoria: Mathematical Association of Victoria.

Miller, L. D. (1990b). Writing to learn mathematics. In K. Milton & H. McCann (Eds.), *Mathematical turning points: Strategies for the 1990s* (Vol. 2)(pp. 404-411). Hobart, Tasmania: Australian Association of Mathematics Teachers.

Miller, L. D. (1991a). When students write in algebra class. *The Australian Mathematics Teacher, 46*(2), 4-7.

Miller, L. D. (1991b). Constructing pedagogical content knowledge from students' writing in secondary mathematics. *Mathematics Education Research Journal, 3*(1), 30- 45.

Miller, L. D., & England, D.A. (1989). Writing to learn Algebra. *School Science and Mathematics, 89*(4), 199-312.

Ministry of Education, Victoria (1988-90). *Production series: Reality in mathematics education*. Melbourne, Victoria: Ministry of Education.

Ministry of Education, Victoria (1989). *Exploring Mathematics in Classrooms [EMIC]*. Melbourne, Victoria: Ministry of Education (Office of Schools Administration).

Mousley, J., & Rice, M. (1990). Pedagogical evaluation and change: teaching and research in mathematics distance education. In T. Evans (Ed.), *Research in Distance Education, 1* (pp. 180-199). Geelong, Victoria: Deakin University.

Mulligan, J. (1988). Evaluation of trialling of Primary RIME: In-service teacher education project. *Teacher Education Mathematics Project*. Sturt: Australian Association of Mathematics Teachers.

Nulty, L., & Williams, K. (1988, p.27). A conference approach to teaching

mathematics. In D. Firth (Ed.), *Maths counts: Who cares?* (pp. 27-29). Melbourne, Victoria: Mathematical Association of Victoria.

Oliver, D., & Byers, L. (1991). A thematic approach to function modelling at Year 9. In J. O'Reilly and S. Wettenhall (Eds.), *Mathematics: Inclusive, dynamic, exciting, active stimulating* (pp. 185-188). Melbourne, Victoria: Mathematical Association of Victoria.

Omar, W., & De Sensi, M. (1988). Modelling 7-10 and above within a goal-based learning approach. In D. Firth (Ed.), *Maths counts: Who cares?* (pp. 90-93). Melbourne, Victoria: Mathematical Association of Victoria.

Participation and Equity Program (1987). *Recreating maths and science for girls*. Melbourne, Victoria: Ministry of Education.

Pateman, N. (1989). Research into practice: Teachers researching their mathematics classrooms. *Arithmetic Teacher 37*(2), 36-38.

Pegg, J. E. (1989). Analysing a mathematics lesson to provide a vehicle for improving teaching practice. *Mathematics Education Research Journal, 1*(2), 17 -33.

Pegg, J., & Redden, E. (1991, July). *Two case studies in the professional in-service development of secondary mathematics teachers: The ITAM Project*. Paper presented to the nineth biennial conference of the Mathematics Education Lecturers' Association, Perth, Western Australia.

Pilley, T. (1988). Maths, science, technology: increasing student interest and participation. *Education Victoria, 3*, 7-10.

Putt, I. (1991, July). *Teaching mathematics at a distance using multimedia information technology*. Paper presented to the nineth biennial conference of the Mathematics Education Lecturers' Association, Perth, Western Australia.

Quinlan, C. (1986). Report on Algebra research group. *Reflections, 11*(3), 49-51.

Quinlan, C. Low, B., & Llewellyn, B. (1987). Action research into teaching algebra. *Research in Mathematics Education in Australia, 3*, 57-61.

Quinlan, C., Low, B. Sawyer, E., & White, P. (1989). *A Concrete Approach to Algebra*. Sydney, New South Wales: Mathematical Association of New South Wales.

Rice, M. (1990, July). *Key Group - How empowering is it? A case study in the professional development of teachers*. Paper presented to the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

Richards, M. J. (1987). Catering for mixed ability classes - An innovative program at Mornington Technical School. *Vinculum, 24*(2), 30-31.

Robinson, I., & Fowler, H. (1989). Parental involvement in mathematics education: Families count. In B. Doig (Ed.), *Everyone counts* (pp. 119-121). Melbourne, Victoria: Mathematical Association of Victoria.

Robinson, I. (1989). The empowerment paradigm for the professional development of teachers of mathematics. In N.F. Ellerton & M.A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 269 -

283). Geelong, Victoria: Deakin University.

Robinson, I., & Alexopoulos, N. (1987). *Keygroup: Empowering teachers to change*. Melbourne, Victoria: Ministry of Education.

Shuttleworth, V. (1987). Maths and science project to improve girl's participation. *Education Victoria, 2*(6), 3.

Skehan, S. (1988, July). *Action research in action at St Michael and St John's Primary School, Horsham*. Paper presented to the eleventh annual conference of the Mathematics Education Research Association of Australasia, Geelong, Victoria.

Skinner, P. (1990). *What's your problem?: Posing and solving mathematical problems in junior classes*. South Melbourne, Victoria: Nelson.

Stacey, K. (1991). The effects on students' problem-solving behaviour of long-term teaching through a problem solving approach. In *Proceedings of the fifteenth annual conference of the International Group for the Psychology of Education* (Vol. 3)(pp. 278 - 285), Assissi, Italy,

Stewart, A. (1989). Thematic studies in the senior curriculum. In B. Doig (Ed.), *Everyone counts* (pp. 327-328). Melbourne, Victoria: Mathematical Association of Victoria.

Stoessiger, R., & Edmunds, J. (1987). *Investigating a process approach to mathematics*. Hobart, Tasmania: Education Department of Tasmania.

Stoessiger, R., & Wilkinson, M. (1991). Emergent mathematics. *Education 3-13, 19*(1), 37-39.

Swan, C. (1988). Children's literature: An aid to mathematical understanding. In D. Firth (Ed.), *Maths counts: Who cares*? (pp. 74-76). Melbourne, Victoria: Mathematical Association of Victoria.

Tan, S. H., Noh, I. M., & Sim, J. T. (1991, July). *Teachers' professional; growth through collaborative action research - the TISM experience*. Paper presented to the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia.

Taylor, P. (1989, November). The influence of teacher beliefs on constructivist teaching practices. *Proceedings of the History and Philosophy of Science in Science Teaching First International Conference*. Miami: Florida State University.

Taylor, P. C., & Fleming, B. (1987). Improving teacher effectiveness: a collaborative approach. In *Educational research: scientific or political?: Proceedings of the joint conference of the Australian Association for Research in Education and the New Zealand Association for Research in Education*, Christchurch, New Zealand.

Tobin, K. (1987). The role of wait time in higher cognitive level learning. *Review of Educational Research 57*(1), 69-95.

van der Spek, A. (1989). Active, enjoyable and noisy mathematics. *Prime Number, 3*(3), 20-23.

Vasey, J. (1989) Setting up a family maths library and resource centre. In B. Doig (Ed.), *Everyone counts* (pp. 139-145). Melbourne, Victoria: Mathematical Association of Victoria.

Waters, M., & Montgomery, P. (1989). Simulating the early learning environment in a maths classroom. In B. Doig (Ed.), *Everyone Counts* (pp. 78-85). Melbourne, Victoria: Mathematical Association of Victoria.

Waters, M., & Montgomery, P (1990). Bringing learners and mathematics together: A community of mathematics. In K. Milton & H. McCann (Eds.), *Mathematical Turning Points: Strategies for the 1990s* (Vol. 2)(pp. 561-565). Hobart, Tasmania: Australian Association of Mathematics Teachers.

Waters, M., & Montgomery, P. (In press). *Language learning and mathematics learning.*

Watson, H. (1990). The Ganma project: Research in mathematics education by the Yolngu Community in the schools of the Laynhapuy (N.E. Arnhemland). In R. Hunting & G. Davis (Eds.), *Language issues in learning and teaching mathematics. Progress in Mathematics Education Series,1* (pp. 33: 50). Bundoora, Victoria: La Trobe University.

Whyte, G. Language learning and mathematics: Cross-age tutoring at Wollert Primary School. *Prime Number, 3*(2), 6-8.

Williams, G. (1990). Extensions (Change and approximation). In M. A. Clements (Ed.), *Whither mathematics?* (pp. 66-70). Melbourne, Victoria: Mathematical Association of Victoria.

Wilson, P.S. (1989). Critical variables for teacher involvement in curricular change. In J. Malone, H. Burkhart, & C. Keital (Eds.), *The Mathematics curriculum: Towards the Year 2000* (pp. 331-336). Perth, Western Australia: Curtain University of Technology.

Wirth, E. (1991). Putting magic into maths and keeping it there. In J. O'Reilly & S. Wettenhall (Eds.), *Mathematics: Inclusive, dynamic, exciting, active stimulating* (pp. 23-29). Melbourne, Victoria: Mathematical Association of Victoria.

PART II

Issues in Learning and Instruction

COGNITIVE STUDIES IN MATHEMATICS EDUCATION

Lyn D. English-Halford
Centre for Mathematics and Science Education
Queensland University of Technology

The 1988 review of mathematics education research in Australia (Blane and Leder, 1988) addressed three broad areas, namely, gender and mathematics, psychology of mathematics education, and problem solving. With the increased number of chapters in the present review, this chapter on cognitive studies will not address the wide range of topics covered in the 1988 psychology section. Several of the 1988 topics, particularly those dealing with social aspects of mathematics education, will not be examined here.

Prior to selecting items for review in this chapter, it was necessary to define the parameters of cognitive studies in mathematics education. Given that other chapters in this volume also address cognitive components, e.g., learning strategies, it was decided to confine this chapter to research dealing specifically with cognition, cognitive models, and mathematics education. Several of the papers reviewed incorporated the terms "cognitive" and "cognition" in their title, yet devoted very little discussion to cognitive issues. It was felt that these papers would be more appropriately examined in one of the other chapters. Hence the research reviewed in this chapter addresses two major areas in cognitive studies and mathematics education: *cognition and problem solving* and *cognitive models of mathematical learning*. A smaller third section dealing with research articles which have some relevance to this chapter but do not belong to either of these two categories has also been included. Although a comprehensive search of the literature was undertaken, there will no doubt be important items of research which have been omitted from this review. Any such omissions are regretted.

Probably the most significant change in cognitive studies in Australian mathematics education during the last decade has been an increased emphasis on in-depth analyses of children's responses to thought-provoking mathematical tasks. Mathematics education has moved away from a reliance on the norm-referenced results of large scale testing in schools. Among other things, mathematics education research in Australia is concerned with the structure of students' mathematical knowledge (including their misconceptions) and the ways in which they acquire this knowledge. Analyses of students' responses to novel mathematical problems has received attention, along with the application of particular cognitive models to students' learning in different

mathematical domains. The remainder of this chapter is devoted to a review of selected research within these areas.

COGNITION AND PROBLEM SOLVING

Continued interest in mathematical problem solving has been evident during the last four years with an emphasis on structured interviews to probe students' thought processes and the use of protocol analysis frameworks to record and analyse observed behaviours (e.g., English, 1988, 1990; Yee, 1990). Yee proposed a framework for analysing think-aloud data from performance on particular non-routine mathematical problems. His taxonomy comprised cognitive, metacognitive, and affective behavioural categories with five major dimensions: problem-orientation heuristics, problem-solution heuristics, domain-specific knowledge, metacognition, and affective behaviour. Yee claimed that such a taxonomy enables in-depth analysis of a student's problem-solving performance as well as a global analysis of coherent episodes of behaviour.

Chinnappan and Lawson (1990) also developed a framework for analysing protocols generated from students' think-aloud solutions to geometric problems. Their framework focused on a number of classes of events: problem analysis, problem representation, planning, monitoring of progress, activation of knowledge and manipulation of that knowledge, review of progress, errors and self-assessment. They observed small groups of low-achieving and high-achieving Year 11 students solving four plane-geometry problems under think-aloud instructions. The results suggested that high-achieving students not only show a more fully developed knowledge base relevant to the geometry problems but are also more active in their analysis and identification of problem features, in regulating their course of problem solving during the solution process, and in particular, in activating knowledge relevant to the problem. Furthermore, these high-achieving students generated more problem-related knowledge through retrieval and manipulate this knowledge more actively than did their low-achieving peers.

Structured interviews and protocol analyses were used by English (1988, 1990, 1991a, 1991b, 1991c, in press) in a series of studies to investigate children's competence in solving novel combinatorial problems. In one of these studies (English, in press) 72 children aged 4 to 9 years were individually administered a set of six combinatorial problems involving the dressing of toy bears in all possible combinations of clothing items. Because the problem domain was novel, the children had to use their existing general strategies to help them solve the problems. Analyses of the children's responses revealed a series of increasingly sophisticated solution strategies - reflecting a knowledge of the combinatorial domain-plus a number of scanning actions serving primarily in a monitoring capacity-reflecting an application of general strategies. Significant associations were found between children's solution

strategies and their scanning actions on each problem, with the children changing the nature of their scanning as they adopted more complex solution strategies. The nature of this association was a key factor in problem success, especially when there was an additional constraint on goal attainment. The findings of the study were examined in terms of changes in children's principled knowledge base and in the nature of their general strategies. Cases in which children failed a problem in spite of their sophisticated domain knowledge highlight the importance of children applying the appropriate domain-general strategies in both novel and routine problem solving.

In another study (English, 1991a), the competence of older children (7 to 12 years) in solving more complex combinatorial problems was investigated. Of interest here was children's ability to transfer from two-dimensional (X x Y) to three-dimensional (XxYxZ) combinatorial problems. The children's responses to the 3-D problems revealed a series of five increasingly sophisticated solution strategies. These extended the five strategies identified previously, yielding a hierarchy of 10 combinatorial strategies. It was found that the 7 to 9 year-olds showed significant improvement as they progressed from the 2-D to the 3-D problems, while the 10 to 12 year-olds did not demonstrate as great an improvement. This latter result was due largely to the older children adopting sophisticated strategies on the 2-D problems (although they had not been formally taught these) and continuing to apply such methods to the 3-D problems. Experience in solving the set of 2-D problems appeared to encourage the children to adopt efficient methods for solving the 3-D problems. This finding was supported by the responses of a control group of children who solved only two 3-D problems. The experimental group performed significantly better than the control group when introduced to the 3-D problems.

Continued interest in students' use of metacognitive or executive processes is evident in the studies of Wong (1989) and Herrington (1990). Wong investigated whether students from different academic settings differ in their use of metacognitive strategies in mathematical problem solving, and if so, the nature of these differences. Like English (1992), Wong argued that knowledge alone, is insufficient for successful problem solving and that learners also need to apply higher-level cognitive skills such as metacognitive strategies. These generalised strategies include those of planning, monitoring, controlling, selecting, and evaluating cognitive activities. In a study spanning two years, Wong investigated the metacognitive strategies used by over a thousand secondary and pre-university students from 12 schools. Students from different grade levels and academic streams were administered a questionnaire requiring them to self-report on their metacognitive beliefs, their usage of metacognitive strategies in mental tasks involving memory, problem solving, and comprehension, and their attitudes towards the learning of various academic subjects. Most students indicated that they were conscious of metacognitive processes and used strategies for monitoring and regulating the processes necessary for problem solving. However it was found that Normal stream

students (5-year secondary school education) exhibited a lower usage of metacognitive strategies than those in the Express stream (4-year secondary course) and Special stream (special assistance program). No significant differences in the frequency of usage of metacognitive strategies were found between students from different academic tracks (General, Science, Arts). It was also found that students from different grade levels in the secondary school exhibited similar frequency of usage of metacognitive strategies in problem solving. Given that students reported less frequent use of monitoring strategies in planning, executing, and orientation, Wong argued for the inclusion of guided instruction in the use of metacognitive strategies for problem solving. In particular, lower ability students required greater assistance in the application of these strategies.

Herrington (1990) used a metacognitive framework based on Flavell's (1979) three categories (person, task, strategy) to explore seventh grade students' beliefs about learning mathematics. He investigated how students perceive themselves and others as learners of mathematics (person awareness), how they perceived the learning of mathematics as distinct from other subjects (task awareness), and how they perceived mathematics could be learnt (strategy awareness). Responses from interviews with students suggested that upper primary school children had well-formed views on the process of learning mathematics and were able to confidently express them. Students appeared to be aware of their mathematical abilities which tended to match results on teacher-developed tests.

A simulated recall technique was used in a study by Southwell (1990) to investigate the problem-solving performance of teacher education students. A number of variables were examined, including personality characteristics, motivation, beliefs, problem-solving strategies, use of concrete materials, language, and reasoning skills. The results showed that all variables appeared to influence students' performance in solving the problems. One interesting finding was the students' belief that mathematics is not being used when mathematical thinking is applied to the resolution of a problem. Southwell is conducting further research on the nature and effect of problem solvers' beliefs on their performance and their use of reflective processes in the problem-solving situation.

While the previous studies have highlighted the importance of general problem-solving strategies, not all of the Australian research fraternity acknowledge their existence or value. Sweller (1988, 1990) and Owen and Sweller (1989) decried the use of general cognitive strategies and emphasise the importance of the domain-specificity of problem-solving skills. Sweller contended that skilled problem solving requires two components: schema acquisition and rule automation. "Without automation, problem solving is difficult, prone to errors, and most of all, very inflexible" (Sweller, 1989, p. 458). Sweller claimed that the popular means-ends strategy has deleterious consequences on schema acquisition and rule automation because it focuses attention inappropriately and imposes a heavy cognitive load. He thus argued

for the use of goal-free problems and worked examples and provided evidence (Sweller, 1988) via a computational model that such problems reduce cognitive load and facilitate learning.

Adopting a somewhat similar stance to Sweller, Long (1989) attributed success in algebra problem solving to the development of a "well-coordinated, multifaceted knowledge base", not to the development of general skills. Long maintained that naive representations or misconceptions feature significantly in learners' construction of mathematical knowledge. In an effort to address this issue, Long developed a program, *Talking Maths*, for use in secondary school algebra classrooms. The program emphasised the development of knowledge organisation within meaningful contexts involving practical applications. While the work of Sweller, Owen, and Long provide valuable insights into students' problem solving, their studies focused largely on routine problems requiring the application of specific rules. Consideration also needs to be given to novel problem solving, given that this is a significant component of the mathematics curriculum.

The claims against generalised skills have not remained unchallenged. In response to Owen and Sweller's (1989) claims, Lawson (1990) argued that general problem-solving strategies play an important role in learning and transfer and should be included in the mathematics curriculum. In support of his argument, Lawson distinguished among three different types of general problem-solving strategies which "have a general sphere of influence on cognitive activity during problem solving and should be seen as distinct from strategies specific to a particular task" (p. 404). Task orientation strategies influence the dispositional state of the student and include the broad affective, attitudinal, and attributional expectations held by the student about a particular task. Executive strategies are concerned with the planning and monitoring of cognitive activity, while domain-specific strategies include heuristics such as means-ends analysis and other procedures developed by the problem solver for organising and transforming knowledge (e.g., constructing a table or drawing a diagram).

COGNITIVE MODELS OF MATHEMATICAL LEARNING

Two prominent models of cognitive development which have been applied to mathematics education are Halford's analogical reasoning model (in press a; in press b) and Biggs and Collis' SOLO Taxonomy (1982, 1991). Halford's developmental theory, which emphasises the role of understanding, has been applied to mathematics education by Boulton-Lewis and Halford (1991) and Halford and Boulton-Lewis (in press). The essence of the theory is that understanding entails having a representation or mental model which mirrors the structure of the material. The mental model can be used to guide learning, including the development of strategies which, once developed, can function independently of the understanding which gave rise to them. Understanding

will be invoked if strategies have to be modified to fit changing circumstances, but it is not routinely invoked because of the processing load it imposes. The approach also draws heavily on the theory of analogies (Gentner, 1983), which are argued to be the source of many mental models. The mapping of a mental model into a problem, which is essential to understanding, imposes a processing load which depends on the complexity of the structure being mapped, where complexity is defined as the number of independent aspects or dimensions that must be processing in parallel.

According to Halford's theory concrete experience with, for example, sets, provides the basis for elementary understanding of number. However the child does not need to know about sets per se, but can use them as analogs of number operations. Analogy theory and complexity theory account for benefits and limitations of concrete analogs in many situations (Halford and Boulton-Lewis, in press). According to this view concrete teaching aids are analogs, and the reason many apparently promising aids fail is that an excessive processing load is imposed in mapping them into the material which the child is attempting to understand. A similar point is made concerning arithmetic word problems (Halford, in press a). The oft-noted paradox that making arithmetic problems "concrete" by casting them into little stories actually makes them harder, is partly due to the processing load entailed in mapping the problem into the mental model. In general, the effectiveness of concrete aids depends on the child recognizing the correspondence between the structure of the aid and the structure of the concept to be understood; in technical terms, it depends on the child being able to map the structure of the aid into the structure of the concept. Children's success in doing this will depend on the way the structures are coded. Thus coding is a critical feature of understanding, and efficient coding can greatly reduce processing load.

Understanding of more advanced concepts, such as those entailed in algebra, incorporates the same principles, but is not based on concrete sets. Instead, arithmetic provides the mental model for algebraic relationships. Again, analogy theory is invoked to show that success in understanding algebra will depend on the degree to which the child can represent the structure of the arithmetic relationships which provide the mental model for algebra (Halford, in press a; Halford & Boulton-Lewis, in press).

The SOLO Taxonomy has been used widely in mathematics education to classify children's responses as well as to classify items or questions. The theory postulates five broad stages of development of increasing levels of abstraction: sensorimotor (from birth), ikonic (from 18 months), concrete symbolic (from 6 years), formal (from 16 years), and post-formal (from 18 years). The theory also maintains that the same learning cycle is repeated within each of these modes. These five levels of response structure, in increasing order of complexity, are: prestructural responses (comprising a single piece of irrelevant or inappropriate information), unistructural responses (use of only one piece of relevant information), multistructural responses (use of several relevant pieces of given information), relational responses

(integration of all relevant information given), and extended abstract responses (generalisation to abstract concepts and principles, in addition to integration of all relevant information).

While the taxonomy has been applied to a number of mathematical domains, only a few are reviewed here. Collis, Watson, & Campbell (1991) explored the interface between logical processes of the concrete symbolic mode and intuitive processes of the ikonic mode in the context of mathematical problem solving. Their ultimate aim was to design guidelines for teaching and assessment procedures in mathematical problem solving. They contended that much of our thinking in solving novel problems is multimodal. Students in the early high school years have available three modes of intellectual functioning which they can apply to the solution of a problem: sensor motor, ikonic, and concrete symbolic. In their study of the problem-solving performance of 14 and 15 year-old students, Collis et al. were particularly interested in the primary strategies that these students use to solve novel problems and whether these strategies are dependent on obvious problem features. Also of interest was whether specific individual characteristics, such as creativity, relate to the nature of the students' strategies. Two basic approaches to novel problem solving were identified, one based on the concrete symbolic, school-taught procedures and one related to everyday life, visualisation, and the use of intuitive experiences. In the course of solving a problem, students tended to move from one mode to the other, but were more likely to shift from the concrete symbolic to ikonic mode when a school-based method did not work. Collis et al. also found that the nature of the problem itself appeared as the basic factor in determining the primary strategy used. Finally, it was noted that students who were able to obtain clear visual representations of a problem and had high levels of creativity were most likely to be comfortable with ikonic approaches.

In another study, Campbell, Watson, and Collis (1990) applied the SOLO Taxonomy to primary and secondary school students' understanding of volume measurement. The students' responses indicated a repeating unistructural-multistructural-relational cycle within the concrete symbolic mode on the single concept of volume measurement, with a final transition to the formal mode marked by extended abstract responses. The results suggested that learners continued to operate in the various modes (sensorimotor, ikonic, and concrete-symbolic) alongside the higher modes as they were developing (Biggs & Collis, 1982). The interaction between modes was seen as beneficial to learning, with the extended development within modes facilitating this exchange.

Watson, Collis, and Campbell (1991) also applied the SOLO Taxonomy to concept development in common and decimal fractions. Their analysis of student performance suggested a two-part developmental sequence for each fraction type. The first culminated in an understanding of the basic fraction concept. The second cycle commenced with this construct and built further applications of the concept as it was required. Watson et al. hypothesised that two modes of functioning may be operating simultaneously in the solution of

problems involving fractions of each type. They further argued that the interaction of modes within each fraction type and between them might help explain the complexities associated with solving problems involving fractions and decimals. While their theory nicely explained some of the difficulties children experience with fractions, their explanation begs the question of whether these difficulties would exist if decimal and common fractions were taught appropriately, that is, as being derived from the same concept (not two separate concepts).

The SOLO Taxonomy has also been applied to preschoolers' ability to solve novel problems (Leder, 1991). A small sample of preschoolers was given a mathematical task in which they had to distribute letters, post letters, and estimate the number of letters in a letter box. They also had to "put something on paper" about the activity they had played. The students' responses were analysed using the SOLO Taxonomy, with the results indicating that children are capable of quite sophisticated mathematical reasoning, before the commencement of their formal schooling.

OTHER RESEARCH

Constructivism and the Mathematics Classroom

The constructivist approach to mathematics education has continued to receive attention from the Australian research community. Mousley (1990) addressed the concerns teachers have with the notion of constructivism where students are free to organise learning activities as well as construct their own mathematical concepts and operations. She reported on a study which supported the view that "there can be organisation amongst the 'chaos' of constructivism" (p. 60). She illustrated how the children's active involvement could be fostered at each stage of problem-posing, re-defining and recording problems, finding and sharing solutions, and looking at alternative ways of investigating mathematical concepts. Given the tension that teachers feel between a traditional curriculum and the more individualised, child-centred environment of the constructivist classroom, Mousley argued for more administrative support and more freedom for teachers willing to take the step towards this type of classroom.

An action research approach to developing constructivist-based classrooms has been advocated by Klein (1991). She claimed that for change to occur in schools, there must be change within pre-service teacher education courses and stressed the view that tertiary learning must be an active, problem-solving process.

Socio-Cognitive Conflict and Success-Based Theories of Learning

An approach which is similar in philosophy to the constructivist framework has been posited by Ellerton and Clements (1990). They reported on a case study of a primary school classroom in which socio-cognitive conflict and success-based theories of learning were applied to foster significant and worthwhile

mathematical learning. The socio-cognitive conflict theory argues that when children work in groups on meaningful tasks, the difference in cognitive levels of the children's responses creates a type of socio-cognitive conflict. Progress results from the need to coordinate these different responses and resolve the conflict. In success-based classrooms, teachers are accepting of what students do and encourage students to learn from other children through the process of osmosis and through discussion. Ellerton and Clements contended that classrooms based on these two theories offer children more than the standard cognitive-conflict teacher / textbook-owned approaches to school mathematics.

Attitudes to Teaching Mathematics

Studies addressing students' attitudes towards mathematics and mathematics teaching continue to appear in the literature although not as frequently as in past years. In a study of student teachers' attitudes towards mathematics teaching, Nisbet (1990) found that while primary student teachers do not have very positive attitudes towards the subject of mathematics, they nevertheless respond positively to questions relating to their confidence in teaching mathematics and their enjoyment in teaching it. He argued that attitudes towards mathematics and attitudes towards teaching mathematics are separate entities and should be measured separately. To this end, Nisbet (1991) has developed a list of new scales for measuring these attitudes.

Dengate (1991) has also developed an instrument which reveals key components of mathematics' teachers perceptions of teaching mathematics. His results indicated an increasing acknowledgment of the process aspects of teaching and learning mathematics, although there appeared to be some conservatism among seventh grade teachers who gave product aspects greatest credence.

CONCLUSIONS

The last decade has witnessed considerable growth in the application of cognitive psychology to mathematics education. However there are many issues within several topic areas that remain untapped (e.g., students' learning of statistics and graphs; students' use of analogical reasoning in solving algebraic examples). A wider range of cognitive theories and models need to be employed in the analyses of students' mathematical learning in both the primary and secondary sectors. The latter sector particularly needs attention.

One aspect of mathematics education which especially calls for increased research is the interplay between process and product. The *National Statement on Mathematics for Australian Schools* (Australian Educational Council, 1991) emphasises the importance of both products (body of knowledge) and processes (ways of knowing) in students' mathematical learning. The processes are the mathematical thinking skills which enable the products to be developed, applied, and communicated. In order to understand the nature of mathematics

and to be effective problem solvers, students need to experience and appreciate the interplay between mathematical products and processes (p. 26). In the past, school mathematics programs have tended to emphasise content at the expense of processes, resulting in fragmented and frequently meaningless learning (Alexander, 1990; National Council of Teachers of Mathematics, 1989; Resnick, 1987). While recent mathematics programs highlight the importance of both product and process, teachers are given little insight into how these two components interact to effect mathematical learning and little assistance in fostering this development. Until mathematics education research gives greater attention to this issue, many of the goals of our new syllabus documents will not be fully realised.

REFERENCES

Alexander, P. (1990). *A cognitive perspective on mathematics: Issues of perception, instruction, and assessment.* Paper presented at the NATO Advanced Research Workshop on Information Technologies and Problem Solving Research, Spain.

Australian Educational Council (1990). *A National Statement on Mathematics for Australian Schools.* Victoria: Curriculum Corporation.

Blane, D., & Leder, G. (1988). *Mathematics education in Australia: A selection of recent research.* Monash University, Victoria: Mathematics Education Research Group of Australia.

Biggs, J., & Collis, K. (1982). *Evaluating the quality of learning: The Solo Taxonomy.* New York: Academic Press.

Boulton-Lewis, G., & Halford, G. (1991). Processing capacity and school learning. In G. Evans (Ed.), *Learning and teaching cognitive skills,* (pp. 70-86). Melbourne, Victoria: Australian Council for Educational Research.

Campbell, J., Watson, J., & Collis, K. (1990, July). *Volume measurement and the SOLO Taxonomy.* Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

Chinnappan, M., & Lawson, M. (1990, July). *Activity during geometry problem solving.* Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

Collis, K., & Biggs, J. (1991). Developmental determinants of qualitative aspects of school learning. In G. Evans (Ed.), *Learning and teaching cognitive skills,* (pp. 185-207). Melbourne, Victoria: Australian Council for Educational Research.

Collis, K., Watson, J., & Campbell, J. (1991, November). *Cognitive functioning in mathematical problem solving during early adolescence.* Paper presented at the annual conference of the Australian Association for Research in Education, Gold Coast, Queensland.

Dengate, R. (1991, July). *Mathematics Teachers' perceptions: Key factors in the early secondary school.* Paper presented at the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia.

Ellerton, N., & Clements, K. (1990, July). *Cognitive conflict, socio-cognitive conflict, and success as bases for mathematical learning.* Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

English, L. D. (1988). *Young children's competence in solving novel combinatorial problems.* Unpublished PhD thesis, University of Queensland, Queensland.

English, L. D. (1990). Children's competence in forming combinations. In L. Steffe & T. Wood (Eds.), *International perspectives on transforming early childhood mathematics education,* (pp. 174-180). Hillsdale, New Jersey: Lawrence Erlbaum Associates.

English, L. D. (1991a). Children's strategies in solving two-and three-dimensional combinatorial problems. Submitted for publication.

English, L. D. (1991b). Young children's combinatoric strategies. *Educational Studies in Mathematics, 22,* 451-474.

English, L. D. (1991c). Young children as independent learners. In G. Evans (Ed.), *Learning and teaching cognitive skills,* (pp. 70-86). Melbourne, Victoria: Australian Council for Educational Research.

English, L. D. (in press). Children's use of domain-specific knowledge and domain-general strategies in novel problem solving. *British Journal of Educational Psychology.*

Flavell, J. (1979). Metacognition and cognitive monitoring: A new area of cognitive-developmental enquiry. *American Psychologist, 34,* 906-911.

Gentner, D. (1983). Structure-mapping: A theoretical framework for analogy. *Cognitive Science, 7,* 155-170.

Halford, G. (in press-a). *Children's understanding: The development of mental models.* Hillsdale, New Jersey: Lawrence Erlbaum.

Halford, G. (in press-b). An analogical reasoning account of cognitive development. *Human Development.*

Halford, G., & Boulton-Lewis, G. (in press). Value and limitations of analogs in teaching mathematics. In A. Demetriou, A. Efkliades, & M. Shayer (Eds.), *The modern theories of cognitive development go to school.* London: Routledge.

Herrington, A. (1990, July). *Children's beliefs about learning mathematics.* Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

Klein, M. (1991, July). *An investigation of constructivism and pre-service teacher education through action research.* Paper presented at the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia.

Lawson, J. (1990). The case for instruction in the use of general problem-solving strategies in mathematics teaching: A comment on Owen and Sweller. *Journal for Research in Mathematics Education, 21*(5), 403-410.

Leder, G. (1991). Preschoolers, problem solving, and mathematics. In F. Furinghetti (Ed.), *Proceedings of the fifteenth International Psychology of Mathematics Education conference* (pp. 277-284). Assisi, Italy.

Long, E. (1989, November). *The role of domain-specific knowledge in classroom problem solving.* Paper presented at the annual conference of the Australian Association for Research in Education, Adelaide, South Australia.

Mousley, J. (1990, July). *Organised chaos: Problem posing in a constructivist classroom.* Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

National Council of Teachers of Mathematics, (1989). *Curriculum and evaluation standards for school mathematics.* Reston, Virginia: USA.

Nisbet, S. (1990, July). *Attitudes to teaching mathematics.* Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

Nisbet, S. (1991). A new instrument to measure pre-service primary teachers' attitudes to teaching mathematics. *Mathematics Education Research Journal, 3*(2), 34-56.

Owen, E., & Sweller, J. (1989). Should problem solving be used as a learning device in mathematics? *Journal for Research in Mathematics Education, 20,* 322-328.

Resnick, L. (1989). *Education and learning to think.* Washington: National Academy Press.

Southwell, B. (1990, July). *Cognitive and affective aspects of mathematical problem solving: A case study.* Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australia, Hobart, Tasmania.

Sweller, J, (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science, 12,* 257-285.

Sweller, J. (1989). Cognitive technology: Some procedures for facilitating learning and problem solving in mathematics and science. *Journal of Educational Psychology, 81,* 57-466.

Sweller, J. (1990). On the limited evidence for the effectiveness of teaching general problem-solving strategies. *Journal for Research in Mathematics Education, 21,* 11-415.

Watson, J, Collis, K, & Campbell, J. (1991, November). *Concept development in fractions and decimals: Toward a theoretical model.* Paper presented at the annual conference of the Australian Association for Research in Education, Gold Coast, Queensland.

Wong, P. (1989, November). *Students' metacognition in mathematical problem solving.* Paper presented at the annual conference of the Australian Association for Research in Education, Adelaide, South Australia.

Yee, F. (1990, July). *A taxonomy of mathematical problem solving behaviours.* Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

RESEARCH IN LEARNING STRATEGIES IN MATHEMATICS

Khoon Yoong Wong
External Studies Unit
Murdoch University

Tony Herrington
Department of Mathematics Education
Edith Cowan University

Students use strategies to help them learn mathematics. However, learning strategies has been a much neglected research area in mathematics education. This is evident in the landmark review conducted by Begle (1979) who found only three studies about study behaviours of students in mathematics and recommended that further exploratory studies should be conducted in this area.

In the previous MERGA review of research in mathematics education, Jones (1988), discussing investigations on student variables, concentrated on attitudinal variables and previous mathematics achievement without mentioning strategies as an appropriate variable in students' learning. This chapter, too addresses this issue by first providing a theoretical model of learning incorporating strategies, reviewing the few relevant studies conducted recently, and concluding with suggestions for future research.

Like many other educational terms, the word *strategy* has many different meanings. Several other terms are also used to describe this research area, for example study approach, study behaviours, study skills, and thinking skills. A simple example will help to clarify the distinction between research on *teaching*, *learning*, *problem solving* and *strategy*.

Suppose the mathematics task is to find 15% of 200. Research on *teaching* will focus on the use of exposition, concrete materials, and group discussion; the emphasis is on activities organised and managed by the teacher. Studies on *problem solving* may identify the strategies used by students to solve this task; for example, direct multiplication, or finding 10% of 200 and then adding its half. Research on *learning* may examine the misconceptions students hold about percentages. In contrast to these studies, research on *strategy* will consider what students *do* with regards to the teaching approach, how they develop the problem solving strategies that have been identified, and what cognitive and metacognitive processes they engage in to develop an understanding of percentages. For example, a common strategy to monitor one's understanding is to ask oneself questions such as `Do I understand this?'.

It is also important to know which strategies lead to a better understanding and whether they can form part of an effective training program.

From a theoretical viewpoint, the role of strategies is an important one for at least three reasons. First, recent emphasis on constructivism argues that students "construct their own meanings from, and for, the ideas, objects and events which they experience" (Australian Education Council, 1991, p. 16). Given the same mathematical experiences in the classroom, students often construct different meanings from their teacher's exposition and the activities they engage in (Leder, 1990a, 1990b). Some of these conceptions may be inappropriate and may become a hindrance to further learning (Davis, 1988). These idiosyncratic constructions also extend to the students' conceptions of the nature of mathematics, of learning and teaching mathematics, and of assessment. A crucial aspect missing from this current discussion on constructivism is the strategies used by students in constructing their own meanings.

The second rationale concerns the goals of education. According to the *National Statement on Mathematics for Australian Schools* all students should "continue to learn mathematics independently and collaboratively" (Australian Education Council, 1991, p. 15). In view of the explosion of mathematical knowledge and its importance for future education and employment, students should acquire the skills of learning to learn. To pursue this objective of students managing their own learning, it is necessary to equip them with a wide range of strategies so that they can select the appropriate ones to achieve the desired learning goals.

Thirdly, learning strategy research in other domains of knowledge has found that successful students are the ones who possess powerful strategies that enable them to solve novel problems and who are able to regulate and monitor their own learning. Weaker students on the other hand do not have a repertoire of strategies and often lack control over their own performance and learning (Campione, Brown, & Connell, 1989). These findings result in a number of mathematics educators suggesting the need to incorporate learning strategy training as part of the mathematics curriculum (e.g., Kilpatrick, 1986; Lindquist, 1987).

An essential accompaniment to strategy development is the associated awareness or beliefs that students hold: about particular strategies, such as when to use certain ones and not others, about the nature of mathematics and how it can be learnt, and about themselves and others as learners of mathematics. Flavell (1979) describes these three aspects as *strategy*, *task* and *person* awareness. His theoretical framework was effectively employed by Lester and Garofalo (1982) researching students' beliefs about word problems. Flavell (1981) also provides a theoretical framework for considering interactions between beliefs and strategies suggesting that together they have a direct influence on cognitive learning outcomes.

From the above discussion, it can be deduced that ideally students should learn four things in mathematics lessons:

- A body of mathematics (concepts, facts and skills),
- Mathematical thinking (processes),
- Mathematical attitudes (feelings), and
- Learning how to learn mathematics (strategies and beliefs).

While there is already a considerable body of literature on the first three types of learning, the fourth type, learning to learn mathematics, has not entered into mainstream mathematics instruction and research in mathematics education. The next section elaborates on this type of learning within a theoretical framework, which is used for this review.

AN INTERACTIVE MODEL OF MATHEMATICS LEARNING

There are several generic models of learning that also incorporate learning strategies (e.g., Flavell, 1981; Ramsden, 1988). However, they are too broad for our purposes and fail to address the particular issues of concern within mathematics education.

A model of learning mathematics that incorporates strategic knowledge is shown in Figure 1. The central focus of the model covers three clusters of variables about the learner: Learning strategies, beliefs and mathematical outcomes that are assumed to interact with one another. For example, when students solve a mathematical problem, the approach taken by the students will be influenced by the beliefs they hold about problem solving and the learning strategies they have at their disposal, such as elaboration and metacognitive processes. While these beliefs impact upon the problem's solution, the resultant activity will influence future beliefs about problem solving and the appropriateness of strategies employed. Furthermore, these elements in turn will be influenced by learner variables such as previous knowledge or age, and contextual factors within which the learners find themselves, for example, in school or in the market place (Saxe, 1988), under examination conditions or in a group situation. Together these factors suggest a dynamic development of learning with numerous interactions worthy of further study.

Learning to learn then, involves two of the three learner outcomes. The first aspect covers beliefs about learning. The second involves specific strategies such as representing information in words or diagrams (elaboration strategies), inking ideas together in the form of tables or diagrams (organisational strategies) or monitoring and evaluating learning (metacognitive strategies) (Weinstein & Mayer, 1985).

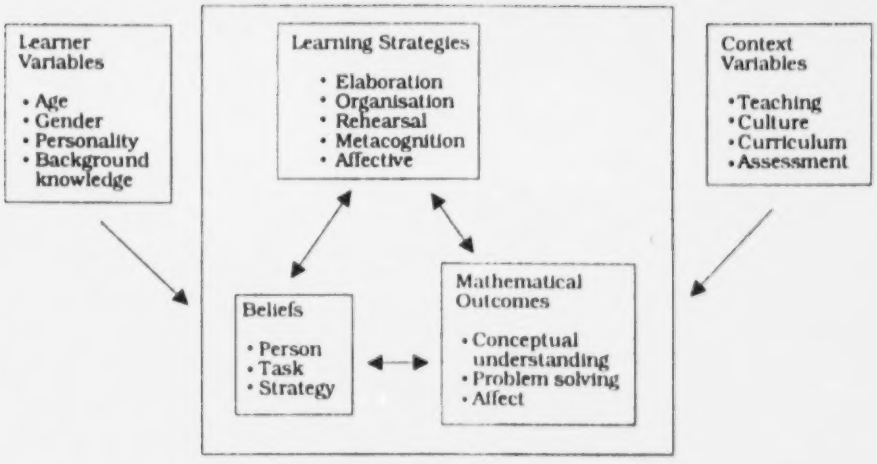


Figure 1: An Interactive Model of Mathematics Learning

The following sections review studies which address some of the relationships indicated in the model above..

STRATEGY AWARENESS

In his doctoral study, Wong (1984) devised a 40-item Likert scale questionnaire to measure self-reported study behaviours of a sample of 216 Year 11 students in Brisbane. The questionnaire covered the following aspects: study organisation, use of notes, problem solving, memorisation, reading, and preparation for tests. These students reported using strategies such as paying attention in class including to hints about tests, handing in homework on time, and learning from mistakes. They concentrated more on how to use a rule rather than how to derive it. On the negative aspect, they seldom tried to solve problems in different ways or read more about mathematics. An overall measure of desirable study behaviours correlated significantly (0.44) with a calculus test. Cluster analysis was used to identify five 'types' of mathematics learners: *conscientious, independent, cue-seeking, rote and minimal*. These student types were associated with different performance on the calculus test.

A modified form of the questionnaire was administered to a large sample (N = 1318) of Singapore students in 1988 (Wong, 1990). This sample consisted of students who were 13 to 17 years old. About 80% of them memorised formulae as a very frequent study activity, and 30% often became nervous before tests because of the many rules to remember. Like the Australian sample, they often paid attention to hints about tests and handed in

homework on time, but seldom tried different ways of solving the same problem or borrowed mathematics books from the library.

These data were consistent with results from the Second IEA Mathematics study that memorising rules and formulae was considered a very important learning activity by about 85% of the students and 80% of the teachers (Robitaille & Garden, 1989). Australian students also reported that listening to the teacher's explanations was the most important aspect of their mathematics experience in schools (Clarke, Stephens & Waywood, in press).

Herrington (1990a) conducted clinical interviews with 19 Year 7 children to identify their beliefs about learning mathematics conceptualised in terms of three aspects of metacognitive awareness (Flavell, 1979). These categories involved children's beliefs about themselves and others as learners of mathematics (person awareness), beliefs about learning mathematics as distinct from other subjects (task awareness) and beliefs about the ways mathematics can be learnt (strategy awareness). Results indicated that beliefs about ability matched test results, and that children could readily identify their difficulties in the subject. Good learners of mathematics were perceived to be born that way, be better listeners, have a supportive home, study or complete homework. Poor learners on the other hand were perceived not to listen, misbehave, be born that way or lack motivation. These students viewed learning mathematics differently to learning other subjects because mathematics involved numbers not words, was difficult and required learning rules. The way to learn mathematics was seen as rehearsing rules by practice, asking others to quiz you, self-testing and using mnemonics.

On the basis of these interviews, Herrington (1991) developed three 12-item Likert scales designed to assess the three categories of students' beliefs about learning mathematics described above. These tests were given to 57 Year 6 students. Many of these students believed that mathematics consisted mainly of arithmetic and had little relevance for later life. They were unaware of their difficulties in mathematics and lacked sufficient skills to check for difficulties. They saw the need for practice and copying from the blackboard as prime strategies for learning mathematics.

McDonough, Spandel and Bourke (1991) are currently developing an instrument that uses children's drawings to identify their beliefs about their preferred environment for learning mathematics. The procedure involves asking children to draw a picture of a situation in which they felt they were learning mathematics well. This is then followed by a questionnaire asking children "to describe their picture, to describe the things they were using, to list the people in the picture and to discuss what in the picture was most helping them to learn maths well" (p. 3). The drawings of 393 children from Years 1 to 6 were categorised according to location, instruction type, teacher's role and tools used in learning.

The majority of students specified the classroom as the location, but year level differences were observed. Commonly, students drew themselves alone without an adult providing instruction and included pencil and paper as the

most frequently depicted tools. While gender differences and year level differences were noted, statistical tests were not employed. Initial findings suggested, however, that this projective and innovative method may provide some important findings.

Southwell (1990) used stimulated-recall interviews to identify, among other variables, the beliefs that three final-year preservice primary teachers held about mathematics and about themselves as learners. As they solved non-routine problems, the students were video-taped and, following completion of the task, the video was replayed. At various points the video was stopped and an interviewer asked questions like: "What were you thinking at that point?", "How did you feel at that point?", "What made you do that?" It was found that the students portrayed mathematics as more than computation and could recognise its usefulness; however, thinking processes used in the solution of a problem were not seen as mathematical. These preservice teachers also saw their own abilities in the subject as low and this perception was likely to influence their ability to teach it. This study holds implications for possible relationships between teachers' beliefs about the subject and their approach to teaching it.

Southwell and Khamis (1991) reported on a survey administered to 185 primary and 310 secondary school students and 75 primary teachers to determine beliefs about mathematics and issues in mathematics education such as assessment and teaching practices. Generally, the three groups believed that mathematics was important for everyday life, that hard work rather than luck resulted in good grades, learning mathematics consisted of memorising facts and procedures, and that teachers allowed only a short time for a question to be answered and then moved on. The students believed that people are naturally good at mathematics, and that good teachers demonstrated correct procedures to follow. The teachers, however, did not support such belief, indicating that students should explore ways of solving problems.

It is apparent from these studies that students hold definite views about ways in which to learn mathematics; in particular, using rehearsal methods to learn mathematics is common to students across the age levels. While Wong's (1984) study looked at the relationship between awareness of strategies and performance for high school students studying calculus, similar studies have also explored the relationships between other belief aspects and performance. These relationships are discussed in the next section.

INTERACTIONS BETWEEN AWARENESS AND PERFORMANCE

Kirby and Ashman (1984) used a battery of planning measures derived from literature on brain-damage to investigate relationships between different aspects of planning and mathematical achievement. In surveys conducted on fifth-grade children, they found significant positive correlations between

measures of planning strategies (selective attention) and metacognitive awareness and arithmetical achievement. Planning strategies were more highly correlated for poor achievers on the mathematical achievement test than for average and more able students. This was not the case for metacognitive awareness which showed a low correlation across ability groups. This result was possibly due to the fact that the metacognitive measures used related to learning in general rather than to mathematics, and that the achievement tests used concentrated on only one aspect of mathematics, namely arithmetical skill performance.

Volet and Lawrence (1989) interviewed nine undergraduate students studying a semester course in introductory statistics to determine relationships between their goals - described as "explicit, individualised intentions generated to guide their study" (p. 497) - and their academic achievement, background knowledge and age. The students' self-reported study strategies consisted of discussion with other students, summarising, revising, practising, identifying requirements and learning from examples. Mature aged students reported using more adaptive and independent learning strategies than did recent school leavers. These strategies were found to be more closely related to the students' goals and their age than to their entering knowledge. Thus, age, learning strategy awareness and goals appear to mediate prior learning and may have a direct influence on performance.

Clarke (1986) provided further evidence of a relationship between performance and beliefs about others as learners of mathematics. Ten Year 7 students were asked to arrange cards on which were written the names of their classmates according to "how good they are at mathematics". The self-ranking of their peers according to this construct of "good at maths" correlated positively and significantly with performance on tests of mathematical achievement.

TEACHING LEARNING STRATEGIES

The evidence from recent studies showing positive relationships between performance and self reported use of learning strategies (strategy awareness) has led a number of researchers to investigate the possibilities of teaching various types of strategies. This following section deals with such studies.

Writing

Research reported in the previous sections show that many students tend to learn mathematics in an instrumental way. While this may lead to short-term success, it will not encourage students to view doing mathematics as a meaningful and creative activity. A recent innovation to engage students in meaningful mathematics learning involves the use of writing as a `mode of learning' (Bell & Bell, 1985). Traditionally the kind of writing in mathematics

is limited to presenting workings in a step-by-step and logical way. Research on writing goes beyond these standard mathematical workings.

The effect of encouraging primary children to write was reported in a study by Mildren, Ellerton and Stephens (1990). The study involved about 120 Year 4, 5 and 6 children in one school. Initially the children were requested to write under the following three headings: "What I did", "What I learnt", and "Problems and difficulties". After it was found that the structure had led to rather superficial reporting of what the children did in mathematics classes, the headings were left out to allow children to write more freely. The content of the children's writings was found to cover three areas: affective behaviours, beliefs about mathematics and themselves, and control of mathematics knowledge. Limited anecdotal evidence was given about children's struggle with mathematical ideas, but the paper did not discuss the effect of writing on mathematics performance.

In the IMPACT project (Clarke, Stephens & Waywood, in press) conducted in 1984, about 700 Year 7 pupils in 36 mathematics classes in 15 Victorian secondary schools were asked once every fortnight to write answers to questions like: "What was the best thing to happen in Maths?", "How do you feel in Maths classes?", "How could we improve Maths classes?" A majority of pupils found such writing useful, in particular to make them think about how they were progressing and to express what they liked and didn't like. While students' writings may suggest appropriate teacher action, the "extent to which participation in the IMPACT program actually facilitated meta-learning and the development of student mathematical thinking remains problematic" (p. 6).

The Vaucluse College Study (Clarke, Stephens & Waywood, in press; Waywood, 1988) began as a pilot study in 1986 and by 1989 was extended to all the classes from Year 7 to 10. The college is a Catholic secondary girls school and its population of about 500 girls had a multicultural mix. After every mathematics lesson all the students were required to write in their journals reflections of their involvement in the lessons. These journals contributed 30% to the assessment in mathematics. Waywood (1988) described a hierarchy of skills which he taught to his students to enhance the quality of writing. These skills covered note taking, reading mathematics, commenting on errors, overviewing and reviewing, writing in own words, and asking questions. The teaching of these skills is consistent with a strategic approach advocated by other researchers in this area.

Evaluation of the Vaucluse Study was carried out in 1988 and 1989. Student journals were classified into Narrative, Summary, and Dialogue, reflecting different taxonomic levels of students' expressions of their mathematical experiences. In the Narrative mode, students described what happened in the math class in a factual way. In the Summary mode, students tried to connect discrete knowledge items into something more coherent. When writing in the Dialogue mode, students were involved in creating and shaping mathematical knowledge. Data were also obtained using student and

teacher interviews and questionnaires. Anecdotal evidence was cited to show the progression of students' writings along the three modes. Although the skills expounded by Waywood (1988) may have effected this transformation, it is not clear from the published evidence the extent to which students had applied these skills successfully and their relationships to mathematical performance.

The use of impromptu writing prompts in mathematics classes was explored by an American study reported in an Australian publication by Miller (1990, 1991). The study involved three algebra classes (about Year 9 to 12) with 25 to 30 students per class. At the beginning of each class, students were asked to write for five minutes using prompts given by the teachers. The objectives were to allow teachers to identify students' understanding through reading their writings and to improve students' attitude towards learning algebra. The study found that teachers had gained insights about students' misunderstanding in specific algebraic concepts, but the effect on students' attitudes was not reported.

At the university level, MacGregor (1990) investigated the effect of writing a sentence in English describing the algebraic relationship before writing an equation on 158 first year mathematics students. Of the 35 students who did not write any sentence only 18 (51%) gave the correct equation. For the remaining 123 students who wrote sentences following the given syntax or their own choice, 97 (79%) got the correct equation. This statistically significant result suggested that "the act of writing a sentence ... encouraged reflection on the meanings of both sentence and equation" (p. 8).

To summarise, the few studies reviewed here share a common enthusiasm of using writing as a mode of learning. There is ample anecdotal evidence to support the use of students' writing to gain insight into their (mis)understanding of mathematics and their struggle to make sense of their mathematical experiences. While Waywood (1988) has described the writing training given to his students, there is insufficient evidence to infer the effect of such training. More controlled studies are needed to validate the proposition that regular writing over an extended period will improve students' performance in mathematics through better attitudes and greater control over their learning (Davison & Pearce, 1990).

Concept Mapping

Concept mapping is a diagrammatic representation that links word concepts in a meaningful way with the use of propositional statements. The technique is a powerful learning strategy in that it enables students to organise existing knowledge, elaborate on that knowledge by making more connections and metacognitively evaluating their own understanding.

Mansfield and Happs (1991) provided examples of concept maps in geometry drawn by primary, secondary and tertiary students indicating the viability of the technique across a wide range of student ages.

Writing and concept mapping appear to be two very important techniques that students can use to elaborate, organise and reflect on their mathematical knowledge with the possibility that such knowledge will be better understood. A study to test this conjecture is described in the next section.

A Learning Strategies Program

A learning strategy program designed to teach elaboration, organisational and metacognitive strategies and awareness was developed, implemented and evaluated by Herrington (1991). The program sought to improve Year 6 students' learning strategy awareness, mathematical intellectual skill achievement and confidence towards learning mathematics using techniques that involved concept mapping, using a thinkboard, self-questioning, and writing (Herrington, 1990b). A range of measures was used to compare the learning outcomes of a class of 31 students undertaking the program with a class of 26 students taught in a traditional way.

The results analysed using ANCOVA indicated that when initial differences in achievement and confidence were taken into account, students undergoing the program displayed a greater extent and quality of learning strategy awareness, confidence towards learning mathematics and higher performance on standardised tests of mathematical achievement. However, only the differences on the extent of strategy awareness were statistically significant. Furthermore, students in the experimental group did not perform as well on tests designed to measure mathematical applications suggesting that training programs should also include learning strategies developed especially for problem solving (e.g., Lester, Garofalo & Kroll, 1989).

CONCLUSIONS

It is evident that the lack of systematic and controlled research on strategy use and beliefs has impeded our understanding of how students come to learn mathematics in a meaningful way. Four areas for further research may be highlighted.

1. Develop a validated and user-friendly instrument to obtain information about students' strategy use. The information will enable teachers to provide counselling to students who are ineffective strategy users. Researchers will be able to test the role of learning strategies as an explanatory factor to account for individual differences in mathematics achievement, in addition to the traditional variables such as attitude, motivation, previous ability, learning environment, social economic backgrounds, and instruction.
2. Investigate the role that contextual factors may have on the beliefs and strategies that children use. This is particularly true for the major influence that assessment techniques may have on beliefs (Schoenfeld,

1988; Leder & Gunstone, 1990). Furthermore, research is needed that systematically identifies and assesses students' strategy usage and awareness in different subject areas and under different forms of instruction. Group work is an example since it has been clearly demonstrated that in primary schools in the United States of America, even the same teachers teaching different subjects employ quite different teaching approaches for mathematics compared to social studies (Stodolsky, 1988).

3. Develop viable intervention strategies specific to mathematics and evaluate their effectiveness. This includes the use of students' writing, concept maps and other techniques that encourage active processing by students about the mathematics they are learning. As an example, first year students at Murdoch University in Western Australia may choose a foundation course that includes learning skills in science. It is not clear to what extent these skills are applied in studying other courses. A difficulty of strategy use is that students are not reminded of these strategies since textbooks and course materials on contents do not deal with study skills. The use of computer-based learning may provide the flexibility to integrate content with study skills.
4. Finally, consider the acquisition of learning strategy development and awareness from the perspective of the teacher as well as the student. The distinction between learning strategies and teaching strategies made at the beginning of this chapter is useful for defining a research perspective but is clearly an artificial dichotomy when the realities of the classroom are considered. If students' learning strategy development is to be effective then the teachers themselves will need to be aware and capable of developing these strategies through activities that are motivating and relevant to students' needs.

REFERENCES

Australian Education Council (1990). *A national statement on mathematics for Australian schools*. Melbourne, Victoria: Curriculum Corporation.

Begle, E. G. (1979). *Critical variables in mathematics education: Findings from a survey of the empirical literature*. Washington, DC: Mathematical Association of America.

Bell, E. S., & Bell, R. N. (1985). Writing and mathematical problem solving: Arguments in favour of synthesis. *School Science and Mathematics, 85*(3), 210-221.

Campione, J. C., Brown, A. L., & Connell, M. L. (1989). Metacognition: On the importance of understanding what you are doing. In R. Charles & E. A. Silver (Eds.), *The teaching and assessing of mathematical problem solving* (pp. 93-114). Reston, VA: National Council for Teachers of Mathematics.

Clarke, D. (1986). Conceptions of mathematical competence. *Research in Mathematics Education in Australia, 2*, 17-23.

Clarke, D., Stephens, M., & Waywood, A. (In press). Communication and the learning of mathematics. In T. A. Romberg (Ed.), *Mathematics assessment and evaluation: Imperatives for mathematics educators*. New York: Suny.

Davis, R. B. (1988). The interplay of algebra, geometry, and logic. *Journal of Mathematical Behaviour, 7*, 9-28.

Davison, D., & Pearce, D. (1990). Perspectives on writing activities in the mathematics classroom. *Mathematics Education Research Journal, 2*(1), 15-22.

Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive-developmental inquiry. *American Psychologist, 34*(10), 906-911.

Flavell, J. H. (1981). Cognitive monitoring. In W. P. Dickson (Ed.), *Children's oral communication skills* (pp. 35-60). New York: Academic Press.

Herrington, A. J. (1990a, July). *Children's beliefs about learning mathematics*. Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

Herrington, A. J. (1990b). Strategies for developing mathematical understanding. In K. Milton & H. McCann (Eds.), *Mathematical turning points: Strategies for the 1990s* (pp. 325-340). Hobart, Tasmania: Australian Association of Mathematics Teachers.

Herrington, A. J. (1991, July). *Evaluation of a learning strategies program in mathematics: A preliminary analysis*. Paper presented at the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia.

Jones, G. A. (1988). Critical variables in mathematics education revisited. In D. C. Blane & G. C. Leder (Eds.), *Mathematics education in Australia: A selection of recent research* (pp. 3-21). Melbourne: Mathematics Education Research Group of Australasia.

Kilpatrick, J. (1986). Reflection and recursion. In M. Carss (Ed.), *Proceedings of the fifth International Congress on Mathematical Education* (pp. 7-29). Boston, MA: Birkhauser.

Kirby, J. R., & Ashman, A. S. (1984). Planning skills and mathematics achievement: Implications regarding learning disabilities. *Journal of Psychoeducational Assessment, 2*, 9-22.

Leder, G. C. (1990a). Does teaching equal learning? In K. Milton & H. McCann (Eds.), *Mathematical turning points: Strategies for the 1990s* (pp. 68-80). Hobart, Tasmania: Australian Association of Mathematics Teachers.

Leder, G. C. (1990b). Talking about mathematics. *Australian Educational Researcher, 17*(2), 17-28.

Leder, G. C., & Gunstone, R. F. (1990). Perspectives on mathematics learning. *International Journal of Educational Research, 14*(2), 105-120.

Lester, F. K., & Garofalo, J. (1982, March). *Metacognitive aspects of elementary school students' performance on arithmetic tasks.* Paper presented at the meeting of the American Educational Research Association, New York.

Lester, F. K., Garofalo, J., & Kroll, D. L. (1989). *The role of metacognition in mathematical problem solving: Final report.* (ERIC Document Reproduction Service No. ED 314 255)

Lindquist, M. M. (1987). Strategic teaching in mathematics. In B. F. Jones, A. S. Palincsar, D. S. Ogle, & E. G. Carr (Eds.), *Strategic teaching and learning: Cognitive instruction in the content areas* (pp. 111-134). Alexandria, VA: ASCD.

MacGregor, M. (1990). Writing in natural language helps students construct algebraic equations. *Mathematics Education Research Journal, 2*(2), 1-11.

Mansfield, H., & Happs, J. (1991). Concept maps. *The Australian Mathematics Teacher, 47*(3), 30-31.

McDonough, A., Spandel, U., & Bourke, D. (1991, December). *What situations do primary school pupils consider help them to learn mathematics well?* Paper presented at the twenty-eighth annual conference of the Mathematical Association of Victoria, La Trobe University, Victoria.

Mildren, J., Ellerton, N., & Stephens, M. (1990). Children's mathematical writing - A window into metacognition. In M. A. Clements (Ed.), *Whither mathematics?* (pp. 356-363). Parkville, Victoria: Mathematical Association of Victoria.

Miller, L. D. (1990). When students write in algebra class. *The Australian Mathematics Teacher, 46*(2), 4-7.

Miller, L. D. (1991). Constructing pedagogical content knowledge from students' writing in secondary mathematics. *Mathematics Education Research Journal, 3*(1), 30-44.

Ramsden, P. (1988). Context and strategy: Situational influences on learning. In R. R. Schmeck (Ed.), *Learning strategies and learning style* (pp. 159-184). New York: Plenum Press.

Robitaille, D. F., & Garden, R. A. (1989). *The IEA Study of Mathematics II: Contexts and outcomes of school mathematics.* Oxford: Pergamon.

Saxe, G. B. (1988). Candy selling and math learning. *Educational Researcher, 17*(6), 14-21.

Schoenfeld, A. H. (1988). When good teaching leads to bad results: The disasters of `well-taught' mathematics courses. *Educational Psychologist, 23*, 145-166.

Southwell, E. (1990, July). *Cognitive and affective aspects of mathematical problem solving: A case study.* Paper presented at the thirteenth annual conference of Mathematics Education Research Group of Australasia, Hobart, Tasmania.

Southwell, E., & Khamis, M. (1991, November). *Beliefs about mathematics and mathematics education.* Paper presented at the annual conference of

Australian Association for Research in Education, Gold Coast, Queensland.

Stodolsky, S. S. (1988). *The subject matters: Classroom activity in math and social studies*. Chicago: University of Chicago Press.

Volet, S. E., & Lawrence, J. A. (1989). Goals in the adaptive learning of university students. In H. Mandl, E. De Corte, N. Bennett, & H. F. Friedrich (Eds.), *Learning and instruction: European research in an international context* (Vol. 2.1, pp. 497-516). Oxford: Pergamon.

Waywood, A. (1988). Mathematics and language: Reflections on students using mathematics journals. In G. Davis & R. P. Hunting (Eds.), *Language issues in learning and teaching mathematics* (pp. 51-59). Bundoora, Victoria: La Trobe University.

Weinstein, C. E., & Mayer, R. E. (1985). The teaching of learning strategies. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (3rd ed.) (pp. 315-327). New York: Macmillan.

Wong, K. Y. (1984). *Mathematical understanding: An exploration of theory and practice*. Unpublished doctoral thesis, University of Queensland, Brisbane, Queensland.

Wong, K. Y. (1990). How secondary students learn mathematics. In K. Milton & H. McCann (Eds.), *Mathematical turning points: Strategies for the 1990s* (pp. 566-572). Hobart, Tasmania: Australian Association of Mathematics Teachers.

CALCULATORS AND COMPUTERS IN THE TEACHING AND LEARNING OF MATHEMATICS

Brian Doig,
Australian Council of Educational Research

Margorie Carss
Department of Education
University of Queensland

Peter Galbraith
Department of Education
University of Queensland

The *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991) provides further endorsement for the National Statement on the use of calculators in Australian Scools. The calculator statement (Australian Association of Mathematics Teachers, AAMT, 1987) was published jointly by the AAMT and the Curriculum Development Centre in 1986 and endorsed by all Australian education systems in 1987. The National Statement also points out that there have been many advances in the computing software appropriate for supporting school mathematics. Both technologies can be used in a variety of contexts in the classroom and each has implications for what is taught and how it is taught. The range of reports of classroom experience with these technologies as teaching tools indicates there is considerable activity in their application. Unfortunately, most reports are of trials of specific use in particular classrooms with few reports of planned research in the incorporation of either calculators or computers in teaching programs. These reports will be considered here as examples of action research and an indication given of the types of activities that they were used.

CALCULATORS

Two major projects on calculator use that have issued reports are:

* the Victoria College Calculator Project, which is a research project investigating the extent to which ready availability of calculators coupled with a systematic program of professional support results in changes to teachers' practice, attitudes and beliefs about the teaching of mathematics; and

* the Calculator-Aware Program for the Teaching of Number (CAPTN) based at the University of Melbourne which seeks to investigate the effects of the availability and use of calculators in the teaching and learning of number.

These projects now work together under the joint title "Calculators in Primary Mathematics".

The Victoria College Calculator Project began by involving Preparatory and Year 1 classes at two schools in Victoria in 1990, and is expanding progressively to higher year levels. In a conference report Groves, Ferres, Bergfeld and Salter (1990) reported on the background to this project. To the end of 1991 more than $50 000 had been allocated to the project by the college. Classes involved in the project have been provided with calculators (Texas Instruments, TI-108) and support from the project team. Schools are visited on a weekly basis and meetings are held between teachers and project staff. Regular inter-school network meetings are supplemented by a regular newsletter. A later report on the project by Groves, Cheeseman, Clarke and Hawkes (1991) concluded with a Preparatory/Year 1 teacher telling of her experiences as a member of the project.

In another report, Beeby and Cheeseman (1991) provided information about a large-scale survey that was conducted in 1990 to assist in setting the Victoria College Calculator Project in the context of the school community. The data from this survey were compared with data from a similar survey undertaken in 1981. The results, inter alia, showed that 75% of the teachers surveyed in 1990 supported the introduction of calculators in the infant school compared with 7% in 1981. Three interview schedules were developed and administered to a sample (20%) of the children at each of the grade levels involved in the Project. The results reported here indicate that the children involved in the Project were able to explore and understand numerical ideas beyond the normal expectations of their teachers for each of the grades. Teacher interviews were also conducted and these revealed that significant changes were occurring in teaching style.

In writing about the Project, Groves (1991) emphasised the role that calculators are playing in the Project schools. Four roles were defined for the calculator; the calculator as a recording device, as a counting device, as a "number cruncher", and as a machine with which to investigate. Teacher comments about their (changed) practice, beliefs and expectation of children were reported. The article included a comprehensive selection of references on this topic.

The CAPTN Project began in 1989 with Preparatory (first year at school) children at four schools. Details of the project were provided by Welsh, Rasmussen and Stacey (1990). Activities used by one teacher involved in the Project were given in McDonald (1990). Welsh (1991) gave details of the background and establishment of the Calculators in Primary Mathematics project and outlined the major issues being investigated. These issues were

identified as: the extent and nature of calculator use, the effects on curriculum content, the degree to which calculators facilitate problem solving, and the long-term implications calculators have for numeracy.

To report on these issues a large amount and variety of data have been collected. To gauge calculator use, teachers' weekly programs of work and samples of children's work have been collected and will be analysed. Interviews with a 10% random sample of children in Years 3 and 4 in the Project schools will be used to assess children's ability to use the calculator as an appropriate tool. Written tests are being used to assess the mathematical skills and understandings of the children in project schools. These tests are in two parts: one using paper and pencil for calculating and the other using a calculator. Data are to be analysed for differences in skills and understandings in specific topic areas according to the length of involvement with the project. The instruments have been trialed and the testing and the interview program commenced in the latter part of 1991.

COMPUTERS

Unlike the calculator, the computer appears to have been more readily accepted as part of the teachers' "tools of trade", especially if the number and variety of reports is any measure. The variety of interests indicated in these articles can be subdivided into several categories, including issues in computer use, computers for mathematical development, using computers to teach specific mathematical topics, and teaching computer programming.

Social issues related to computer use were discussed in four quite diverse reports. Nelson (1988) surveyed primary and secondary students (N=201) in Western Australia. He used a twenty item questionnaire designed to assess attitudes to computers based on a semantic defferential scale. Scoring was devised to maintain consistency with previous United States and British surveys. The results indicated an overall positive attitude towards computers, with primary students slightly more positive than secondary. There were no significant differences in attitudes between boys and girls. Students whose parents owned a computer were more positive in their attitudes than those whose parents did not.

In a study conducted in an all-boys secondary school in England, Houghton (1990) was concerned with the quality and quantity of work produced where the incentive was access to computer games. Students in the sample were 13 and 14 year olds in mixed ability classes. Baseline data of student behaviour were collected using the Observing Pupils and Teachers in Classrooms (OPTIC) schedule. All students were given the same experimental treatment, but one group had a week's delay before entering the experimental program. The results showed that the incentive of computer games produced an 8% to 11% increase in levels of on-task behaviour with an accompanying increase in both the quantity and quality of responses. This was contrasted

with little or no increase in performance when non-computer mathematics games were the incentive.

Using a computer was also popular with the 10 pre-schools studied by Elliot and Hall (1990) in an early intervention program. The study employed a modified pre-test post-test design together with observational data. The purpose of the study was to investigate gains in mathematical knowledge after participation in computer-based activities. The pre-test was developed by the authors and contained ten items focusing on counting and numerical recognition, shapes and comparison of numbers. Concrete materials were used in the test. As measured by the post-test, the six children with computer access showed significant gains over the four in the control group without access. The authors suggested that the use of the computer as a teaching-learning resource can bring about positive changes to achievements in the specific areas of mathematics investigated.

A similar belief is held by the sponsors of the Sunrise School Project where all students were issued with their own laptops. Nevile (1988) outlined the philosophy behind the school's existence and how it functions. An account of an investigation of Dinosaurs is given as an example of the teaching-learning methods employed. Although quite brief, this report does give a hint of future possibilities.

Projects such as the one above emphasise the need for new approaches to teacher education. An outline of one study of computer use with pre- and in-service teacher education was given by Schibeci (1988). Both pre- and in-service teachers were offered the opportunity to work with computers with the language Logo. All students were required to keep a log of their experiences. Initially, students who were first-time users reported frustration with the computers, but as their ability increased, so did their confidence. Quotes from the students' logs indicated that many were aware that Logo was indeed mathematical although no specific mathematics emphasis was inherent in the study.

In contrast, the study by Herbert and Crouch (1988) took a definite mathematical stance. They reported the use of Logo with both primary school students and pre-service teachers. Again journals were used to record experiences of reflection. The aspects of mathematics learning emphasised by this study included continuous goal setting, action, reflection and evaluation.

Another research study by Hunting, Bigelow and Davis (1989) investigated the use of a computer to enhance the learning of a specific topic that usually causes difficulty. The objective of the study was to reveal the levels through which children pass on their way towards a more complete understanding of fractions. Two computer environments were developed. Firstly, the Ferris Wheel was designed to allow the distribution of people icons into the basket icons of the Ferris Wheel. This situation focused on a whole-to-part relation. Secondly, the Copy Cat machine was designed as a black box machine which takes a whole number quantity of discrete items as input and outputs a fraction of the input quantity. The pilot study report was

based on interviews with 40 Year 1 and Year 2 children using Ferris Wheel, Copy Cat and an independence-checking counting protocol. The findings from the pilot study suggested that children at this level work in absolute (additive) differences, not in ratios. The report suggested that knowing this could provide a good springboard for progress to more abstract relations.

Whether learning from one environment can transfer to others is problematic. In their study, Au and Leung (1988) set out to investigate the relationships, if any, between programming in Logo and problem solving. Sixty children (8 to 12 years of age) were selected from Hong Kong primary schools. To establish a baseline, all children were given Raven's Standard Progressive Matrices, then matched and randomly assigned to one of the two experimental groups (process oriented Logo or content oriented Logo) or the control group (typing). The Tower of Hanoi and Rule Learning Task were administered to pre-test problem solving performance. The intervention program was conducted one hour per week for 25 weeks. The three tests were then re-administered following the completion of the intervention program.

The three groups did not differ significantly between pre and post-test scores. The lack of significant differences among the three groups on the Progressive Matrices and Rule Learning Task imply that learning to program in Logo does not enhance the learner's logical reasoning. However, by contrast, there were significant differences between the process oriented Logo children and the other two groups on the tower of Hanoi task. The content oriented Logo group also outperformed the control group on this task. The results of this study were compared with other studies using Logo, Raven's matrices and the Rule Learning Task. Differences in methodology and outcomes between these studies were noted.

A study by Edmonds (1989) addressed issues related to gender specific differences in computer use by utilising a sample of 500 Year 9 students attending Catholic co-educational secondary schools in the Newcastle region. The research was designed to measure: how often students used computes at school and at home, how long they had been using a computer, their perceived mathematics and English abilities, and their self esteem.

Findings indicated that boys used computers more often than girls, and that boys claimed to be more competent than girls at specific tasks that involved using a computer. For girls, however, there was no relationship between computer use and the perception they had of their mathematics achievement.

The aim of a study by Ryan (1989) was to develop an effective computational environment suitable for students of mathematics aged 13 to 15. Three major aims were addressed in this project:

* the production of resources (computer software support materials and instructional approaches) that were effective in learning and teaching elements of secondary school mathematics;

* the evaluation of a software development model that was characterised by continual development, refinement and modification based on reflection (by teachers, students and researcher) in a practical environment; and
* the clarification of theoretical issues concerning the role of technology in mathematics education and also in the wider educational context.

Barnsley (1988) described an investigation to assess the effects of Computer Aided Instruction (CAI) used in conjunction with concrete materials. A first study using male trained primary school teachers as subjects compared three treatment groups: symbolic, pictorial and manipulative modes of presentation. The expected superiority of the manipulative mode was not apparent with the results suggesting that the pictorial mode might be the most effective. A second study compared only two treatments, symbolic and pictorial presentations, but used a larger sample that also included mature aged College entrants. A significant superiority for the pictorial mode of presentation was observed. Mature aged entrants benefited more from the CAI mode than ordinary entrants. A third study extended the research to female students, both ordinary and mature aged entrants, and confirmed the superiority of the pictorial mode previously noted. No significant sex differences were found with CAI presentation.

An investigation utilising primary school data was conducted by White (1989). The study focused on the integration of computers with curriculum, the uses of computers, and the activities of learners who used computers. In particular the study focused on the ways computers were used, arranged either as stand-alone computers in classrooms, or as networked computers in special rooms. A multi-method research design using both empirical and case study methods was employed involving the analysis of survey questionnaires from three selected primary schools and case study interviews of teachers in five primary schools. Although most schools rotated stand alone computers from classroom to classroom, more computers were located in networks in special rooms. Drill and practice programs were used most often on networked computers in lower year levels whereas using the computer as a tool increased in higher year levels. Stand alone computers used in classrooms were used mostly for language activities, and computers in networks for mathematics.

CALCULATORS, COMPUTERS AND CLASSROOM PRACTICE

The *National Statement on the Use of Calculators for Mathematics in Australian Schools* made some recommendations for the use of calculators in mathematics teaching. Its major recommendations, as far as resources allow, were that teachers should

* ensure that all students use calculators at all levels K - 12;
* ensure that the calculator is used both as an instructional aid and a computational tool in the learning process;
* be actively involved in the curriculum change in content and methods arising from calculator use;
* take full advantage of the potential of calculators for mathematics in the total curriculum;
* initiate discussion locally regarding the role of calculators in school and society.

Several small projects were located that reported on practice involving calculators. These focused both on the calculator as a tool for teaching and as a device for improving technical facility. A number of papers relating to the use of calculators and computers are included in "Maths Counts: Who Cares?", (Firth, 1988), "Everyone Counts", (Doig, 1989), "Whither Mathematics?", (Clements, 1990) and "Mathematics: IDEAS", (O'Reilly & Wethenhyall, 1991).

The introduction of graphic calculators was heralded by Groves in an article reporting the use of these devices and computer plotting devices in the United Kingdom (Groves, 1989). Details of the types of activities undertaken by and the reactions of students were given. Groves' personal reactions, included in the paper, give a good idea of the impact of the then new technology. She concluded with a prophetic note "there can be no question that graphic calculators are here to stay". This comment has been borne out by the increase of reports of graphic calculator use by 1991 (seven in all being reported at the annual Mathematics Association of Victoria conference). Research into the role and use of graphic calculators is yet to be reported.

More than 20 examples of computer use in classrooms from primary through tertiary level were identified from the literature. These ranged across applications such as the use of spreadsheets, computer graphing packages, computer assisted instruction and a variety of specific software packages. Three articles addressed issues related to the use of a computer technology within mathematics bridging courses at early tertiary level. These reports, however, did not include a research component and are not discussed further.

FUTURE RESEARCH

Given than there has been a dearth of formalised research in the areas of computers and calculators in mathematics education, it would seem that further research in either area would be most welcome. There are, however, some directions which appear to promise long-term benefits. These directions apply equally to calculators and computers, since in many cases the differences may lie in portability or availability rather than in kind. Similarly the comments

apply to students and teachers at both the primary and secondary levels; it is not a case of one or the other but both!

The first direction for future research is the consideration of how teachers are affected by the impact of the technology. In what ways is the quality of teacher-life changed? Are there effects upon the way in which we teach and the content of our courses? Do some topics of the curriculum no longer need the same emphasis, or indeed a place in the curriculum at all? Will topics retain their current position in the sequence of the curriculum? Teaching styles that employ exposition only and do not allow for student exploration may be challenged by the use of technology. What effects would this have on teachers?

Assessment practices will need to change to suit the use of technology. What will be assessed if the routine skills that are now being assessed are performed by machine? Efforts need to be made to explore alternatives to current assessment practices, to see where these new practices are strong and where they are deficient. Comparisons between the old and new are needed to convince the profession that alternatives are valid and reliable.

The long-term benefits for children may be at odds with the wishes of teachers. What does using a calculator do for one's basic number facts acquisition and retention? Is there a need to learn these by rote methods? Can young children develop a working knowledge of negative and decimal numbers beyond the currently accepted level of performance? Are there topics in mathematics now within the reach of children of a younger age? Can better conceptual understandings be developed through the use of the new technology? Can, or should, mathematics education return to the use of graphical methods rather than analytic methods?

A further aspect of the use of technology that needs attention relates not to the research itself but rather the way in which it is conducted. Since most of the research issues listed above are directly concerned with the classroom, it is important that teachers take on a major role in carrying out such research. Classroom-based research in this area would appear to be the single most sensible approach to research into calculator and computer use and effects in schools.

REFERENCES

Au, W. K. & Leung, P. (1988). Transfer of Problem solving in Logo Programming. In P. Alp (Ed), *Proceedings of the Australian Computers in Education Conference* (pp. 46-59). Perth, Western Australia.

Australian Association for Mathematics Teachers. (1987). A National Statement on the use of calculators for mathematic in Australian schools. Canberra, Australian Capital Territory: Curriculum Development Centre and the Australian Association of Mathematics Teachers.

Australian Education Council. (1991). *A National statement on smathematics for Australian schools*. Carlton, Victoria: Curriculum Corporation.

Barnsley, G. J. (1988). *A Study of effects of institutional modes in a sequence of instructional lessons with an introduction to computer aided diagnosis of arithmetic errors*. Unpublished doctoral thesis, University of New South Wales, New South Wales.

Beeby, T., & Cheeseman, J. (1991, July). *Calculators as an agent for Change: The Victoria College Calculator Project*. Paper presented at the fourteenth annual conference of the Mathematics Education Reseearch Group in Australasia, Perth, Western Australia.

Clements, M. A. (Ed.). (1990). *Whither mathematics?* Melbourne, Victoria: Mathematical Association of Victoria.

Doig, B. (1989). *Everyone counts*. Melbourne,Victoria: Mathematical Associations of Victoria.

Edmonds, C. A. (1989). *Computer use, self esteem, gender and perceived mathematics achievement*. Unpublished masters thesis, University of New England, Armidale, New South Wales.

Elliott, A., & Hall, N. (1990). Computer-based mathematical experiences in an early intervention program. Australian. *Journal of Early Childhood, 15*(3), 37-45.

Firth, D. (Ed). (1988). *Maths counts: Who cares?* Melbourne, Victoria: Mathematical Association of Victoria.

Groves, S. (1989). Graphic calculators - an introduction. In B. Doig (Ed), *Everyone counts* (pp. 395-400). Melbourne, Victoria: Mathematical Association of Victoria.

Groves, S. (1991, November). *The Calculator as an agent for change in the teaching of primary mathematics: The Victoria College Calculator Project*. Paper presented at the annual conference of the Australian Association for Research in Education, Surfers Paradise, Queensland.

Groves, S., Cheeseman, J., Clarke, C. & Hawkes, J. (1991). Using calculators with young children. In J. O'Reilly & S. Wettenhall (Eds.), *Mathematics: Inclusive, dynamic, exciting, active, stimulating* (pp. 325-333). Melbourne, Vivtopria: Mathematical Association of Victoria.

Groves, S., Ferres, G., Bergfeld, S., & Salter, S. (1990). Calculators in the infant school: the Victoria College Project. In M. A. Clements (Ed.), *Whither mathematics?* (pp. 244-250). Melbourne, Victoria: Mathematics Association of Victoria

Herbert, G. & Crouch, W. (1989). Logo in the classroom: a graphic view. In B. Doig (Ed.). *Everyone counts* (pp. 111-116). Melbourne, Victoria: The Mathematical Association of Victoria.

Houghton, S. (1990). Increasing on-task behaviour and the quantity and quality of work produced through group contingent access to computer mathematics and arcade games. *Education Research and Perspectives, 17*(1),76-84.

Hunting, R. P., Bigelow, J. C. & Davis, G. E. (1989). The trouble with fractions: Can computer based learning environments help?. In B. Doig

(Ed.), *Everyone Counts* (pp. 130-133). Melbourne, Victoria: The Mathematical Association of Victoria.

McDonald J. (1990). Calculator Aware Program for the teaching of number. In M. A. Clements, (Ed.), *Whither mathematics?*(pp. 251-255), Melbourne, Victoria: Victoria Association of Mathematics.

Nelson, L. (1988). Attitudes of Western Australian students towards micro-computers. *British Journal of Educational Technology, 19*(1), 53-57.

Nevile, L. (1988). The Sunrise School - A school of the Future Project. Australian Computer Society. *Victorian Bulletin*, June, 5-9.

O'Reilly, J., & Wettenhyall, S. (Eds.). (1991). *Mathematics: IDEAS*. Melbourne, Victoria: Mathematical Association of Victoria.

Ryan, M. (1989). T*he development of a computational environment in secondary school mathematics*. Unpublished masters thesis, Queensland University of Techniology, Brisbane, Queensland.

Schibeci, R. A. (1988). Logo in preservice and inservice teacher education. In P. Alp (Ed.), *Golden Opportunities, proceedings of the sixth Computers in Education conferece. Perth, Western Australia: Educational Computing Association of Western Austaslia.*

Welsh, R. (1991, July). *The CAPTN Project: The long term effect of calculator use on primary school children's development of mathematical concepts*. Paper presented at the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia.

Welsh, R., Rasmussen, D., & Stacey, K. (1990). *A Calculator Aware Program for the teaching of number*. Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia.

White, G. K. (1989). *A Study of theintegration of the uses of computers in different locations and configurations with curriculum in primary education*. Unpublished masters thesis, Flinders University of South Australia, Adelaide, South Australia.

LANGUAGE FACTORS IN MATHEMATICS LEARNING

Nerida F. Ellerton
Faculty of Education
Deakin University - Geelong Campus

Philip C. Clarkson
Faculty of Education
Australian Catholic University - Mercy Campus

Australian mathematics education researchers have consistently argued that, contrary to common belief, language factors have a particularly strong influence on the learning of mathematics (see for example, Ellerton & Clements, 1991, p. 141). Over the past 20 years, and especially over the past four years, Australian mathematics educators have investigated ways and means of establishing mathematics learning environments in which learners interact with each other and with their teachers in ways which will improve the quality of mathematics learning. It is the aim of this chapter to review recent Australasian research on these issues, and to provide a brief summary of the main underlying themes.

LANGUAGE RESEARCH 1988-1992: AN OVERVIEW

A useful indicator of the amount of research carried out is the number of major publications dealing with issues in that area. Since 1990, three books by Australian authors have focused on how language and mathematics learning are inextricably intertwined (Bickmore-Brand, 1990a; Davis & Hunting, 1990; Ellerton & Clements, 1991). Another book (Durkin & Shire, 1991), an edited collection published by the Open University in the United Kingdom, had an Australian as one of the editors and three Australian researchers among the contributors. In addition, monographs by Australian authors (for example, Clarkson, 1991a; MacGregor, 1991a; Mousley & Marks, 1991; Pengelly, 1992; Sullivan & Clarke, 1991a) as well as specific topic books (for example, Griffiths & Clyne, 1990; MacGregor & Moore, 1991) have appeared. At least 50 journal articles or book chapters written by Australians have been published in the last four years, many in quality journals or books of international standing (such as Steffe & Wood, 1990); overviews of research and classroom implications (Bickmore-Brand, 1990b; Ellerton & Clements, 1990a, 1991), and

a paper identifying ten major issues for future research (Davis, 1990) have also been published.

Language and mathematical learning was the focus of a special issue of the *Australian Journal of Reading* (Vol. 12, No. 2) which was published in 1989, and of a special conference which was held at La Trobe University in 1988. The Mathematics Education Research Group of Australasia's Language and Mathematics special interest group, established in 1979, has continued to be active, and a recently formed special interest group of the Australian Reading Association has attracted a large teacher membership.

An important stimulus for this sustained activity in the language and mathematics field has been the interest shown by language educators in mathematics in recent years (Ellerton & Clements, 1991, pp. 114-128). Primary school teachers, in particular, have been greatly influenced by language education approaches, and such teachers have, in their turn, influenced the direction of mathematics education research. In this context it is not surprising that a number of the studies reviewed in this chapter had their genesis in a desire to explore factors that contribute to good classroom practice rather than in the achievement of a well planned research study. In fact, one might speculate that it has been the interplay between classroom practice and designed research that has given a unique strength and impetus to the language and mathematics movement in Australia.

Such was the impact of the theories of Graves (1983) and Cambourne (1989) on language teaching that various primary teachers and mathematics educators across Australia attempted to restate Cambourne's principles for learning so that they were appropriate in the context of school mathematics (McIntosh, 1988; Robinson, 1986, 1990). Indeed, the notion of a "process approach" to mathematics teaching was taken up in many primary classrooms, promoted both through Exploring Language in Classrooms (ELIC) and Basic Learning in Primary Schools (BLIPS), well funded government-sponsored professional development programs.

However, while it is true that the "natural" approach to mathematics teaching has strongly influenced the thinking and actions of many Australian primary school teachers, and that a number of highly successful, innovative "natural" learning mathematics programs have been reported (see, for example, Baker, Semple & Stead, 1990; Richards, 1990; Stoessiger, 1988; Waters & Montgomery, 1989, 1990), it is doubtful whether such ideas have had a lasting influence on more than a small proportion of primary school mathematics teachers in Australia (Ellerton & Clements, 1991). Recent studies have suggested, for example, that initial positive effects of the ELIC and BLIPS programs in Victoria may not have been enduring (see Ellerton & Clements, 1989a; Mousley & Clements, 1990; Mousley, Sullivan, & Clements, 1991).

Marks and Mousley (1990) argued that the process writing approach to mathematics learning led to limited expectations of narrative recounts in students' writing about mathematics. According to Marks and Mousley (1990), activities designed by teachers and textbook authors allow "very few

opportunities for students to talk, create or record non-algorithmic mathematical ideas," and although process mathematics approaches did involve the use of students' language and experiences, "they were often used as add-on teacher-directed extension activities completed after the 'real' mathematics has been done" (p. 121).

Marks and Mousley added that "emphasis was on doing the activity, perhaps for variety or to 'teach cooperation,' rather than for learning and sharing mathematics." A process approach to mathematics tended to generate narrative genres from students and, according to Marks and Mousley (p. 134), the narrative genre is of limited use in adult life; in any case it is desirable that students observe and explore the use of a wider variety of mathematical genres than a process approach typically provides.

Considering the expanded research base available to inform mathematics educators about different issues, it is with some concern that we draw attention here to the apparent lack of recognition of the importance of language factors in the recently released *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991). Only about two pages of the *Statement* are specifically concerned with language factors - there is a statement under the heading "mathematics learning is likely to be enhanced by using and developing appropriate language" (pp. 19-20), and a brief section on "mathematical expression" (p. 37).

The aim of this chapter is not only to make teachers and teacher educators more aware of key issues in the field of language and mathematics, but also to stimulate further research into implications for the classroom.

MAJOR CONTRIBUTIONS BY AUSTRALIAN RESEARCHERS

Research on how language factors affect mathematics learning has always been a particularly important focus of Australian mathematics education research. Indeed, while it is undoubtedly true that researchers in many countries have contributed to the various pertinent literatures, it could be argued that Australians have led the way in identifying and researching key aspects of how language factors are involved in the teaching and learning of mathematics. These key aspects will be briefly reviewed under the following five major headings:

1. Development of models relating language and mathematics;
2. Expanding the modes of communication in mathematics classrooms;
3. Language, culture and mathematics learning;
4. Children writing mathematics;
5. Mathematics in context: Story maths.

Australian contributions have not been confined to these areas, however, and following the discussion of these five key components, a brief overview of other recent Australasian research will be presented.

Development of Models Relating Language and Mathematics

During the 1970s and 1980s mathematics educators became increasingly aware of the diverse strands of research that are concerned with the interface between language, mathematics, and mathematics learning. The model put forward by Ellerton (see Ellerton, 1989a; Ellerton & Clements, 1990a, 1991) was developed in response to the obvious need to develop links among the various research thrusts already well established in the mathematics education literatures. The model is summarised in Figure 1, and reflects the complexity of the multi-faceted relationships between mathematics, language, the mathematics classroom, and the underlying cultural context.

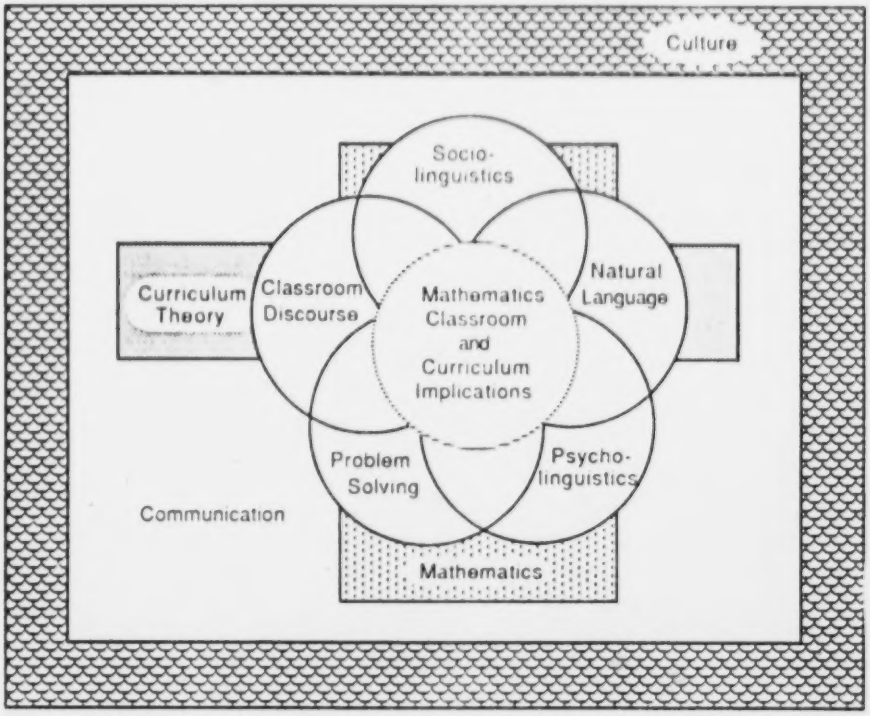


Figure 1. The interface between mathematics and language

While the definitions of, and relationships between, the components of a model such as this are open to debate, the theorising of the links between the various areas of research in language and mathematics is seen not only as an

important way of unifying and directing research, but also as something likely to influence curriculum development as well as the teaching and learning of mathematics.

The socio-psycho-linguistic model proposed by Gawned (1990) arose from a desire to find ways of organising classroom data being collected by teachers who were trialing the implementation of language learning principles in the mathematics classroom. Her model has a narrower focus, emphasising the need to link real-world language and that of the classroom with specialised domains of mathematical language, and the construction of meaning in mathematics. The Carawatha Language Development Centre (Donovan et al., 1990) provides an instance of how Gawned's model can be applied in mathematics classrooms.

Expanding the Modes of Communication in Mathematics Classrooms

In traditional mathematics classrooms, children often have lengthy exposure to expository teaching (during which they are expected to listen in silence), with little opportunity to express themselves except for formal written solutions or oral answers to specific questions. When they respond to questions, they interpret someone else's communication; they rarely have opportunity to initiate communication, or exercise their own imagination (Clarke, 1990; Clarke, Stephens, & Waywood, in press). According to Clarke (1990), unless the various modes of communication are enhanced, learners will be limited in their articulation and development of mathematical meaning.

Clements and Del Campo, in a series of papers exploring relationships between the modes of communication in mathematics classrooms, have maintained that communication in any classroom should be viewed from a multi-modal perspective, and that usually a better balance between the receptive and expressive language forms is needed (Clements & Del Campo, 1989; Del Campo & Clements, 1990). Table I (from Del Campo & Clements, 1990, p. 59) summarises possible relationships between the different language modes. The table provides a useful framework for later discussions in this chapter concerning Australasian research on children writing mathematics, story maths, and the role of improved questioning techniques in the mathematics classroom.

The distinction in Table 1 between receptive and expressive communication modes raises the controversial issue of the extent to which mathematical ideas, concepts, and whole chunks of knowledge can be transported from one person to another via the various modes of communication. The radical constructivist perspective, so popular around the world in the second half of the 1980s, is that whatever anyone says or writes or does will be subjectively interpreted by another (von Glasersfeld, 1991, p. xiv), and that therefore the only mathematical knowledge that people acquire is that which they have personally constructed. In challenging this widely accepted point of view, Ellerton and Clements (1991, p. 55) contend that

words and other forms of communication *do* have the power to communicate unambiguously, *provided* the person receiving the communication has previously established cognitive structures that facilitate the links intended by the person making the communication.

Ellerton and Clements (1991) do agree, however, that what students actually learn is not necessarily congruent with what their mathematics teachers want them to learn. On this matter, Hunting, Pitkethly and Pepper (1990) draw attention to the contrasting roles of communicator and communicatee, and regard the extent to which the interpretations of the receiver coincide with the intentions of the sender as a measure of the effectiveness of the communication process. Their research on language factors in early fraction learning suggested that the teacher's choice of language played a crucial role in the verbal explanations young children gave of their approach to a practical situation involving the notion of sharing.

Table 1.
Receptive and Expressive Modes of Communication (from Del Campo & Clements 1990, p. 59)

Language Mode	Receptive Language (processing someone else's communication)	Expressive Language (your own language)
Spoken	listening	speaking
Written	reading	writing
Pictorial	interpreting diagrams, pictures	drawing
Active	interpreting others' actions	performing, demonstrating
Imagined	_	imagining*

* Note that "imagining" has been included as an expressive form of language because by imagining, one can communicate with oneself.

The role of language in linking the personal worlds of learners with formal mathematical language. The idea of linking the personal worlds of children with formal, verbal and symbolic language, has become a unifying theme in recent Australasian research on language factors in mathematics learning. Unless the experiences children have in mathematics classrooms are linked with real-life examples from the personal worlds of the children, there is a real danger that the concepts they form will remain fragmented and stored in long-term memory only as formal mathematical ideas to be preserved for school. Current Australasian research is exploring how, through the appropriate use of language, teachers can create learning environments that will facilitate the development and reinforcement of such links (e.g. Bickmore-Brand, 1990b; Clements & Del Campo, 1989, 1990; Gawned, 1990; Harris, 1991; Reeves, 1990; Watson, 1990). Figure 2 (from Clements & Lean, 1988) suggests how language can be used to link possible real-life embodiments with formal mathematical language and symbol manipulation.

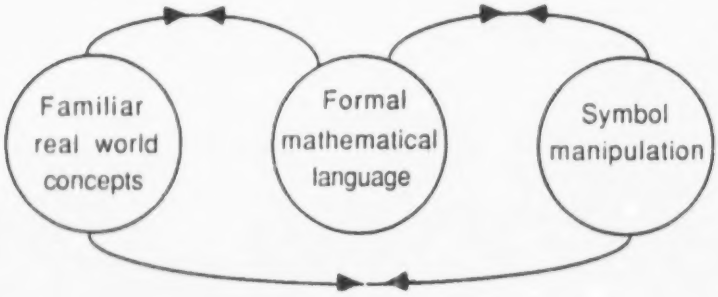


Figure 2. Establishing Links in Cognitive Structure (from Clements & Lean, 1988)

The importance of providing children with the opportunity to talk as well as to listen to other students talk about mathematics was confirmed by Leder (1990) in a study that focused on a Year 3 lesson aimed at extending children's knowledge of fractions beyond a half and a quarter. She suggests that such strategies are not only useful in monitoring student learning, but also providing a base for subsequent error analysis.

Newman error analysis. A number of contemporary studies have adopted an error analysis approach first developed by Newman (1977, 1983), used extensively by Clements (1980), and applied by others (for example, Clarkson, 1980; Watson, 1980). The approach uses a structured interview technique to identify the types of errors first made by students as they attempt to solve

written mathematical problems. Newman classified these initial errors into six categories (reading, comprehension, transformation, process skills, encoding, and careless).

In her initial study, Newman (1977) found that reading, comprehension and transformation errors made by 124 low-achieving Grade 6 pupils accounted for 13%, 22% and 12% respectively of all errors made. Thus, almost half the errors made occurred before the application of standard algorithms. Clements (1980) reported similar results. The findings of these early studies obviously had important implications for the teaching and learning of mathematics, especially for the development of appropriate diagnostic and remedial programs, and throughout the 1980s, Australian researchers continued to use the Newman methodology, both in Australia and in surrounding nations. The Newman approach also attracted much attention in the international mathematics education literature (see, for example, Dickson, Brown & Gibson, 1984; Mellin-Olsen, 1987; Zepp, 1989).

Clarkson (1983, 1991b, 1991c, 1991d) applied Newman techniques in a series of research projects carried out in Papua New Guinea (PNG). The Newman approach was first used in Malaysia in 1990, when Marinas and Clements (1990) found that over 90% of initial errors made by a sample of Year 7 students from Penang were either careless, or of the comprehension or transformation types. Recently the method has been applied to both mathematics and science education research studies in Brunei (Radiah, 1991), Indonesia (Ora, 1992), Malaysia (Ferrer, 1991; Teoh, 1991; Kownan, 1992), the Philippines (Jimenez, 1992), and Thailand (Singhatat, 1991; Sobhachit, 1991).

In most of the Newman studies reported, the highest error frequency occurred in the comprehension category, and it is clear that this category is language-based. Clarkson (1991c) provided some support for the notion of linking the number of comprehension errors made with measures of language competency. Further, for bilingual students, there was evidence that the frequency of such errors was related to competency in both the mother tongue and the language of instruction. Clarkson (1991d) examined the relationship between careless/unknown errors, as defined by Newman (1983), and various achievement and psychological measures. In a comparison between the errors made by PNG and Western students, Clarkson concluded that the apparently lower occurrence of careless/unknown errors for PNG students could be attributed to a higher systematic error rate.

Faulkner (1992) used Newman techniques in research investigating the errors made by nurses undergoing a calculation audit. She found that the majority of errors the nurses made were of the comprehension or transformation type. This result, based on adult data, is interesting in that it extends and confirms the findings of recent research (see, for example, Clarkson, 1991c; Marinas & Clements, 1990) that a deeper understanding of the sources of the comprehension and transformation categories of errors is vital.

The high percentage of comprehension and transformation errors found in studies using the Newman procedure in widely differing contexts has, perhaps more than any other body of research, provided unambiguous evidence of the importance of language in the development of mathematical concepts. The research, however, raises the difficult issue of what educators can do to improve a learner's comprehension of mathematical text or ability to transform, that is to say, to identify an appropriate sequence of operations that will solve a given word problem. At present, little progress has been made on this issue, and it should be an important focus of the Australian mathematics education research agenda during the 1990s.

Questioning in the mathematics classroom. It has been well documented that one of the ways in which teachers dominate the communication that occurs in many classrooms is through the questions they pose for their students (see, for example, Clarke, 1990, p. 20). Through the use of carefully designed, often open-ended questions, Sullivan and Clarke (1991a, 1991b) describe how students of mixed ability are able to give correct responses at individually appropriate levels. Questions such as "A rectangle has a perimeter of 30 units. What might be its area?" (Sullivan & Clarke, 1991a, p. 15) clearly encourage more than simple recall, and provide students with varied learning opportunities, at the same time giving the teacher an opportunity to observe how students approach the problem.

Language, Culture and Mathematics Learning

Some of the most difficult questions facing mathematics educators arise in the context of how language, culture and mathematics learning should be related (Ellerton & Clements, 1990b). For example, should the same mathematics be taught to all children regardless of ethnic background? Should similar teaching approaches be used, regardless of context? Should the mathematics taught in the villages of Papua New Guinea, or in traditional Aboriginal towns of outback Australia, be the same as that taught to immigrant children learning in their second language in an inner city school in Sydney, Australia? And should this be the same mathematics as that taught to native English speakers attending upper middle-class suburban schools in Melbourne? Should the mathematics attempted by Maori students using their own Maori language be the same as that for Pakeha students?

The culture of mathematics classrooms. It would be a mistake to regard the "culture and school mathematics" issue as something that is mainly concerned with schools where the majority of children have non-English-speaking backgrounds. Australasian mathematics educators have looked seriously at the extent to which the teaching and learning of mathematics in *any* school is a culture-free endeavour. They have recognised that language is clearly the vehicle by which much of the interpretation and renegotiation of culture takes place for, as Bruner (1986, p. 123) pointed out, culture cannot be defined by a

static description of traditions and ways of working and thinking; rather, it is "constantly in the process of being created as it is interpreted and renegotiated by its members."

Research by Bourke (1984), Clarke (1984), Ellerton and Clements (1989a), Mousley and Clements (1990), and Mousley, Sullivan and Clements (1991) has demonstrated that there is a well-established culture of school mathematics that strongly resists change. Mathematics lessons in Australian schools, at all levels, usually take the form of the teacher presenting an introduction, before the students are set to work on exercises from their books, worksheets, or the blackboard.

Constructivist ideas about how mathematics should be taught and learned, although fashionable in mathematics education circles, are often at variance with the established culture of the mathematics classroom (Harris, 1989; Robinson, 1989; Siemon, 1989). Indeed, there are forces at all levels of society that consciously and unconsciously resist attempts to make school mathematics more meaningful for children from different sub-cultures (Ellerton & Clements, 1988; Lidster & Watson, 1990; Thomas, 1989).

Non-English-speaking-background students and mathematics learning. Stacey and MacGregor (1991) reported that, because of the linguistic demands of a mathematics lesson, non-English-speaking-background (NESB) immigrant pre-service teachers needed extra help in coping with the English language when they taught mathematics. It is hardly surprising, then, that NESB students also find mathematics lessons difficult. MacGregor and Moore (1991) reviewed and interpreted much of the research relevant to the teaching of mathematics to NESB children, and have developed a number of workshops for use in teacher-training courses (see also Clarkson, 1991a; Clarkson & Kaleva, in press; MacGregor, 1991b).

Western mathematics in Aboriginal and Maori cultures. When immigrants choose to live in a new culture they do so knowing that they will have to contend with an unfamiliar dominant language and life patterns. In contrast, however, Australian Aborigines and New Zealand Maoris did not choose to live within a dominant English-speaking culture - this was imposed on them. Although in the past Aboriginal and Maori children were forced to study, in English, the same school curricula as their largely Anglo-Saxon peers, in recent years it has been increasingly recognised that such an approach fundamentally denies the principle of equal educational opportunity for all.

Research reported by Harris (1989, 1991), Graham (1988), Kearins (1991), Ohia, Moloney and Knight (1990), and Watson (1990) has not only amply demonstrated that many Western mathematical ideas are irrelevant to Aboriginal and Maori cultures, but also that these cultures have their own unique mathematical ideas that have never been taken into account in traditional school mathematics programs. As Clements, Grimison and Ellerton (1989, p. 72) have observed, such colonialist attitudes have been accepted in

the past by both the leaders and representatives of the colonising power *and* by those who are being colonised. Or, as Clarkson (in press a), and Ellerton and Clements (1989b) have pointed out, in many developing countries there is a real danger that expatriate advisers will recommend practice that is appropriate for their own culture but of questionable value in other cultures.

While Aboriginal and Maori communities are opposed to their children being taught a form of school mathematics that effectively denies their cultural heritage, many recognise that in the global world of the future their children are likely to need appropriate mathematical skills that will enable them to survive with dignity in a Western, problem-oriented society. Culturally appropriate classroom material has been developed by Harris (1989, 1991) for use with Aboriginal children, while Watson (1989, 1990) has provided a brief rationale for working wholly within an Aboriginal culture and has suggested that links between Aboriginal and Western technological cultures can be sensitively developed. In New Zealand, there have been strong moves for students to study all subjects, including mathematics, in their own language (Ohia, Moloney & Knight, 1991).

Bilingual programs. Some New Zealand secondary school bilingual programs attempt to bring Maori organisational structures into the schools (Spolsky, 1987). Thus, for example, schools that have adopted the *tuakana* and *teina* principle (that is to say, using a traditional form of interacting in which older and younger students are grouped together), have reported improved achievement and behaviour patterns (Ohia, 1991). Such academic success has been attributed "to a simple Maori concept which states that a person will learn a particular topic properly only when given the chance to teach it" (p. 119). Perhaps this is what Vygotsky (1986) meant when he described concept attainment as being complete only when a student could frame the concept in appropriate words. While some teachers have observed that a considerable amount of informal Maori language can be incorporated into discussion about mathematics problems, it is clear that this needs to be tempered by the fact that a serious lack of resources in the Maori language inhibits the introduction of *kaupapa* Maori (that is, a Maori basis) into the mathematics classroom (Ohio, Moloney & Knight, 1991).

In Papua New Guinea schools the language of instruction is English at all year levels beyond Year 1. English, although the official language of the country, is often a student's second, third or even fourth language, in a country in which at least 740 distinct languages (or 16% of all languages in the world) are spoken (Souviney, 1983). The mathematics curriculum at both primary and secondary levels, however, is similar to curricula found in Australian schools (Clarkson & Kaleva, in press), and little effort has been made to draw on indigenous mathematical ideas, even though many of these are now well documented (Lean, 1991).

The effect of bilingualism on mathematical achievement has also been investigated. While it is well known that students' competence in English

affects mathematical achievement (Clarkson, 1987), research with Year 6 students in Papua New Guinea suggests that competence in a student's first language is also equally important (Clarkson, in press b; Clarkson & Galbraith, 1992). On this basis, the wisdom of restricting the teaching of mathematics in the mother tongue to Year 1, with English or some other second language being used as the language of instruction thereafter, is questionable.

In summary, although research on the effects of bilingualism in mathematics learning is in its infancy, Australasian mathematics educators have already made significant contributions to identifying key issues in the field. There are at least two areas which can be built on fruitfully: First, there is a need to relate the outcomes of this research more closely with models of how the brain operates; second, although the interplay between language and culture is more transparent in research carried out with bi- or multi-lingual students, or with students who belong to a minority culture, the same issues are pertinent with respect to the mathematics learning of monolingual students from different sub-cultures within a single dominant culture.

Children Writing Mathematics

While in the 1980s there was a growing international interest in the notion of giving children more opportunities to *write* mathematics in order to achieve greater ownership on the part of learners, it was in the second half of the decade that Australian educators began to make important contributions to the "children writing mathematics" literature. These contributions can be classified under the broad headings: (a) children creating mathematics, and (b) children writing about mathematics. The latter category covers a wide range of genre, from students' simple verbal recounts of what they have done in mathematics classes to metacognitive, in-depth reflections by students on how and why they approached a mathematical problem-solving task in the way they did (Mousley & Marks, 1991; Waywood, 1991).

Children creating mathematics. In most mathematics classrooms children are asked to solve problems designed by other people (usually adults); they rarely have the opportunity of creating or communicating mathematics problems of their own. In a study investigating how children acquire mathematical concepts, Ellerton (1989b) asked a large sample of 10 500 secondary school students in Australia and New Zealand to make up a difficult mathematics problem, and then to solve it. Students' responses reflected the language of the mathematics classrooms in which they worked - while there were relatively few words, extensive use was made of symbolic mathematical language and abstract contexts. The problems the students created rarely made reference to real-world contexts. Clearly the students had been exposed, predominantly, to receptive language, and had had little opportunity to operate in expressive modes (see Figure 2).

Pengelly (1990a, b) believes that children should be immersed in a mathematical environment in which they have access to materials and activities

so that children can mathematise their ideas and structures. Through the children's mathematical writings, it is possible to follow the development of their mathematical language and understanding. While her approach allows children to be exposed to all aspects of expressive language (see Figure 2) as well as to as much receptive language as they choose to take in, there is an ever-present tension concerning the need to ensure that children *own* the mathematics with which they are engaged, yet at the same time ensuring that mathematical ideas underlie the environment that the teacher has provided (Pengelly, 1992).

Children writing about mathematics. Research in a number of countries has suggested that considerable educational benefits derive from having children write, on a regular basis, about what they perceive to be happening when they have mathematics lessons (see, for example, Ellerton & Clements, 1991, pp. 136-140, for a brief review). Students can be asked to make daily entries in their journals in which they write frankly, not only about the topic they are studying, and about why they are studying it, but also about their successes, difficulties, frustrations, sudden inspirations, strategies, and plans for action, for example. They can be given a relatively free hand in what they say (Mildren, Ellerton & Stephens, 1990) or they can be asked to express their understanding of particular mathematical concepts, skills or generalisations (Miller, 1990, 1991 calls these "impromptu prompts"). Lovitt and Clarke (1988) also suggest a variety of classroom contexts and activities for encouraging children to write about mathematics.

In a longitudinal study, Waywood (1986, 1990, 1991) introduced junior secondary school students at Vaucluse College to the idea of making journal entries about the mathematics they were doing. Waywood found that journal writing represents a powerful instrument for aiding the metadevelopment of students (Clarke, Stephens & Waywood, 1989, in press). Mildren (1992) has investigated the use of journals in the primary school: children in Grades 4, 5 and 6 were asked to write under three headings *What I did, What I learnt,* and *Problems and difficulties.* He used the children's entries to exemplify metacognitive knowledge and experiences, that is to say, the awareness, control, and regulation of an individual's cognitive processes (Mildren, Ellerton & Stephens, 1990).

In a large study, Clarke (1987) asked 700 Grade 7 students to provide written, confidential (but not anonymous) responses to a series of questions probing their reactions to the mathematics they were experiencing in the classroom. He coined the term *Impact program* to describe his data-gathering technique, and provided rich data in support of his claim that the program is able to generate fascinating and important data that can shed new light on the mathematics classroom.

Projective techniques, in which children are asked to respond, in writing, to a particular situation relating to mathematics or to the teaching of mathematics, have been found to be fruitful in helping mathematics educators

understand children's perception of mathematics and of what goes on in the mathematics classroom (Ellerton, 1988; Ellerton & Clements, 1989a).

Davison and Pearce (1990, p. 19), in their investigation of the effectiveness of writing activities in the mathematics classroom, found that students identified two writing activities as being particularly beneficial to their understanding of mathematical concepts: explaining why certain mathematical exercises were missed, and writing definitions in their own words. As a note of caution, Davison and Pearce observed that teachers implementing writing activities in the mathematics classroom should not only have substantial in-service preparation, but also need to believe in the effectiveness of the approach.

Mathematics in Context: Story Maths

A dilemma facing mathematics teachers who are attempting to focus their mathematics program on activities that students see as personally interesting and relevant, is that what is most interesting for one learner could well be boring or threatening for another. One approach is to capitalise on the fact that almost everyone likes a good story. It is in that sense that stories are ideal vehicles for extending the personal worlds of children, and for linking these worlds directly with important mathematical concepts and powerful problem-solving strategies.

The idea of well known stories and poems being used as mathematical "springboards" for stimulating mathematical discussion, activities and projects for primary school students was taken up by Australian mathematics educators (see for example, Clyne & Griffiths, 1990; Griffiths & Clyne, 1988, 1990, 1991; Lovitt & Clarke, 1988, pp. 453-465). Lovitt and Clarke (1988, p. 439) also suggested that stories such as *King Kaid of India*, and Lewis Carroll's *Alice* books and E.A. Abbott'e *Flatland*, can be used as stimulus material for secondary mathematics students.

Sometimes placing a mathematical topic in the context of a brief story, created specifically for the purpose of illuminating that topic, can provide the context which will motivate students to carry out sustained and creative problem-solving activities. Thus, for example, ratio concepts might be introduced through a story about glasses of lemonade and orange juice, "leftover" after a party, all being poured into a jug; the children might consider the relative strengths of the orange taste, given different proportions of glasses of lemonade and orange juice (Lovitt & Clarke, 1988, pp. 182-184). Lovitt and Clarke have referred to this as providing a "story shell framework" (pp. 437-466).

One practical difficulty with this idea is that most teachers have insufficient time to create a range of appropriate, mathematically rich stories. While several Australian mathematics educators have attempted to provide suitable story shell frameworks for a range of topics (see, for example, Ellerton & Clements, 1992; Lovitt & Clarke, 1988), it is nevertheless true that

someone else's story shell is not as likely to be as effective as one created by classroom teachers with the needs and interests of their own students in mind.

Other Australian Research

There have been a number of other important Australasian research studies conducted during the past four years that, although also having language and mathematics as their major thrust, did not obviously fit into any of the five major areas just reviewed. This other Australasian research will now be briefly summarised under the headings: textbooks, the nature of teacher-student interactions, the influence of semantic structure on children's understanding of word problems, and early childhood issues.

Textbooks. While textbooks are clearly a major resource in many mathematics classrooms, Shield (1988, 1989, 1991a) demonstrated that students are often unable to read textbooks with understanding. It is possibly for this reason that teachers rarely use the theoretical explanatory sections in textbooks as a primary reference source for students, but rather use them to provide exercises and problems for students to complete in class or at home. Shield (1991b) complemented his research on the role of textbooks by providing guidelines to assist teachers to select appropriate mathematics textbooks for their students.

The nature of teacher-student interactions in mathematics classrooms. Leder's (1987, 1988, 1989) work on the discourse in mathematics classrooms showed that both the quantity and quality of teacher's and students' interactions are influenced by gender factors and the perceived ability of students. Readers are referred to the chapter on gender issues in this volume for more detailed discussion on this issue.

Mousley, Clements, and Ellerton (in press) and Ellerton and Clements (1991), reported studies on teacher perceptions of their role in constructivist mathematics learning environments. Ellerton and Clements (1991, pp. 53-87) looked at the quality of teacher-student, student-teacher, and student-student interactions in seven examples of mathematics learning environments, and then placed the seven environments on an ownership/content continuum that had "teacher/textbook/examiner" at one pole, and "student deciding content and method without guidance" at the other pole. They concluded that the quality of the mathematics learning environment was not necessarily a function of its position on the continuum.

The influence of semantic structure on children's understanding of word problems. Lean, Clements and Del Campo (1990) suggested that the widely accepted information processing models explaining how students process change, combine and compare problems (for which additive and subtractive operations are used) are not adequate. Their data, collected in a cross-cultural study involving children from Australia and Papua New Guinea, were more in accord with psycholinguistic theories than with North American information

processing theories. Mulligan (1991, in press) developed similar ideas to the change, combine, compare classification of addition and subtraction tasks, by first classifying students' answers on multiplication and division problems, and then using this scheme to suggest more appropriate teaching strategies. Watson and Mulligan (1990) used the SOLO taxonomy to investigate students' processing of multiplication problems, and Boulton-Lewis and Halford (in press) explored the processing loads of young children who were attempting to come to terms with place value concepts.

Early childhood studies. Bell (1990), who investigated children's early counting and particularly the dependence of immigrant children on their first language when learning to count, suggested that the Piagetian contention that children should not be introduced to double digit numbers too early because of the difficulties inherent in the naming process does not necessarily apply to many Asian children, for whom the language structures are different. Macmillan (1990) and Wright (1991) found that children beginning school have often acquired a wide range of number concepts, and have varying abilities to convey their understandings linguistically. Taken together, these studies suggest that early childhood school curricula may be under-valuing the knowledge that young children bring to school. Reference to other Australasian research investigating young children's learning of mathematical concepts can be found in the chapter in this book on *Early Mathematics Development.*

CONCLUSIONS

There can be little doubt that the upsurge in the number of publications reporting research on issues associated with language and mathematics reflects an increasing awareness of the importance of the influence of language factors on mathematics learning. The two main messages of the Australasian research reported in this chapter are that (a) educators have taken up the exciting challenge of creating learning environments for school children, and for pre-service and in-service teachers, in which the modes of communication in mathematics teaching and learning contexts are greatly expanded; and (b) Australasian mathematics education researchers are evaluating the effects of the learning environments that have been created.

The ongoing challenge for mathematics teachers and teacher educators is to develop programs that not only succeed in "dejargonising" school mathematics as much and as quickly as possible, but also provide learning environments that create links between the personal worlds of children and both the formal language (verbal and written) and applications of the mathematics they study (Ellerton & Clements, 1991, p. 143).

An associated challenge is for mathematics educators to stimulate interest, especially among policy makers and curriculum developers in state and national

bodies, in the fundamental issues raised in this chapter. Without an active response from policy makers and curriculum developers to these issues the resources and support needed to generate appropriate action at all levels will not be forthcoming, and mathematics classrooms will not become more productive places of learning.

While the intertwining of language, mathematics and culture is now widely accepted as inevitable, a deeper understanding of the interrelationships between the various strands of the model depicted in Figure 1 is vital if we are to succeed in creating education environments that will allow learners to become fluent in the language of mathematics and confident in their ability to apply this fluency in their daily lives. This provides a foundation on which future research into language factors in mathematics learning should be built.

REFERENCES

Australian Educational Council (1990). *A national statement on mathematics for Australian schools*. Melbourne, Victoria: Curriculum Corporation.

Baker, A., Semple, C., & Stead, T. (1990). *How big is the moon?* Melbourne, Victoria: Oxford University Press.

Bell, G. (1990). Language and counting: Some recent results. *Mathematics Education Research Journal, 2*(1), 1-14.

Bickmore-Brand, J. (Ed.).(1990a). *Language in mathematics*. Melbourne, Victoria: Australian Reading Association.

Bickmore-Brand, J. (1990b). Implications from recent research in language arts for mathematical teaching. In J. Bickmore-Brand (Ed.), *Language in mathematics* (pp. 1-9). Melbourne, Victoria: Australian Reading Association.

Boulton-Lewis, G., & Halford, G. (in press). The processing loads of young children's and teachers' representations of place value and implications for teaching. *Mathematics Education Research Journal*.

Bourke, S. (1984). Insights into the teaching of mathematics in primary schools. *Victorian Institute of Educational Research Bulletin, 52*, 6-18.

Bruner, J. (1986). On teaching thinking: An afterthought. In J. W. Segal, S. F. Chipman, & R. Glaser (Eds.), *Thinking and learning skills: Research and open questions* (pp. 597-608). Hillsdale, NJ: Lawrence Erlbaum.

Cambourne, B. (1989). *The whole story: Natural learning and the acquisition of literacy in the classroom*. Gosford, New South Wales: Ashton Scholastic.

Clarke, D. J. (1984). Secondary mathematics teaching: Towards a critical appraisal of current practice. *Vinculum, 21*(4), 16-21.

Clarke, D. J. (1987). The interactive monitoring of children's learning of mathematics. *For the Learning of Mathematics, 7*(1), 2-6.

Clarke, D. J. (1990). Communication and the learning of mathematics. In K. Milton & H. McCann (Eds.), *Mathematical turning points: Strategies for*

the 1990s (pp. 20-32). Hobart, Tasmania: Australian Association of Mathematics Teachers.

Clarke, D. J., Stephens, W. M., & Waywood, A. (in press). Communication and the learning of mathematics. In T. A. Romberg (Ed.), *Mathematics assessment and evaluation: Imperatives for mathematics educators*. Albany, NY: The State University of New York Press.

Clarkson, P. C. (1980). The Newman error analysis: Some extensions. In B. A. Foster (Ed.), *Research in mathematics education in Australia, 1980*(Vol. 1, pp. 11-22). Hobart, Tasmania: Mathematics Education Research Group of Australia.

Clarkson, P. C. (1983). Types of errors made by Papua New Guinea students. *Educational Studies in Mathematics, 14*, 355-367.

Clarkson, P. C. (1987). Language and mathematics: A brief review of literature from Papua New Guinea. *Papua New Guinea Journal of Education, 23*(2), 155-162.

Clarkson, P. C. (1991a). *Bilingualism and mathematics learning*. Geelong, Victoria: Deakin University Press.

Clarkson, P. C. (1991b). Mathematics in a multilingual society. In K. Durkin & B. Shire (Eds.), *Language and mathematical education* (pp. 237-247). Milton Keynes, UK: Open University Press.

Clarkson, P. C. (1991c). Language comprehension errors: A further investigation. *Mathematics Education Research Journal, 3*(2), 24-33.

Clarkson, P. C. (1991d, July). *Unknown/careless errors in a mathematical language context: A further investigation*. Paper presented at the annual conference of the Mathematics Education Research Group of Australasia, University of Western Australia, Perth, Western Australia.

Clarkson, P. C. (in press a). Mathematics and language: Culture and implementation. In N. F. Ellerton & M. A. Clements (Eds.), *Mathematics and science education: Cultural contexts*. Geelong, Victoria: Deakin University.

Clarkson, P. C. (in press b). Language and mathematics: A comparison of bi and monolingual students in mathematics. *Educational Studies in Mathematics*.

Clarkson, P. C., & Galbraith, P. (1992). Bilingualism and mathematics: Another perspective. *Journal for Research in Mathematics Education, 23*(1), 34-44.

Clarkson, P. C., & Kaleva, W. (in press). Mathematics education in Papua New Guinea. In G. Bell (Ed.), *Asian perspectives on mathematics education*.

Clements, M. A. (1980). Analysing children's errors on written mathematical tasks. *Educational Studies in Mathematics, 11*(1), 1-21.

Clements, M. A., & Del Campo, G. (1989). Linking verbal knowledge, visual images, and episodes for mathematical learning. *Focus on Learning Problems in Mathematics, 11*(1), 25-33.

Clements, M. A., & Del Campo, G. (1990). How natural is fraction knowledge? In L. Steffe & T. Wood (Eds.), *Transforming children's mathematics education: International perspectives* (pp. 181-188). Hillsdale, NJ: Lawrence Erlbaum.

Clements, M. A., Grimison, L., & Ellerton, N. F. (1989). Colonialism and school mathematics in Australia, 1788-1988. In N. F. Ellerton & M. A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 50-78). Geelong, Victoria: Deakin University.

Clements, M. A., & Lean, G. A. (1988). Discrete fraction concepts and cognitive structure. In A. Borbas (Ed.), *Proceedings of the Twelfth International Conference for the Psychology of Mathematics Education* (Vol. 1, pp. 3-23). Vészprem, Hungary: International Group for the Psychology of Mathematics Education.

Clyne, M., & Griffiths, R. (1990). Metre and measure: Mathematical poetry. In M. A. Clements (Ed.), *Whither mathematics?* (pp. 152-155). Melbourne, Victoria: Mathematical Association of Victoria.

Davis, G. (1990). Research issues in language aspects of mathematics. In G. Davis & R. Hunting (Eds.), *Language issues in learning and teaching mathematics* (pp. 120-125). Bundoora, Victoria: Institute of Mathematics Education, La Trobe University.

Davis, G., & Hunting, R. (1990). *Language issues in learning and teaching mathematics*. Bundoora, Victoria: Institute of Mathematics Education, La Trobe University.

Davison, D. M., & Pearce, D. L. (1990). Perspectives on writing activities in the mathematics classroom. *Mathematics Education Research Journal, 2*(1), 15-22.

Del Campo, G., & Clements, M. A. (1990). Expanding the modes of communication in mathematics classrooms. *Journal für Mathematik - Didaktik, 11*(1), 45-79.

Dickson, L., Brown, M., & Gibson, O. (1984). *Children learning mathematics: A teacher's guide to recent research*. Oxford: Schools Council.

Donovan, M., Blamey, C., George, E., Bishop, M., Crookham, J., Gyford, A., Manook, J., Strang, W., & Leitao, S. (1990). The development of mathematical understanding. In J. Bickmore-Brand (Ed.), *Language in mathematics* (pp. 59-73). Melbourne, Victoria: Australian Reading Association.

Durkin, K., & Shire, B. (Eds.).(1991). *Language and mathematical education*. Milton Keynes, UK: Open University Press.

Ellerton, N. F. (1988). Exploring children's perception of mathematics through letters and problems written by children. In A. Borbas (Ed.), *Proceedings of the twelfth annual conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 280-287). Veszprém, Hungary: International Group for the Psychology of Mathematics Education.

Ellerton, N. F. (1989a). The interface between mathematics and language. *Australian Journal of Reading, 12*(2), 92-102.

Ellerton, N. F. (1989b). *The development of abstract reasoning in mathematics*. Unpublished PhD thesis, Mathematics Department, Victoria University of Wellington, New Zealand.

Ellerton, N. F., & Clements, M. A. (1988). Reshaping school mathematics in Australia 1788-1988. *Australian Journal of Education, 33*(3), 387-405.

Ellerton, N. F., & Clements, M. A. (1989a). Students' perceptions of mathematics, science and English classes. In B. Doig (Ed.), *Every one counts* (pp. 221-227). Melbourne, Victoria: Mathematical Association of Victoria.

Ellerton, N. F., & Clements, M. A. (1989b). *Teaching post-secondary mathematics at a distance: A report to the Commonwealth Secretariat*. Geelong, Victoria: Deakin University.

Ellerton, N. F., & Clements, M. A. (1990a). Language factors in mathematics learning. In K. Milton & H. McCann (Eds.), *Mathematical turning points: Strategies for the 1990s* (pp. 230-258). Hobart, Tasmania: Australian Association of Mathematics Teachers.

Ellerton, N. F., & Clements, M. A. (1990b). Culture, curriculum and distance teaching of mathematics. In T. Evans (Ed.), *Research in distance education* (pp. 200-218). Geelong, Victoria: Deakin University Press.

Ellerton, N. F., & Clements, M. A. (1991). *Mathematics in Language: A review of language factors in mathematics learning*. Geelong, Victoria: Deakin University.

Ellerton, N. F., & Clements, M. A. (1992). *Story maths: Putting mathematics into context*. Geelong, Victoria: Deakin University

Faulkner, R. (1992). *Research on the number and type of calculation errors made by registered nurses in a major Melbourne teaching hospital*. Unpublished M.Ed. Research Paper, Deakin University, Victoria.

Ferrer, L. M. (1991). Diagnosing children's conceptions of fractions and decimals. *Understanding and Doing: What Research Says, 1*(1), 1-7.(A publication of the Regional Education Centre for Science and Mathematics, Penang, Malaysia.

Gawned, S. (1990). The emerging model of the language of mathematics. In J. Bickmore-Brand (Ed.), *Language in mathematics* (pp. 27-42). Melbourne, Victoria: Australian Reading Association.

Graham, B. (1988). Mathematical education and aboriginal children. *Educational Studies in Mathematics, 19*, 119-135.

Graves, D. (1983). *Writing: Teachers and children at work*. London: Heineman.

Griffiths, R., & Clyne, M. (1988). *Books you can count on: Linking mathematics and literature*. Melbourne, Victoria: Nelson.

Griffiths, R., & Clyne, M. (1990). *More than just counting books*. Melbourne, Victoria: Nelson.

Griffiths, R., & Clyne, M. (1991). The power of story: Its role in learning mathematics. *Mathematics Teaching, 135*, 42-45.

Harris, P. (1989). Contexts for change in cross-cultural classrooms. In N. F. Ellerton & M. A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 79-96). Geelong, Victoria: Deakin University.

Harris, P. (1991). *Mathematics in a cultural context*. Geelong, Victoria: Deakin University.

Hunting, R., Pitkethly, A., & Pepper, K. (1990). Knowing and telling: Language carriers and barriers in early fraction learning. In G. Davis & R. P. Hunting (Eds.), *Progress in mathematics education 1* (pp. 100-111). Melbourne, Victoria: Institute of Mathematics Education, La Trobe University.

Jimenez, E. C. (1992). *A cross-lingual study of Grade 3 and Grade 5 Filipino children's processing of mathematical word problems*. Unpublished manuscript, SEAMEO-Regional Education Centre Science and Mathematics, Penang, Malaysia.

Kearins, J. (1991). Number experience and performance in Australian Aboriginal and western children. In K. Durkin & B. Shire (Eds.), *Language and mathematical education* (pp. 247-255). Milton Keynes, UK: Open University Press.

Kownan, M. B. (1992). *An investigation of Malaysian Form 2 students' misconceptions of force and energy*. Unpublished manuscript, SEAMEO-Regional Education Centre for Science and Mathematics, Penang, Malaysia.

Lean, G. (1991). *Counting systems of Papua New Guinea* (2nd ed., Vols. 1-17). Lae, Papua New Guinea: Papua New Guinea University of Technology.

Lean, G. A., Clements, M. A., & Del Campo, G. (1990). Linguistic and pedagogical factors affecting children's understanding of arithmetic word problems. *Educational Studies in Mathematics 21*(2), 165-191.

Leder, G. C. (1987). Teacher student interaction: A case study. *Educational Studies in Mathematics, 18*, 255-271.

Leder, G. C. (1988). Teacher-student interactions: The mathematics classroom. *Unicorn, 14*(3), 161-166.

Leder, G. C. (1989). Do girls count in mathematics? In G. C. Leder & S. N. Sampson (Eds.), *Educating girls: Practice and research* (pp. 84-97). Sydney: Allen & Unwin.

Leder, G. C. (1990). Talking about mathematics. *Australian Educational Researcher, 17*(2), 17-28.

Lidster, S., & Watson, J. (1990). Is it time to move from sex differences to SES differences in mathematics education? In K. Milton & H. McCann (Eds.), *Mathematical turning points* (Vol. 2 pp. 376-380). Hobart, Tasmania: Australian Association of Mathematics Teachers.

Lovitt, C., & Clarke, D. M. (1988). *MCTP Professional development package: Activity bank* (Vols. 1 & 2). Canberra, Australian Capital Territory: Curriculum Development Centre.

MacGregor, M. (1991a). *Making sense of algebra: Cognitive processes influencing comprehension*. Geelong, Victoria: Deakin University.

MacGregor, M. (1991b). Teaching mathematics in the multicultural classroom. *Vinculum, 28*(3), 6-8.

MacGregor, M., & Moore, R. (Eds.).(1991). *Teaching mathematics in the multicultural classroom*. Melbourne, Victoria: Institute of Education, University of Melbourne.

McIntosh, A. (1988). Process approach to mathematics - Whence and whither? *Prime Number, 3*(1), 13-21.

MacMillan, A. (1990). Constructivism in a kindergarten mathematics class. *Mathematics Education Research Journal, 2*(2), 12-27.

Marinas, B., & Clements, M. A. (1990). Understanding the problem: A prerequisite to problem solving. *Journal of Science and Mathematics Education in Southeast Asia 13*(1), 14-20.

Marks, G., & Mousley, J. A. (1990). Mathematics education and genre: Dare we make the process writing mistake again? *Language and Education, 4*(2), 117-135.

Mellin-Olsen, S. (1987). *The politics of mathematics education*. Dordrecht: Reidel.

Mildren, J. (1992). *Metacognition in mathematics education: A view through an examination of children's mathematics journals*. Unpublished M.Ed. Research Paper, Deakin University, Victoria.

Mildren, J., Ellerton, N. F., & Stephens, M. (1990). Children's mathematical writing - A window into metacognition. In M. A. Clements (Ed.), *Whither mathematics?* (pp. 356-363). Melbourne, Victoria: Mathematical Association of Victoria.

Miller, L. D. (1990). When students write in algebra class. *Australian Mathematics Teacher, 46*(2), 4-7.

Miller, L. D. (1991). Constructing pedagogical content knowledge from students' writing in secondary mathematics. *Mathematics Education Research Journal, 3*(1), 30-44.

Mousley, J., & Clements, M. A. (1990). The culture of mathematics classrooms. In M. A. Clements (Ed.), *Whither mathematics?* (pp. 397-406). Melbourne, Victoria: Mathematics Association of Victoria.

Mousley, J. A., Clements, M. A., & Ellerton, N. F. (in press). Teachers' interpretations of their roles in mathematics classrooms. In G. Leder (Ed.), *Curriculum planning + Assessment = Learning?* Sydney, New South Wales: Allen & Unwin.

Mousley, J., & Marks, G. (1991). *Discourses in mathematics*. Geelong, Victoria: Deakin University.

Mousley, J. A., Sullivan, P., & Clements, M. A. (1991, July). *The perceptions which student teachers have of teaching practices in classrooms observed*

during field experience. Paper presented at the annual conference of the Mathematics Education Lecturers' Association, Perth, Western Australia.

Mulligan, J. (1991, July). *The role of intuitive models in solving multiplication and division word problems*. Paper presented at the annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia.

Mulligan, J. (in press). Children's solutions to multiplication and division word problems: A longitudinal study. *Mathematics Education Research Journal*.

Newman, M. A. (1977). An analysis of sixth-grade pupils' errors on written mathematical tasks. In M. A. Clements & J. Foyster (Eds.), *Research in mathematics education in Australia, 1977* (Vol. 2, pp. 269-287). Melbourne, Victoria: Swinburne College Press.

Newman, M. A. (1983). *Strategies for diagnosis and remediation*. Sydney, New South Wales: Harcourt, Brace Jovanovich.

Ohia, M. (1991). The unresolved conflict and debate: An overview of bilingual education in New Zealand secondary schools. In A. Begg, B. Bell, F. Biddulph, M. Carr, M. Carr, J. McChesney, & J. Young Loveridge (Eds.), *Science and Mathematics Education Papers 1990* (pp. 111-132). Auckland, New Zealand: Longman Paul.

Ohia, M., Moloney, M., & Knight, G. (1991). A survey of mathematics teaching in New Zealand secondary school bilingual units. In A. Begg, B. Bell, F. Biddulph, M. Carr, M. Carr, J. McChesney, & J. Young Loveridge (Eds.), *Science and Mathematics Education Papers 1990* (pp. 133-158). Auckland, New Zealand: Longman Paul.

Ora, M. (1992). *An investigation into whether senior secondary physical science students in Indonesia relate their practical work to their theoretical studies*. Unpublished manuscript, SEAMEO-Regional Education Centre for Science and Mathematics, Penang, Malaysia.

Pengelly, H. (1990a). Mathematical learning beyond the activity. In L. P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education: International perspectives* (pp. 89-107). Hillsdale, NJ: Lawrence Erlbaum.

Pengelly, H. (1990b). Acquiring the language of mathematics. In J. Bickmore-Brand (Ed.), *Language in mathematics* (pp. 10-26). Melbourne, Victoria: Australian Reading Association.

Pengelly, H. (1992). Mapping mathematical meaning. Melbourne, Victoria: Ashton Scholastic.

Radiah Mohidin (1991). *An investigation into the difficulties faced by the students of Form 4 SMJA secondary school in transforming short mathematics problems into algebraic form*. Penang, Malaysia: SEAMEO-Regional Education Centre for Science and Mathematics.

Reeves, N. (1990). The mathematics-language connection. In J. Bickmore-Brand (Ed.), *Language in mathematics* (pp. 90-99). Melbourne, Victoria Australian Reading Association.

Richards, L. (1990). "Measuring things in words": Language for learning mathematics. *Language Arts, 67*(1), 14-25.

Robinson, I. (1986). Exploring mathematics in classrooms. In N. F. Ellerton (Ed.), *Mathematics: Who needs what?* (pp. 80-85). Melbourne, Victoria: Mathematical Association of Victoria.

Robinson, I. (1989). The empowerment paradigm for the professional development of teachers of mathematics. In N. F. Ellerton, & M. A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 269-283). Geelong, Victoria: Deakin University.

Robinson, I. (1990). Mathematics and language: The experiences of EMIC and Key Group. In G. Davis & R. Hunting (Eds.), *Language issues in learning and teaching mathematics* (pp. 84-89). Bundoora, Victoria: Institute of Mathematics Education, La Trobe University.

Shield, M. (1988). The use of mathematics textbooks: Research and directions. *Research in Mathematics Education in Australia, May*, 57-63.

Shield, M. (1989). Mathematics teachers' preferences in textbook characteristics. *Mathematics Education Research Journal, 1*(1), 11-15.

Shield, M. (1991a). Mathematics textbooks: How are they used? *Australian Journal of Reading, 14*(1), 60-68.

Shield, M. (1991b). Mathematics textbooks: Much maligned much neglected. *Australian Mathematics Teacher, 47*(3), 24-27.

Siemon, D. E. (1989). Knowing and believing is seeing: A constructivist's perspective of change. In N. F. Ellerton, & M. A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 250-268). Geelong, Victoria: Deakin University.

Singhatat, N. (1991). *Analysis of mathematics errors of lower secondary pupils in solving word problems.* Penang, Malaysia: SEAMEO-Regional Education Centre for Science and Mathematics.

Sobhachit, S. (1991). *An investigation into students' understanding of the electrochemical cell and the electrolytic cell.* Penang: SEAMEO-Regional Education Centre for Science and Mathematics.

Souviney, R. (1983). Mathematical achievement, language and cognitive development: Classroom practices in Papua New Guinea. *Educational Studies in Mathematics, 14*, 183-212.

Spolsky, B. (1987). *Report on Maori-English bilingual education.* New Zealand: Department of Education.

Stacey, K., & MacGregor, M. (1991).Difficulties os students with limited English language skills in pre-service mathematics education courses. *Mathematics Education Research Journal, 3*(2), 14-23.

Steffe, L., & Wood, T. (Eds.).(1990). *Transforming children's mathematics education.* Hillsdale, NJ: Lawrence Erlbaum.

Stoessiger, R. (1988). Using language learning conditions in mathematics. *Pen 68* (Primary English Teaching Association), 1-4.

Sullivan, P., & Clarke, D. (1991a). *Communication in the classroom: The importance of good questioning.* Geelong, Victoria: Deakin University.

Sullivan, P., & Clarke, D. (1991b). Catering for all abilities through "good" questions. *Arithmetic Teacher, 39*(2), 14-18.

Teoh, K. S. (1991). *An investigation into three aspects of numeracy among pupils studying in Year three and Year six in two primary schools in Malaysia*. Penang, Malaysia: SEAMEO-Regional Education Centre for Science and Mathematics.

Thomas, J. (1989). Understanding disadvantage: The social context of teaching mathematics. In B. Doig (Ed.), *Every one counts* (pp. 184-186). Melbourne, Victoria: Mathematical Association of Victoria.

von Glasersfeld, E. (1991). *Radical constructivism in mathematics education*. Dordrecht: Kluwer.

Vygotsky, L. (1986). *Thought and language*. Cambridge, MA: MIT Press.

Waters, M., & Montgomery, P. (1989). Stimulating the early learning environment in a maths classroom. In B. Doig (Ed.), *Every one counts* (pp. 78-85). Melbourne, Victoria: Mathematical Association of Victoria.

Waters, M., & Montgomery, P. (1990). Language learning and mathematics learning: A parallel approach. In J. Bickmore-Brand (Ed.), *Language in mathematics* (pp. 74-89). Melbourne, Victoria: Australian Reading Association.

Watson, H. (1989). A Wittgensteinian view of mathematics: Implications for teachers of mathematics. In N. F. Ellerton & M. A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 18-30). Geelong, Victoria: Deakin University.

Watson, H. (1990). The Ganma project: Research in mathematics education by the Yolngu Community in the schools of Laynhapuy (N.E. Arnhemland). In G. Davis & R. Hunting (Eds.), *Language issues in learning and teaching mathematics* (pp. 33-50). Bundoora, Victoria: Institute of Mathematics Education, La Trobe University.

Watson, I. (1980). Investigating errors of beginning mathematicians. *Educational Studies in Mathematics, 11*, 319-330.

Watson, J., & Mulligan, J. (1990). Mapping solutions to an early multiplication word problem. *Mathematics Education Research Journal, 2*(2), 28-44.

Waywood, A. (1986). English expression: A precursor to mathematics. In N. F. Ellerton (Ed.), *Mathematics: Who needs what?* (pp. 173-176). Melbourne, Victoria: Mathematical Association of Victoria.

Waywood, A. (1990). Mathematics and language: Reflections on students using mathematics journals. In G. Davis & R. Hunting (Eds.), *Language issues in learning and teaching mathematics* (pp. 51-59). Bundoora, Victoria: Institute of Mathematics Education, La Trobe University.

Waywood, A. (1991). *Journal writing and learning mathematics*. Melbourne, Victoria: Australian Catholic University Mathematics Teaching and Learning Centre.

Wright, B. (1991). What number knowledge is possessed by children beginning the kindergarten year of school? *Mathematics Education Research Journal, 3*(1), 1-16.

Zepp, R. (1989). *Language and mathematics education.* Hong Kong: API Press.

PART III

Specific Topics in Mathematics Learning

RESEARCH ON EARLY CHILDHOOD MATHEMATICAL DEVELOPMENT

Robert W. Perry
School of Education
University of Western Sydney

Joanne T. Mulligan
Faculty of Education
Macquarie University

Robert J. Wright
Faculty of Education, Work and Training
University of New England - Northern Rivers

In this chapter, the authors review research publications in the field of early mathematical development which have arisen from Australasian mathematics education researchers over the over the period 1988-1991. No doubt, there are some such publications which were missed and to those researchers the authors apologise. Nonetheless, the chapter does review many papers from authors who are either citizens of the Australasian countries or were working in those countries at the time of publication.

This review has been divided into a number of sections in order to make it more coherent. In each section, relevant papers are discussed, critically reviewed and clearly referenced so that interested readers may pursue the issues to greater depth.

CHILDREN CONSTRUCTING MATHEMATICS

Research in Australasia into the mathematical development of young children has been greatly affected by the work of the constructivists and, in particular, by the Action Group of International Congress of Mathematics Education (ICME-6) which dealt with early childhood mathematics. In fact, of the 46 papers in the report of this Action Group, ten were authored by Australians. This work has continued in the intervening four years and has resulted in much of the work discussed here. The importance of constructivism in Australasian mathematics education research was given focus at a special workshop held during the 1989 Annual Conference of the Australian Association for Research in Education. Papers presented by Bell (1989), Wright (1989c), Davis and

Hunting (1989) all reported on the use of constructivist methodologies in mathematics education research. The view was then put by Perry (1989) that

> The constructivist methodology facilitates the development of models about how children learn mathematics by allowing researchers a detailed view of children constructing and making meaning of mathematical ideas. However, as the children develop their ideas, their interactions with the researcher as teacher allow the researcher to construct models of learning. That is, the processes of reflectiveness are facilitated for both the subject and the researcher. (p. 4)

The use of constructivist approaches in the classroom was given proper recognition by both Macmillan (1990) and Mousley (1990). Macmillan (1990) showed how young children might be encouraged to consider the concept of elegance of solutions to problems through the development of a closer awareness of their own mathematical thinking. Macmillan reported on and analysed her exploration, with her class, of mathematical problems. She concluded that teachers and children can benefit from the exploration of the children's thinking as they attempt to solve the problems set by the teacher and that:

> A powerful opportunity exists for the teacher to contribute to the achievement of higher levels of conceptual understanding during the process of pupil engagement in thinking and the construction of meaning. (p. 25)

Mousley (1990) reported on observations of a series of lessons and a teacher which bring together the notions of constructivist approaches and problem-posing. The paper explored the encouragement of the active involvement of all children in the classroom in problem-posing, redefining and recording problems, attempting to find solutions, sharing these attempts and solutions and thinking about alternative ways of moving towards solutions. The role of the teacher in all of these stages was analysed and a case was made for the encouragement of such approaches by educational authorities, in spite of the appearance, perhaps, of a slightly chaotic classroom organisation.

Of course, early mathematics education research in Australasia has not been uni-dimensional-it would be difficult to argue that constructivism itself is such-and other approaches to the quest for knowledge of young children's mathematical development have been undertaken vigorously.

Learning Mathematics Through Resource Production

Makin and her colleagues (Geoghegan, Makin & Rogers, 1988; Makin, 1990) developed an approach to mathematics education in the early years of primary school which they called Approaches to Student-Produced Instructional

Resources in Education (ASPIRE). The ASPIRE Program involved teachers and children in the following sequence of activities:

* decision-making about the mathematics concepts to be explored and the approaches to this exploration,
* development of the content of the resources to be produced,
* development and practice of any new skills required,
* editing of production material,
* development of accompanying material,
* presentation of resources to the audience, and
* evaluation of the process and product.

Makin claimed that, through the production of the resources, students learned a great deal of mathematics, both directly and indirectly.

Davis (1989) reported on a particular methodology using children's reflections on their own or other children's mathematical activities. Video records of the mathematical activities were made and the children watch and report on these records. Davis pointed out the value of the approach for both the researcher and the teacher and suggests that it can provide unexpected insights into both sets of children's thinking. He did, however, warn that the approach requires careful planning both before the initial video records are made and at the stage of the children reflecting on it. The technique was used by Davis and Pilkethly (1990) to investigate cognitive aspects of sharing in young children.

PROBLEM SOLVING

The Australian Journal of Early Childhood devoted one of its 1990 issues to mathematics. In this forum, Groves and Stacey (1990) discussed the role of problem solving as a link between mathematics and children's realities. Based on their extensive work with young children, they reported on numerous problem solving activities and the reactions of young children to them. This allowed the development of a rationale for using problem solving as a vehicle in the mathematical development of the children. The concluding paragraphs of the paper summarised the findings.

> Many of the major issues in mathematics teaching such as the role of language, the use of co-operative group work, children taking responsibility for their own learning and the use of calculators and computers can be addressed through problem solving....
>
> ... By creating a classroom climate in which mathematics is strongly linked to children's reality, teachers can build on children's natural curiosity and mathematical experiences outside the classroom. (p. 11)

English's research (e.g., English, 1988, 1990, 1991a, 1991b, 1991c) focused on young children's strategies and competence in solving novel problems in the combinatoric domain. One study by English (1991c) reviewed relevant Piagetian theory, and hypothesised that young children would display more sophisticated combinatoric skills at an earlier age than suggested by Piagetian studies. English's study involved individual interviews with 50 children whose ages ranged from 4 years 6 months to 9 years 10 months, and aimed to determine the strategies children use when producing combinations of two items drawn from discrete sets of items. The interview involved presenting seven tasks within a context of dressing toy bears in all possible different outfits, that is, tops and pants. Task 1 had the purpose of familiarising the children with the materials and terminology. Tasks 2 and 3 were 2x3 combination problems. Task 2 for example, involved presenting the child with sets of yellow and blue tops and sets of yellow, blue and pink pants. Task 4 was a 3x3 combination problem, while Task 5 was a 2x3 combination with an additional constraint that the third bear must have a blue top. The constraint was aimed at assessing robustness of children's solutions. Task 6 was also a 2x3 combination problem, but involved variation in the number of buttons rather than colour. Task 7 was similar to Task 6 but involved forming all possible different total numbers of buttons. Tasks 6 and 7 were designed to assess children's flexibility in generating procedures.

A pilot study identified a series of six, increasingly sophisticated, solution strategies. Thus, of interest in this study, were how these strategies vary across children's ages, and the changes in children's strategies as they solve a series of increasingly complex combinatorial problems. The solution strategies in increasing order of sophistication are:

* random selection of items with no rejection of inappropriate items;
* trial-and-error with rejection of inappropriate items;
* emerging pattern in item selection with rejection of inappropriate items;
* consistent and complete cyclical pattern in item selection, with rejection of inappropriate items;
* emergence of an "odometer" pattern in item selection, with possible selection; and
* complete odometer pattern in item selection, with no rejection of items.

The findings of this study included confirmation of a result from an earlier study. The sophistication of the solution strategy increased with age and the older children demonstrated considerable improvement in their solution strategies over the course of the tasks. In concluding, English highlighted the potential of combinatorics as an area of study for young children. The study demonstrated what young children can do, rather than what they cannot do and

the utility of learning paradigms in which children can discover and reflect on a range of different procedures.

English (1991a) reported the results of a study of similar focus to the one described above, involving interviews with 65 children and five tasks set in the context of dressing toy bears. As well as describing the six solution strategies or paths, English outlined two principles, which she hypothesised, are implicit in children's knowledge, and specific to the combinatorial domain - the principle of difference and the odometer principle. Children acquire a more sophisticated knowledge of the principle of difference during task execution, viz, the disjunctive concept that, in the context of dressing toy bears, if the tops or the shorts are different, then the combinations are different. The odometer principle refers to the emergence of systematic procedures for selecting items, and is analysed in terms of the properties of systematic variation, constancy, exhaustion, and completion. English emphasised that young children have the capacity to acquire independently, efficient algorithmic procedures, and that early mathematics programs should provide appropriate problem-solving experiences to capitalise on this.

English (1990) argued for greater recognition of young children's cognitive competence, and advocated increased emphasis in early childhood mathematics curricula on the teaching of logical reasoning, and flexible and creative thinking, because of their use in problem solving and other areas of mathematics. Examples of appropriate learning activities were provided. English (1991b) provided a further extension of the reports discussed earlier. With respect to the combinatoric tasks described above, children's scanning actions and achievement levels, as well as their solution paths were discussed. Thus this report focused on changes in solution paths as children progressed along the tasks, interactions between solution paths and scanning actions, and the effects of these interactions on achievement levels. Scanning actions referred to eye focus and head movements during task execution, for example: focusing on one versus both types of items, on used versus unused items, or before versus after item selection. Achievement levels were differentiated by presence or absence of errors, need for assistance, and voluntary verbal awareness of their means of goal achievement. Six scanning actions and five achievement levels were identified. Demonstrated in English's analysis is growth in competence as children progress across the series of tasks. This growth is attributed to the acquisition of more sophisticated domain-specific knowledge principles. English develops the following argument:

> The major moderator of children's independent learning in a novel domain is the interaction between their conceptual and procedural knowledge structures and their higher-order cognitive processes. (p. 85)

The reports discussed above focus on the empirical analyses by English. Her work also includes extensive theoretical analyses and the development and

application of a theoretical framework of children's competence in the combinatorial domain (English, 1988).

DEVELOPMENT OF MATHEMATICAL CONCEPTS

Number Learning

Leder (1989, 1990) investigated pre-schoolers' mathematics and the effect of home influences on acquiring mathematical competency. Leder addressed the propositions that children acquire considerable mathematical knowledge before they start formal schooling, and that parents play a critical part in this process. The data illustrated children's views about numbers, and described the extent to which pre-school children were able to use their own written representations of small groups of objects to solve a specific problem - to select from a group of four closed and identical tins the one which contained the number of blocks specified. Data pertaining to parents were obtained from responses to a questionnaire and through interviews and diaries. Results indicated that the children had a sound grasp of the various ways in which numbers are typically used in real life experiences. Approximately 75% of the children surveyed were able to use written representations which allowed them to select repeatedly and with 100% accuracy the tin with the designated number of blocks, both at the time at which the representations were first made, and again one week later.

While parents claimed that they used mathematics extensively at home and at work, there was relatively little overlap in the activities they described in the two settings. Parents' own fears about mathematics and the reasons cited for the importance of the subject for their children, revealed rather narrow views of mathematics. These views were reflected in the mathematical activities parents introduced to their children at home. As well as such parent initiated activities, all the children were regularly exposed to various children's programs on T.V. Their mathematical content, however, was not acknowledged specifically by parents.

Sierink and Watson (1991) report a study on place value using a series of tasks to assess understanding and determine common errors with a sample of 60 children in Years Prep to 4. Children were classified as operating on several levels on each task and overall performances were used to allocate them to one of the five stages in the interpretation of two-digit numbers. These stages were briefly identified as whole number, positional property, face value, construction zone, and understanding.

Results for Tasmanian children were compared with those obtained for children in Switzerland and the U.S.A. In some cases, the levels of performance were higher for the Australian children. It was not possible, however, for the researchers to make detailed comparisons of the studies. The results showed that using these tasks provided valuable indicators for

differentiating difficulties children may experience and for providing instructional devices which could aid understanding.

Bell (1988b) extended Steffe, von Glasersfeld, Richards and Cobb's (1983) theory of counting types by undertaking an analysis of counting from a linguistic viewpoint. Bell posited three meaning systems underlying children's numerical concepts - extension, count and linguistic. These correspond respectively to Leech's (1981) denotative, associative and thematic meaning types. Drawing on the work of Steffe .. al. and also Thomson (1982), Bell outlined eight routines or schemes (e.g. integrating, patterning, extending), which "were depicted as being driven by different cognitive levels corresponding approximately to Piaget's pre-conceptual, intuitive and operative developmental stages" (p. 107).

Bell applied his model of children's numerical meaning system in the construction of case studies for each of eight children who participated a six-month long teaching experiment (see also Bell, 1990). Bell (1988a, 1988b, 1990) investigated the influence of number naming grammars on young children's initial place value concepts. In reviewing the extensive body of research associated with the Piagetian notion of conservation, Bell observed that previous research "seemed to take the Standard Number Word Sequence (SNWS) as an invariant" (p. 3), Observing that there is ample evidence of cross-cultural differences in attainment on arithmetic tasks" (p. 3), Bell also investigated the role in children's numerical development, of number naming systems in particular languages. Of interest were structural differences in number naming systems of Germanic or Romance languages, versus those of Asian languages such as Vietnamese, Mandarin and Japanese. Bell reviewed a separate investigation of similar focus conducted by Miura in the U.S. which concluded that, "although socialisation could account for some of the cultural differences in mathematical attainment, there appeared also to be differences in basic mental representation of numbers as they are linguistically represented" (p. 6)

Bell's study involved individual teaching sessions, once per week, over a 15 week period, with four mono-lingual English-speaking (ME) children and four bilingual Vietnamese-speaking (BVE) children. The teaching sessions focused on developing SNWS skills and their application to addition and subtraction. The theoretical basis for the study was Steffe and von Glasersfeld's model of conceptual structures (see Steffe et al. 1988) for the unit 10, which posits eight levels, the first of which is a specified perceptual collection and the last of which is an abstract composite unit. The study focused on differences between the two groups of children in the exhibited conceptual structures for 10. Clear trends were evident in Bell's results: three BVE children and no ME child exhibited 10 as an abstract composite unit; and, with the exception of one ME child, on six of the conceptual structures for 10, BVE children were in advance of ME children. Bell rejects the current deferment of the teaching of 2-digit numbers and advocates for teachers, the use of base 10 blocks to model two-digit numbers with simultaneous use of

decimalised number names (e.g., "two tens and three") with the corresponding numerals. In recommending the use of specific activities to overcome the inherent linguistic difficulties of the English SNWS, Bell's suggestions include exposing children to Vietnamese or Japanese number names.

Boulton-Lewis (1988a, 1988b, 1989, 1990, 1991, Boulton-Lewis and Halford, 1990a, 1990b, Halford and Boulton-Lewis 1989, in press) reported on a large scale suite of research projects which have applied Halford's structure mapping theory of cognition to the analysis of processing loads of representations and strategies used in mathematics learning in Years 1 to 3 of school. Place value, counting, operations on whole numbers, fraction concepts, spatial and measurement ideas have all been tested. In 1992, the study will be extended to include Years 4 to 6.

Potential for school learning was found to be closely related to processing capacity. Boulton-Lewis asserted that children's capacity to process information as well as their knowledge base develop with age. In order to cognize a new concept a child must have mapped the task into a mental model that corresponds to the structure of the task. The difficulty that children experience in cognizing many concepts may be due to the process of mapping the task into a mental model and the processing load that it imposes.

Boulton-Lewis drew attention to the use of concrete materials, often used by children and teachers as representations of mathematical concepts. She explained that mapping the concept into the mental model entails a processing load, the size of which depends on the complexity of the task and the nature and familiarity of the representation. It has been noted increasingly in recent literature that the concrete representations often fail to produce expected positive results. Boulton-Lewis proposed that teachers often take insufficient account of the difficulties that children experience with the increase in processing load that using concrete materials entails.

Halford's theory allows the level of complexity of selected mathematical tasks to be determined so that any additional processing load may be assessed. The 1989 sample consisted of 29 children aged between 5 and 8 years and their teachers in suburban Brisbane schools in low, medium and high socioeconomic areas. The children were interviewed and videotaped individually, before and after they were taught specified numerical concepts, to determine knowledge of concepts, representations and procedures. Teachers were interviewed and asked to describe their objectives, teaching strategies and representations for selected numerical concepts. Similar methodologies have been used in later studies.

Certain representations and procedures used by teachers, with the very best of intentions, increased memory loads or caused inefficient behaviour or responses. The results suggested that whether or not teacher-determined strategies and representations facilitate or interfere with learning depends on their goodness of fit with the child's existing procedures, how much they increase the processing load for each individual child, how motivated the child is, and how good the teaching is.

For example, with place value, the teacher representations and strategies which caused the most confusion or increased the processing load were using a variety of materials to represent three-digit numbers so that the children did not reach a level of expertness with any one material and allowing the children to use inaccurate or meaningless terms to describe two-digit numbers. For subtraction, the greatest memory or processing loads seemed to arise from focusing for a long time on addition before introducing subtraction, the counting backwards procedure, children not being taught to read all of the symbols, assuming complete knowledge of representations and relationships between them and teaching subtraction algorithms without regrouping before those requiring grouping.

The Boulton-Lewis/Halford (1989) Structure Mapping Theory of cognition has resulted in many questions having to be asked about approaches which have been traditionally taken in mathematics education. For example, the belief in the importance of the generative rule for counting and its strong relationship to levels of knowledge of place value places great emphasis on children not only knowing the Standard Number Word Sequence as far as possible but, also, being able to understand this rule.

Wright (1988, 1989b, 1990, 1991a, 1991c) extended the theoretical models of Steffe et al. (1983) in a teaching experiment which charted the numerical development of four children in the first year of school. The study involved weekly teaching sessions from six to eight months, with four children initially classified as counters of perceptual unit items. The teaching activities included; counting visible and partially or totally screened collections, recognising spatial and auditory patterns, counting sequences of sounds and movements, and number word sequence tasks. Retrospective analyses of videotaped teaching sessions led to the construction of individual case studies focusing on the children's counting types, meanings of number words, accommodations and, distinctive to this study, forward and backward number word sequence developments and their comparison with progression across the counting types. That the counting types progress from greater to less dependence on sensory experience was wholly supported by the study, and each of the five counting types was frequently observed. Two children advanced to the abstract stage, a third gave indications of being abstract in the last teaching session, and the fourth advanced from perceptual to motor. The hypothesis that experiences in counting sequences of sounds and movements, and visualising spatial patterns can contribute to prenumerical children's advancement to the abstract stage was supported.

Included in Wright's study was a review and critique of Fuson's (Fuson, Richards, & Briars, 1982) model of children's elaboration of the number word sequence. Wright posited separate models of the construction of forward number word sequences (FNWSs) and backward number word sequences (BNWSs) in the range one to thirty and argued that "although there are many experiential factors which promote the development of FNWSs in preference to BNWSs, there appears to be no theoretical reason why the construction of

BNWSs must necessarily lag behind" (Wright, 1989b, p. 284). Wright concludes that "children can develop facility with BNWSs which is somewhat independent of and parallel to their facility with FNWSs" (p. 286).

Wright (1989a and in press) outlined British and U.S. factors that have influenced Australian curricula in early childhood mathematics during the last 30 years:

* Cuisenaire - which de-emphasised children's counting
* Piaget's theories - still a dominant influence on prenumber and early number
* the back-to-basics movement in the U.S. which continues to strongly influence introductory topics on arithmetical operations
* the re-emphasis on counting.

(Wright, in press, p. 4)

In Wright's terms, shortcomings of current curricula and dominant text series include: lack of challenge for many children, an overemphasis on topics such as sorting, matching and classifying, and an overemphasis on activities with concrete materials and pictorial representations at the expense of more challenging arithmetical tasks. Features which are prominent in some state curricula and text series of which Wright is critical include: a limited view of the role of children's arithmetical thinking strategies, and when using base ten blocks to teach algorithms, "training the children to verbalise steps which correspond closely to the steps of the standard algorithm" (p. 4).

In Wright's terms these features highlight the influence of the U.S. back-to-basics movement of the mid-70s on current Australian mathematics curricula. Drawing on the work of Steffe and Cobb (1988), Wright urged greater emphasis on the development of children's verbal number sequences rather than paper-and-pencil algorithms, and underlines the need for research-driven, classroom-based interventionist studies in line with those of Cobb, Wood, and Yackel (in press a, in press b) in the U.S. Additionally, Wright argues for the inclusion of activities based on spatial and auditory patterns, number word sequence activities, and counting in problematic situations - activities which are prominent in Steffe's work. Examples of appropriate instructional activities, and their significance for young children's numerical development are described in detail in Wright (1991c).

Wright's two-phase study of children's number knowledge in the first year of school focused on beginning number knowledge (Wright, 1991d) and its development through the year (Wright, 1991b). The first phase involved a videotaped interview with each of 45 children, 15 from each of two kindergarten year classes from different schools, and fifteen from a Year 1 class - the second year of school - from one of those schools. The interview focused on assessing the child's knowledge of number word sequences and spatial patterns, and ability to recognise numerals and use counting to establish the numerosity of visible and screened collections. The schools were selected

as typifying contrasting socio-economic levels. Five models, each of a different aspect of number development, and each containing up to five levels or stages, were used to generate for each child, a profile of numerical development. This led to consideration of the variations of children's number knowledge, within and between year levels and schools. Findings included a wide variation in the number knowledge of children beginning the kindergarten year, greater variation in the lower socio-economic school, and strong indications that the kindergarten year mathematics curriculum is most suited to the least advanced. Wright (1991b) provided a preliminary report of the second phase of this study. The second phase involved subsequent interviews of 41 of the children at the middle and end of the school year. Tabulated results showed the gains over the school year, of each of the 41 children, in terms of the five models referred to above. The study demonstrated that children make significant advancements during the first year of school, but also highlighted mismatches between children's developing numerical knowledge and current curricula. In conclusion, Wright urged curricula reform which makes appropriate use of explanatory models of children's mathematical learning.

Young-Loveridge (1989), a New Zealand researcher, studied the development of children's number concepts in the first year of school. In this study, 80 children aged from 5.0 to 5.1 years, were individually interviewed on each of two occasions. For each child, the first interview occurred when they had been at school for no longer than one month, and the second occurred one year later. "The schools attended by the subjects covered a wide range in terms of geographic area and socio-economic status" (p. 48).

The first interview involved 36 tasks, ranging over topics such as rote counting, understanding the number sequence, enumeration, pattern recognition and addition and subtraction with and without objects. The second interview involved an additional nine tasks. Two teacher questionnaires were also administered. The first focused on teachers' abilities to judge their child's correctness on each task in the initial interview, and the second focused on whether each interview task had been taught during the school year. Data analysis involving the matching of each child's performance on the initial interview with their teacher's responses on the initial questionnaire, showed an overall teacher accuracy of 70%, indicating that, "considering how short a time the children had been at school, teachers were remarkably accurate in their judgements about mathematical knowledge" (p. 58).

Tabulated results included percentages of children who already knew or learned a number concept over the first year of school, according to whether or not the teacher had taught the concept. For example, 85% of the children already knew but were taught to produce a set of two objects, and 40% learned but were not taught the task of 5-2, in a form labelled as "subtraction-imaginary objects".

Young-Loveridge argued that, although teachers can make accurate judgements about what children know, they typically underestimate children's

mathematical skills. Additionally, she was critical of the current curricula in New Zealand which overemphasises activities such as sorting, matching, comparing, ordering and classifying, at the expense of number topics. Given the close similarity between early childhood mathematics curricula in Australia and New Zealand, there was a strong accordance of the results and conclusions of Young-Loveridge and Wright (1991d).

Young-Loveridge (1989c) used case-studies to investigate the relationship between children's home experiences and their mathematical skills on entry to school. This involved interviewing the mothers of six children categorised according to measures of socio-economic status (SES) and achievement on a Number Tasks Interview - two children were categorised as high SES and low achievement, two as high SES and high achievement and two as low SES and high achievement. The study indicated that high achievers, in addition to having many number related experiences, "seemed to find working with numbers intrinsically motivating" (p. 55).

Also significant in explaining differences between high and low scorers were the family's orientation to numeracy, the extent to which parents modelled the use of numbers and mothers' negative attitudes towards mathematics. For example, the family of one of the low scorers seemed to have a basic misconception about numeracy while the other emphasised literacy development. The study highlighted the need for parent education about the importance of numeracy and using everyday experiences to help children learn number.

Young-Loveridge (1990b) provided preliminary results of a study which compared school- and home-based programs of intervention in the number learning of five-year olds. Both intervention programs used number games and number stories. Both programs were successful and greater gains were made in the school-based program. The school-based program involved 26 children, daily sessions of 30 minutes duration over a seven-week period, and was not based on individualised instruction. Young-Loveridge (1990a) also examined gender differences in a longitudinal study of number development of children aged five to nine. For children who started school in the lowest of three achievement groups, the boys began with a very small advantage over the girls but, by the end of fourth full year at school, this advantage had increased considerably. This suggested that girls who begin school with low levels of number knowledge are more likely than boys to remain as low achievers.

Young-Loveridge (1989a) also examined the mathematical content of interactions between eleven preschool children and their mothers during baking sessions of duration ranging from 41 to 77 minutes. Tabulated results showed, separately for mothers and children, the frequencies of words in the categories of number, size, shape, position. In contrast to an earlier study involving studio recording of interactions rather than recording in a familiar setting, such as one in which mother and child are at home, the children in this study used number words considerably less than their mothers did, whereas in the earlier study children's number word use exceeded that of their mothers.

Young-Loveridge (1988) argued against "the anti-interventionist perspective which has tended to dominate many mathematics programs in recent years" (p. 39). Rather than wait for children to discover mathematics on their own "early childhood teachers clearly have a role to play in helping children learn number skills, particularly for those children whose families do not provide them with appropriate experiences" (p. 39). Young-Loveridge further argued that it is essential "that every child is given the opportunity to participate in experiences involving numbers, and counting particularly" (p. 39).

Young-Loveridge (1990c) discussed and illustrated the use of interviews to explore children's knowledge about numbers, arguing that use of a combination of quantitative and qualitative approaches provides "richer and more powerful data" (p. 85).

Pengelly's (1988, 1989, 1990a, 1990b) research work advocating a "process" approach gained much impetus through the Basic Learning in Primary Schools (BLIPS) funded by South Australian Mathematics Project R-4, and the Catholic Education Office Sydney, Numeracy Project. This action research was conducted largely through the on-going and careful analysis of observations and recordings of children as they worked in mathematical situations. The research approach aimed at giving children freedom to explore and acquire their own written mathematics, and become familiar with principles underlying the written conventions of mathematics.

Pengelly supported the view that "children learn to write mathematics by writing mathematics". She asserts that her research shows children learn mathematics if immersed in an environment that has materials and activities which model mathematical ideas and structures. Commonly, children were working as individuals or pairs on specific activities such as patterning. Standard conventions seem to be gradually acquired by children, through their observations. Pengelly's work challenged traditional styles of teaching mathematics that rely on text book and teacher-directed learning. Her newly coined term Mapping Mathematical Meaning (M3) reflected her approach to teaching and research. Pengelly described M3 as about children inventing in their minds, mathematical understandings, and that their mathematical thinking will eventually begin to approximate more established mathematical ideas. Pengelly's research was influential in causing practitioners to reflect on their overall approach to their children's mathematical learning and the relationships between this learning and language development.

Addition and Subtraction

The large scale comparative study by Lean, Clements and Del Campo (1990) examined linguistic and pedagogical factors affecting children's understanding of arithmetic word problems. This study integrated aspects of a linguistic approach while using various semantic categories and different number sizes for addition and subtraction word problems. The sample contained 1195 children in Years K through 6 in Victorian schools and 1298 children in Years

4 through 6 in Papua New Guinea (PNG). Children were asked addition and subtraction word problems based on Change, Combine, Compare categories with another comparative category for Pure Numbers (which number is 3 more than 4?).

While the test instrument was based on familiar information processing models of how children solve arithmetic word problems, the data were not so much in accord with these models as with psycholinguistic theories. These theories proposed that children acquire linguistic terms of addition and subtraction in polarised comparative pairs such as "more" and "less". The main factor determining difficulty between problem classes was the semantic structure of the questions. While the children from both Australian and PNG samples showed similar strategies and errors, differences in performance between corresponding grades from the two samples were attributed to differences in degree of English language competence rather than to numerical facility.

In analysing errors, the prevalence of the Type 1 and Type 2 strategies showed that children were influenced by the linguistic cues. A Type 1 strategy looks for the larger "more" or smaller "less" of the two given numbers and state this as the answer. A Type 2 strategy applies an additive process (counting-on) "more" or a subtractive process "less" to the two given numbers to obtain an answer. The samples tended to use Type 1 and Type 2 strategies even when the words "more" or "less" were not specifically mentioned such as when "got", "found", "altogether" were associated with "more" and, "lost" and "gave" were associated with "less". Also, there was a tendency to fall back on erroneous Type 1/Type 2 linguistic strategies when more semantically complex problems (e.g. Compare) had to be processed.

It is worth noting here that while Lean et al. propose that the process of representing a problem and the process of drawing a solution are separable, they assert that the child's mental representation is a function of many variables which are not independent of each other. These variables include the syntax and semantics of the problem, the linguistic background of the child and links which the wording of problems provides with the cognitive structures of the child. The influence of the child's preference for analytic as distinct from visual thinking is also acknowledged. Furthermore, Lean et al. point out that recent classroom teaching or other experiences the child has had that relate to the questions being asked are other variables that may affect solution process.

Langford (1988) examined the role of conceptual and linguistic factors on children's solutions to arithmetic word problems. Langford presented a model stating that when young children solve word problems using concrete materials, they do not form a mental schema to represent the information. His study of 6 year olds' solution methods and errors involving addition and subtraction word problems supported the view that much symbolic manipulation occurs "on the table" rather than "in the head".

Langford presented an alternative view to the Riley, Greeno and Heller (1983) model which assumes that all symbolic manipulations take place

internally, and argues that for many problems much of the manipulation is accomplished externally. Several aspects of the results suggested that problems associated with semantic comprehension and previously unrecognised confusions need to be given more attention in models of arithmetical problem solving in children.

Young-Loveridge (1989d) studied the abilities of seven-year-olds to form equations to represent addition and subtraction operations they carried out with materials and to explain the meanings of formed equations. The study found that approximately half the children could explain equations but only about a third could represent operations and, contrary to expectations, equations representing subtraction operations were easier to explain than those representing addition.

Multiplication and Division

Mulligan (1988, 1990, 1991, in press a, in press b, in press c) has conducted research investigating the relationship between informal and formal strategies in the development of multiplication and division processes. Children's solutions to simple multiplication and division word problems were analysed initially in a *cross-sectional pilot study* of 34 children from Years K-2. More than 50% of the sample were able to solve the problems even though they had not received formal instruction in multiplication and division. A wide variety of solution strategies was found involving modelling with counters, counting and additive procedures. Seven problem structures were trailed showing that the semantic structure of the problems strongly affected solution type. Use of counters and size of number combinations used also influenced performance and strategy use. These exploratory findings provided a basis for a 2-year longitudinal study which examined how and why children use particular strategies.

Children's solutions to a variety of multiplication and division word problems were analysed at four interview stages. The study followed 70 children from Year 2 into Year 3, from the time where they had received no formal instruction in multiplication and division to the stage where they were being taught basic multiplication facts. Ten problem structures, five for multiplication and five for division, were classified on the basis of differences in semantic structure. The relationship between problem condition (small or large number combinations and use of physical objects), on performance and strategy use was also examined.

Results indicated that 75% of the children were able to solve the problems using a wide range of strategies even though, for most of the 2-year period, they had not received formal instruction in multiplication or division. Performance level generally increased for each interview stage, but few differences were found between multiplication and division problems except for low performance on Cartesian and Factor problems. Further analysis of individual profiles revealed that 25% of the children were unable to solve two or more of the easiest eleven small number problems at any interview stage.

Solution strategies were classified for both multiplication and division problems at three levels:

* direct modelling with counting
* no direct modelling with counting, additive or subtractive strategies
* use of known facts (addition, multiplication).

Counting strategies were classified as counting-all, skip(multiple) counting, and double counting.

Secondary analysis of the data revealed new evidence of underlying intuitive models for multiplication and division. More than one model was found for multiplication but a strong preference for the repeated addition model was shown. Analysis of partition, quotition and rate division problems across the four interview stages showed widespread evidence of a building-up model for division and this was based on counting or additive strategies where the child built-up to the dividend. For example, the child built up groups of 2 until 16 was reached and verbalised counting-all, skip counting or double counting "2,4,6,8...". Another model, building-down, was distinctive because the child always modelled or counted the dividend first such as "16 take away 2, take away 2....". Sharing one-by-one was rarely used. A change in problem type may have influenced a change in the intuitive model shown, which supports the notion that children can develop more than one intuitive model. The data indicated that children who were more successful showed different intuitive models depending on the problem, and were more efficient and flexible in the strategies used. Mulligan asserted that these findings raise important questions for teaching and learning methods that rely on the sharing and/or repeated subtraction models for division. Informal, but efficient solution strategies may be restricted rather than building on children's intuitive understandings.

Mulligan (in press a) also conducted a constructivist teaching experiment on representing multiplication and division situations with 10 Year 3 children over an 8-week period of the school term. Children were encouraged to represent word problems through oral language, writing, modelling, drawing and symbolising. Further evidence of children's underlying intuitive models for multiplication and division was found and recorded. The use of the skip (multiple counting) patterns and the hundred square were found to be influential in assisting children to develop understanding of multiplication and division patterns. Mulligan concludes that spontaneous use of conflict-based questions to assess conceptual understanding is an important part of facilitating learning of multiplication and division.

Fraction Concepts

Davis and Pitkethly (1990) investigated the logical aspects of 17 Year 2 pre-school children's sharing and counting activities. In the videotaped episodes, the pre-schoolers routinely counted or checked the heights of the stacks when

asked how they knew there was a fair share. The Year 2 children, with some exceptions, expressed the view that the counting after dealing was essential to tell if there was a fair share. The major features of the study suggested that dealing without counting is an implicit, widespread sharing strategy but that children do not have a conscious awareness that it is an adequate procedure. When questioned as to its validity, they will resort to other checking procedures. Counting was also found to be a predominant strategy for checking fair shares.

Building on the work of Hunting and Sharpley (1988a, 1988b), Hunting and Davis (1991) conducted structured interviews with 75 pre-school children engaged in sharing biscuits among dolls. Sharing by dealing was found to be the most common strategy demonstrated by 85% of the sample. However, children were unaware that dealing was sufficient to ensure equality of shares and resorted to counting or measuring to check.

In a further study by Davis and Hunting (1990), on spontaneous sharing of jelly beans by 46 preschoolers in unstructured sharing situations, the children did not use dealing at all. Two possible explanations were discussed by Davis and Hunting, but neither was considered sufficient to explain the results. The first was that the children did not see that dealing or counting would help them in the task. The children, however, used these strategies in an almost identical situation. The second possible explanation was that the children did not see any need to establish fair shares. Again, the researchers had some difficulty with this. There is an important pedagogical issue raised as the result of this research. It is the apparent overtaking of the dealing procedure, used by many preschoolers, by a counting procedure seen in primary children. "This seems to be a real loss, because dealing can be used by a pre-school or infant teacher as an almost universal action scheme for introducing concrete experiences that deal with fractions and ratios of discrete items" (p. 372).

The work of Hunting, Davis and their collaborators has been collected and consolidated into the book *Early Fraction Learning* (Hunting and Davis, 1991). This book is a milestone in the collaborative work in which the editors have been involved for many years.

Pepper's (1991) investigation focusing on preschool children's knowledge of counting and sharing in discrete quantity settings is reported as a chapter in Hunting and Davis (1991). The study involved interviewing, on two occasions, 75 children from two preschool centres, whose ages ranged from 4 years 5 months to 5 years 8 months. The interviews lasted about 15 minutes. One interview was designed to elicit children's sharing strategies while the other was designed to elicit children's counting abilities. At question was whether current practice, in the first years of school, of focusing almost exclusively on whole number knowledge rather than also focusing on early fraction knowledge, was justified in terms of the numerical knowledge of preschool children. In the first mentioned interview the child was asked to share 12 biscuits between two dolls. Following this a third doll was

introduced and the child was required to re-distribute the biscuits among the three dolls. On the latter task, children were classified as good, average or poor sharers. A different classification system, also containing three levels, was applied to the first task. The second mentioned interview was designed to investigate levels of counting development according to Steffe's (see Steffe et al., 1983) theory of counting types, and included verbal counting tasks, quantitative comparisons, and a set of simple additive tasks set in the context of a farm yard. Children were classified as poor counters - counters of perceptual unit items only-, developing counters - counters of figural unit items-, or good counters - counters of motor, verbal or abstract unit items. For each of the two sharing tasks, a 3x3 contingency table shows frequencies according to the counting and sharing classification systems. The study showed that dealing competence (sharing) did not relate directly to counting skills. Pepper suggested that because, for example, poor counters can equally divide groups of discrete items, their sharing may involve prenumerical skills such as subitizing or rudimentary measurement skills. Pepper advocated for preschool children, tasks in which small discrete quantities are hidden from view. Such tasks can further develop children's counting and dealing skills, and informal understanding of addition and subtraction.

The work of Clements and Lean (1988) and Clements and Del Campo (1990) helped distinguish between fractional knowledge which is imposed on children and that which is "natural". Clements and Lean reported on the almost universal incidence of dealing as a strategy for sharing discrete items by children in Papua New Guinea and suggested that the Western ideas of fraction knowledge were very much imposed on these children. This position is further emphasised by the suggestion that "fraction knowledge of the kind taught in Western schools certainly does not appear to be innate. ... For the large majority of school children, fraction knowledge is something that is crammed into their heads in school mathematics lessons, which outside the classroom has little relevance. It is not natural, and is needed only for the purpose of studying more mathematics" (Clements and Del Campo, 1990, pp. 187-188).

Spatial Concepts

Owens (1990a, 1990b, 1991a, 1991b) developed a series of techniques for investigating spatial thinking processes in young children. Her use of introspection, retrospection, observation and paper and pencil tests allowed her to delve into the solutions of spatial problems and the processes which accompany these solutions. When asked to solve three spatial problems, designated as the tangram, pentomino and matches puzzles, it was found that the tangram puzzle helped the children to pose their own problems, use spatial visualisation to show how shapes fitted together and how they compared. The matches puzzle encouraged long-term memory and alternative approaches to solving problems while the pentomino puzzle developed the concept of shape, grid and orientation imagery. The use of videotaping techniques aided the

investigation by allowing the retrospective analysis of the children's manipulations.

The paper and pencil test developed by Owens has been designed to enable her to investigate the thinking of children as they endeavour to answer multiple choice questions. There are six subtests dealing with the following topics: recognition of equality of shapes in rotated or reflected positions, recognition of shapes made by tessellations of a smaller shape, transformation of two-dimensional shapes into three-dimensional shapes by mental folding, recognition of embedded shapes in outlines and completion of shape outlines, construction of models made of parts from memory and/or under transformation, and recognition of equal angles in different orientations or contexts.

This comprehensive test, which takes 90 minutes to administer, is introduced using concrete materials in practice examples but is completed without the aid of such manipulatives. Owens trailed the test and reported that item analyses using both Rasch and traditional item difficulty procedures were equivalent, that the two forms of the test were equivalent and that the concrete introduction and practice items were essential. This test was used in later research on spatial thinking completed by Owens.

In his extensive survey of the research on the development of children's concepts of angle, Mitchelmore (1989, 1990) categorised angles in three ways: a ray-pair giving the difference between two directions, a region of a plane bounded by two rays with a common end-point, and the amount of rotation. He has surveyed the research in which the word "angle" was used both in its mathematical sense and when it is used in the study of figures which contain angles. His findings suggested that children's perceptions of angles develop rapidly from about age 5 until about 12 culminating in the establishment of an angle as a class of equivalent figures. This culmination, Mitchelmore conjectured, takes place independently for the three categories of angle established earlier. An extensive program for future research was proposed, some of which Mitchelmore has begun.

Outhred and Mitchelmore (1991), investigated the transition from concrete to pictorial representations of area in young children. In the pilot project for this study, children from two quite different schools were interviewed individually as they worked through a sequence of tasks designed to investigate the skills involved in representing square coverings of rectangular regions. The tasks involved the children counting dots and squares in arrays, drawing of rectangular arrays of squares from cardboard tile models and diagrams where no measurement was required and drawing of rectangular arrays where either measurement or accurate estimation were required.

The results of the pilot study indicated the difficulty many children have in moving to the pictorial equivalent of their actions in the construction of arrays. They also suggested that young children do not automatically interpret arrays of squares as rows and columns. This may hinder their learning about the measurement of area using diagrams of rectangular regions. To some

extent, this may be avoided through the linking of counting by groups to the structure of the arrays. The pilot study resulted in the researchers hypothesising that both array multiplication and drawing arrays of parallel lines are independent and prerequisite to measurement in two dimensions.

Linear Measurement

Willis (1988a, 1988b, 1990, 1991) examined the linear measurement understandings of young children and the relationship between the ability to measure, understanding of selected dimensional adjectives and conservation of length. Willis tested 160 children in grades 1, 3 and 5 in eight Sydney schools and found that children possess a much broader concept of measurement at a younger age than that postulated by Piaget and continuously maintained by the neo-Piagetians.

Willis found that all the children tested had quite well developed notions about linear measurement prior to being able to conserve length. Each child completed six tasks - two each for conservation of length, measurement of length and understanding of the dimensional adjectives, big, small, long and short. One of the measurement tasks required children to devise a strategy to measure and compare the length of the table tops of three small tables. The measurement tests required children to justify their length decisions with appropriate strategies supported by reasons for their choice. The tests were a strong, positive indication that young children were able to measure using a strategy with perceptual or numerical clues and justify their actions with a reason. Willis examined the data set using a non-parametric analysis of variance based on a program of multi-level chi-square and a probabilistic model for the validation of hierarchies present in dichotomous data sets.

Use of a probabilistic model proved extremely effective in examining the relationships between linear measurement and conservation of length and between conservation of length and selected dimensional adjectives. A sequence of difficulty was generated closely resembling the order of acquisition found in other studies. Two patterns were revealed in the data set: Most of the selected dimensional adjectives precede conservation of length and are linked in sequence; and measurement precedes and is linked to conservation of length ($p < .001$) but is not linked to the selected dimensional adjectives.

Willis asserted that these findings have major implications for the teaching of measurement to children in Years K-2. Although major revisions of elementary mathematics curricula have occurred in most states in Australia, Piagetian theory, particularly the notion of "ages" and "stages", still dominates aspects of the way teaching learning programs in measurement are being inserted into schools. Consequently, children who have well developed notions about measurement, albeit informal, are not being "allowed" to measure until they reach the appropriate age or stage.

OTHER TOPICS IN EARLY MATHEMATICS

Mathematics and Music

Geoghegan (1991) reported on his initial work with a group of five year old children, 40 of whom are participating in a selective music program and 40 of whom have had little, if any, musical training. His aim was to compare their development of number concepts with the hypothesis, based on pilot work over a number of years, that those children undertaking music training will have enhanced number concepts. The study is only in its formative stages but does promise to provide an interesting insight to relationships between mathematics and creative arts.

Teacher Education

Reeves (1990) focused on action research by early childhood teachers as they tried to familiarise themselves with new approaches to mathematics teaching and the relationship between these approaches and those used in the children's language development. Groups of teachers participating in professional development activities in mathematics education became involved in a variety of action research projects which gave them the opportunity to observe and analyse the mathematics learning which was going on in their own classrooms. There were a number of benefits to be had from this approach. The teachers involved become more aware of the nature of mathematics, children's thinking about mathematics and their development of it. There are close links made between mathematical learning and the learning of language and the researchers have a lot more first hand research on which to base their teaching decisions than they would have had if they had relied on traditional sources of research information.

Mansfield (1990) offered a challenge to teachers of young children when she considered the implications of a constructivist view of learning on the role of these teachers. Based on earlier research on children's learning, she suggested approaches for teachers which arise from the key task of ascertaining the knowledge and beliefs possessed by each learning. What followed was an extensive discussion of approaches which have been shown to have benefit in some situations. The point was made, however, that the children will not always bring sufficient ideas to the classroom to allow them to proceed to where the teacher feels they should.

The challenge is to encourage students to construct, through sharing and negotiation, a common mathematical meaning or process that is valid, without undermining their own abilities or distorting the mathematics. If this constructed view appears to be restricting the students' further learning, the teacher can again challenge the students' view and help them construct a new meaning that is more useful to them`.

Following a careful description of distancing strategies, the paper concluded with the observation that the teacher's task is not easy if the choice

is to subscribe to the constructivist approach to learning but the rewards can be great.

REFERENCES

Bell, G. (1988a). Number naming grammars and the concept of "ten". In A. Borbas (Ed.), *Proceedings of the twelfth annual conference of the International Group for the Psychology of Mathematics Education, Volume I* (pp. 154-161). Veszprem: PME.

Bell, G. (1988b). *The acquisition of initial place value concepts in different linguistic environments.* Unpublished doctoral thesis, Macquarie University, Sydney, New South Wales.

Bell, G. (1989, November). *The effect of the linguistic structure of standard number word sequences on children's numerical cognition.* Paper presented to the annual conference of the Australian Association for Research in Education, Adelaide, South Australia.

Bell, G. (1990). Language and counting: Some recent results. *Mathematics Education Research Journal, 2*(1), 1-14.

Boulton-Lewis, G. M. (1988a). The relationship between capacity to process information and levels of mathematical learning. *Proceedings of the twelfth annual conference of the International Group for the Psychology of Mathematics Education.* Veszprem: Psychology of Mathematics Education.

Boulton-Lewis, G. M. (1988b, August). *School learning, knowledge and capacity to process information.* Paper presented to Australian Council for Education Research Seminar on Intelligence, Melbourne, Victoria.

Boulton-Lewis, G. M. (1989). Mathematical learning and children's developing capacity to process information. *The Australian Mathematics Teacher. 45*(2), 30.

Boulton-Lewis, G. M. (1990). Young children's thinking strategies and levels of capacity to process information. In L. P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education - international perspectives* (pp. 156-160). Hillsdale: Lawrence Erlbaum.

Boulton-Lewis, G. M. (1991, July). *The values and limitations of using concrete representations to teach mathematics to young children.* Paper presented to the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia.

Boulton-Lewis, G. M., & Halford, G. S. (1990a, July). *An analysis of the value and limitations of mathematical representations used by teachers and young children.* Paper presented to the fourteenth annual conference of the International Group for the Psychology of Mathematics Education, Mexico.

Boulton-Lewis, G. M., & Halford, G. S. (1990b, November). *Young children and teachers negotiating mathematics.* Paper presented at the annual conference of the Australian Association for Research in Education, Sydney, New South Wales.

Boulton-Lewis, G. M., & Halford, G. S. (in press). Processing capacity and school learning. In G. Evans (Ed.). *Learning and teaching cognitive skills*. Melbourne, Victoria: Australian Council for Educational Research.

Clements, M. A., & Del Campo, G. (1990). How natural is fraction knowledge? In L. P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education - international perspectives* (pp. 181-188). Hillsdale: Lawrence Erlbaum.

Clements, M. A., & Lean, G. A. (1988). Discrete fraction concepts and cognitive structure. In A. Borbas (Ed.), *Proceedings of the Twelfth International Conference on the Psychology of Mathematics Education, Volume 2*. (pp. 215-222). Veszprem: Psychology of Mathematics Education.

Davis, G. E. (1989). Children talking about children's mathematics. *Mathematics Education Research Journal*, *1*(2), 36-42.

Davis, G., & Hunting, R. (1989, November). *Jayme and 1/3*. Paper presented to the annual conference of the Australian Association for Research in Education, Adelaide, South Australia.

Davis, G. E., & Pilkethly, A. (1990). Cognitive aspects of sharing. *Journal for Research in Mathematics Education*, *21*(2), 145-153.

Davis, G. E., & Hunting, R. P. (1990). Spontaneous partitioning: pre-schoolers and discrete items. *Educational Studies in Mathematics*, *21*, 367-374.

English, L. D. (1988). *Young children's competence in solving novel combinatorial problems*. Unpublished doctoral thesis, University of Queensland, Brisbane, Queensland.

English, L. D. (1990). Mathematical power in early childhood. *Australian Journal of Early Childhood*, *15*(1), 37-42.

English, L. D. (1991a). Children's competence in forming combinations. In L. P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education - international perspectives* (pp. 174-180). Hillsdale: Lawrence Erlbaum.

English, L. D. (1991b). Young children as independent learners. In G. Evans (Ed.), *Learning and teaching cognitive skills* (pp. 72-88). Melbourne, Victoria: Australian Council for Education Research.

English, L. D. (1991c). Young children's combinatoric strategies. *Educational Studies in Mathematics*, *22*(5), 451-474.

Geoghegan, N., Makin, L., & Rogers, T. (1988). *ASPIRE: Resources for children by children*. Sydney, New South Wales: Institute of Early Childhood Studies.

Geoghegan, N. (1991, July). *The effects of early childhood music on mathematical development*. Paper presented to the ninth biennial conference of the Mathematics Education Lecturer's Association, Perth, Western Australia.

Groves, S., & Stacey, K. (1990). Problem solving - a way of linking mathematics to young children's reality. *Australian Journal of Early Childhood, 15*(1), 5-11.

Halford, G. S., & Boulton-Lewis, G. M. (1989, September). *Value and limitations of concrete analogs in teaching mathematics.* Paper presented to the third European Conference for Research on Learning and Instruction, Madrid, Spain.

Halford, G. S., & Boulton-Lewis, G. M. (in press). Values and limitations of analogs in teaching mathematics. In A. Demetriou, A. Efkliades, & M. Shayer (Eds.), *Modern theories of cognitive development go to school.* London: Routledge and Kegan Paul.

Hunting, R. P., & Sharpley, C. F. (1988a). Fraction knowledge in preschool children. *Journal for Research in Mathematics Education, 19*(2), 175-180

Hunting, R. P., & Sharpley, C. F. (1988b). Preschoolers' cognitions of fractional units. *British Journal of Educational Psychology,. 58,* 172-183.

Hunting, R. P., & Davis, G. E. (1991). *Early fraction learning.* New York: Springer-Verlag.

Langford, P. (1988). Arithmetical word problems: evidence for thinking on the table. *Research in Mathematics Education in Australia,* June, 1-7.

Lean, G. A., Clements, M. A., & Del Campo, G. (1990). Linguistic and pedagogical factors affecting children's understanding of arithmetic word problems: a comparative study. *Educational Studies in Mathematics, 21,* 165-191.

Leder, G. C. (1989). Number concepts of pre-school children. *Perceptual and Motor Skills, 69,* 1048-1050.

Leder, G. C. (1990, July). *Pre-schoolers' mathematics: the effect of home influences.* Paper presented to the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

Macmillan, A. (1990). Constructivism in a kindergarten mathematics class. *Mathematics Education Research Journal, 2*(2), 12-27.

Makin, L. (1990). Learning mathematics through resource production. *The Australian Mathematics Teacher, 46*(1), 22-23.

Mansfield, H. (1990). The role of the teacher in early childhood mathematics. In L. P. Steffe & T. Wood, *Transforming children's mathematics education - international perspectives* (pp. 383-388). Hillsdale: Lawrence Erlbaum.

Mitchelmore, M. (1989). The development of children's concepts of angle. *Proceedings of the thirteenth international conference for the Psychology of Mathematics Education,* (Vol. 2, pp.304-311). Paris: Psychology of Mathematics Education.

Mitchelmore, M. (1990). Psychologische und mathematische schwierigkeiten beim lernen des winkelbegriffs. *Mathematica Didactica, 13,* 19-37.

Mousley, J. (1990, July). *Organised chaos: problem posing in a constructivist classroom.* Paper presented to the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

Mulligan, J. T. (1988). Children's solutions to story problems. *Prime Number, 3*(2), 5-10.

Mulligan, J. T. (1990). Developing mathematical understanding through word problems, *Prime Number, 5*(4), 3-7.

Mulligan, J. T. (1991, July). *The role of intuitive models in young children's solutions to multiplication and division word problems.* Paper presented to the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia.

Mulligan, J. T. (in press a). Multiplication and division. What we can use for the classroom. *Challenging children to think when they compute.* Brisbane, Queensland: Queensland University of Technology, Centre for Mathematics and Science Education.

Mulligan, J. T. (in press b). Research directions: developing number operations with understanding. *Report of the international symposium on number sense, computation and estimation.* Brisbane, Queensland: Queensland University of Technology, Centre for Mathematics and Science Education.

Mulligan, J. T. (in press c). Children's solutions to multiplication and division word problems: A longitudinal study. *Mathematics Education Research Journal.*

Outhred, L., & Mitchelmore, M. (1991, July). *The transition from concrete to pictorial representation of area.* Paper presented to the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia.

Owens, K. (1990a, July). *Getting inside the problem solver's head: Using retrospection and observation to access spatial and problem thinking processes.* Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

Owens, K. (1990b, November). *When qualitative and quantitative analysis are complementary: An example on the use of visual imagery by primary school children.* Paper presented at the annual conference of the Australian Association for Research in Education, Sydney, New South Wales.

Owens, K. (1991a, July). *Children's responses to a "paper-and-pencil" test of spatial thinking processes.* Paper presented at the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia.

Owens, K. (1991b, September). *Assisting young children with the spatial aspects of mathematics.* Paper presented to the annual conference of the Mathematics Association of New South Wales, Wollongong, New South Wales.

Pengelly, H. (1988). Representing mathematics: from materials to symbols. In D. Plummer (Ed), *Planning for thinkers and learners: The early years* (pp. 89-107). Melbourne, Victoria: Australian Reading Association.

Pengelly, H. (1989). Mapping mathematical meaning through writing. *Australian Journal of Reading, 12*(2), 129-140.

Pengelly, H. (1990a). Mathematical learning beyond the activity. In L. P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education - international perspectives* (pp. 357-376). Hillsdale: Lawrence Erlbaum.

Pengelly, H. (1990b). Acquiring the language of mathematics. In J. Bickmore-Brand (Ed.), *Language in mathematics* (pp. 10-26). Victoria: Australian Reading Association.

Pepper, K. (1991). Preschoolers' knowledge of counting and sharing. In R. P. Hunting & G. Davis (Eds.), *Early fraction learning* (pp. 103-129). New York: Springer-Verlag.

Perry, B. (1989, November). *The teaching of mathematics by adults. The learning of mathematics by children.* Paper presented at the annual conference of the Australian Association for Research in Education, Adelaide, South Australia.

Perry, B. (1990). Cultural perspectives on success in early childhood mathematics. In L. P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education - international perspectives* (pp. 451-464). Hillsdale: Lawrence Erlbaum.

Reeves, N. (1990). Action research for professional development: informing teachers and researchers. In L. P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education - international perspectives* (pp. 436-447). Hillsdale: Lawrence Erlbaum.

Sierink, T. M., & Watson, J. M. (1991). Children's understanding of place value. *Australian Journal of Early Childhood, 16*(4), 33-42.

Willis, A. (1988a, May). *Measurement abilities of young children-A new developmental hierarchy.* Paper presented at the centenary conference of the Australian New Zealand Association for the Advancement of Science, Sydney, New South Wales.

Willis, A. (1988b, December). *Early experiences in measurement: Future directions for mathematics programs in primary schools.* Paper presented at the annual conference of the Australian Association for Research in Education, Armidale, New South Wales.

Willis, A. (1990). *The place of measurement operations in cognitive development.* Unpublished doctoral thesis, Macquarie University, Sydney, New South Wales.

Willis, A. (1991, November). *Assessing linear measurement concepts.* Paper presented at the first national assessment in mathematics conference of the Australian Council for Educational Research, Geelong, Victoria.

Wright, R. J. (1988). The development of the counting scheme of a five year old child. In A. Borbas (Ed.), *Proceedings of the Twelfth Annual Conference of the International Group for the Psychology of Mathematics Education,* (Vol. 2)(pp. 649-656). Veszprem: Psychology of Mathematics Education.

Wright, R. J. (1989a, July). *A commentary on the theoretical bases of early childhood mathematics programs.* Paper presented at the annual

conference of the Mathematics Education Research Group of Australasia, Bathurst, New South Wales.

Wright, R. J. (1989b). Numerical development in the kindergarten year: a teaching experiment. (Doctoral dissertation, University of Georgia). *Dissertation Abstracts International, 50A*, 1588.

Wright, R. J. (1989c, November). *Investigating mathematics learning from a constructivist paradigm.* Paper presented to the annual conference of the Australian Association for Research in Education, Adelaide, South Australia.

Wright, R. J. (1990). Interactive communication: Constraints and possibilities. In L. P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education - international perspectives* (pp. 235-243). Hillsdale: Lawrence Erlbaum.

Wright, R. J. (1991a). An application of the epistemology of radical constructivism to the study of learning. *The Australian Educational Researcher, 18*(1), 75-95.

Wright, R. J. (1991b, July). *A study of mathematical development in the first year of school.* Paper presented at the annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia.

Wright, R. J. (1991c). The role of counting in children's numerical development. *The Australian Journal of Early Childhood, 16*(2), 43-48.

Wright, R. J. (1991d). What number knowledge is possessed by children entering the kindergarten year of school? *The Mathematics Education Research Journal, 3*(1), 1-16.

Wright, R. J. (in press). Number topics in early childhood mathematics curricula: historical background, dilemmas, and possible solutions. *The Australian Journal of Education.*

Young-Loveridge, J. M. (1988). Is greater autonomy always in the best interests of children's mathematics learning? *Australian Journal of Early Childhood, 13*(3), 37-40.

Young-Loveridge, J. M. (1989a, August). *Mathematics in early childhood: The mathematical content of interactions between preschool children and their mothers during baking.* Paper presented at the inaugural biennial conference of the New Zealand Association of Mathematics Teachers.

Young-Loveridge, J. M. (1989b). The development of children's number concepts: The first year of school. *New Zealand Journal of Educational Studies, 24*(1), 47-64.

Young-Loveridge, J. M. (1989c). The relationship between children's home experiences and their mathematical skills on entry to school. *Early Child Development and Care, 43*, 43-59.

Young-Loveridge, J. M. (1989d, December). *Young children's understanding of the formal written symbolism of arithmetic.* Paper presented at the eleventh annual conference of the New Zealand Association for Research in Education, Wellington, New Zealand.

Young-Loveridge, J. M. (1990a, July). *Gender differences in number knowledge: A longitudinal study of five to nine year olds.* Paper presented at the annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

Young-Loveridge, J. M. (1990b, December). *The use of games and books to improve children's number concepts: The EMI-5s study.* Paper presented at the twelfth annual conference of the New Zealand Association for Research in Education, Auckland, New Zealand.

Young-Loveridge, J. M. (1990c). The use of interviews to explore children's knowledge about numbers. In A. Begg, B. Bell, F Biddulph, M. Can, J. McChesnay, & J. Loveridge (Eds.), *Science and mathematics education papers - 1990* (pp. 74-88). Auckland, New Zealand: Longman Paul.

RESEARCH IN MATHEMATICAL PROBLEM SOLVING

Ian J Putt
School of Education
James Cook University

Ian Isaacs
Faculty of Education
Northern Territory University

The previous review produced by the Mathematics Education Research Group of Australasia (Blane & Leder, 1988) reported developments in research in mathematical problem solving under seven broad areas: innovative practice, the importance of problem solving, projects, teaching problem solving, cognitive processes, modelling and applications, and assessment .

A selection of innovative practices from primary to tertiary level were included in the review. These ranged from early work aimed at giving students the opportunity to experience genuine problem solving first hand, to attempts to involve students and teachers in problem solving. The importance of problem solving in the mathematics curriculum was emphasised in a number of articles which discussed the role of problem solving in developing mathematical thinking and in the learning and teaching of mathematics. The review also stressed the importance of the research and development activities being carried out around Australia by mathematics educators working at all levels.

The need for a major shift away from expository teaching was canvassed. It was illustrated with reference to projects such as Reality in Mathematics Education (RIME), Mathematics Curriculum and Teaching Program (MCTP), Practical Applications of Mathematics (PAM), as well as other materials and courses developed by a number of different authors around the country. The changes in approach to teaching mathematics through problem solving were discussed as well as the need for changes in classroom organisation to allow implementation of these approaches, some of which included the use of calculators and computer software. A number of research projects which sought to evaluate the effectiveness of different programs of instruction were outlined. These projects included controlled experiments in regular classrooms and also students' attendance at task centres set up specifically for mathematical problem solving.

An increasing emphasis on the role of cognitive processes was evident from the large number of studies investigating children's use of strategies,

visual imagery, and mathematical reasoning. Included in these studies were those which investigated children's difficulties in problem solving. The interest in this area appeared to stem from a desire to develop and/or refine models of the problem-solving process, or to produce effective instructional programs. A number of studies reported on the influence both cognitive and affective factors have on problem-solving performance. Within most of these studies one of the variables investigated was gender differences in performance and its relation to other variables such as attitudes, perseverance, socioeconomic level, and confidence.

One section of the review was devoted to the process of mathematical modelling. This section examined the definition of mathematical modelling and explored a number of articles which reported on courses on mathematical modelling at secondary and tertiary levels, and the types of real world problems that readily lend themselves to modelling. The final section in the 1988 review focused on the question of assessment of problem solving. Articles included the development and trialing of items which purported to measure problem-solving performance. A number of procedures and techniques for assessment were also reported.

The 1988 selection of recent research in problem solving was quite comprehensive and serves as a useful introduction to the work which has been undertaken in the ensuing four years. Major sources for the 1992 review were the ERIC, PSYCHLIT and AUSTROM databases, mathematics education journals published in Australia and overseas, and proceedings from Australian and overseas conferences. The same broad areas used in the 1988 review will be used here.

THE IMPORTANCE OF PROBLEM SOLVING

A National Statement on Mathematics for Australian Schools (Australian Education Council, 1991) lists as one of the goals for school mathematics that "students should develop their capacity to use mathematics in solving problems individually and collaboratively" (p. 12). It stresses the need to give students "considerable experience in dealing with non routine mathematical problems and unfamiliar situations" (p. 12). The justification for this was clearly stated by Clarke and McDonough (1989) who argued that the mathematics classroom must have a problem-solving focus because "in today's society the major advances will result from the collaborative efforts of groups of individuals who pool their expertise to solve problems which may not even have existed when those same individuals were at school" (p. 20).

There is no doubt that mathematics educators see the centrality and importance of problem solving but one study which investigated high school students' perceptions of the role of problem solving in mathematics found that some more able students perceived problem solving as a separate branch of mathematics which is applied only to puzzle type problems (Isaacs, 1990a,

1990b). While average and above average students were keen to tackle problems, they were not willing to persevere for longer than 15 minutes on a difficult problem (Stacey, 1990b). For students to develop into life long problem solvers who have acquired the knowledge, skills, and attitudes necessary for this, it appeared that teachers should be teaching, and children should be learning, FOR, ABOUT, and THROUGH problem solving (Siemon & Booker, 1990) . They argued that:

> Teaching and learning FOR problem solving is needed to ensure the availability of appropriate knowledge, skills and strategies, built on understanding and exercised with confidence. Teaching and learning ABOUT problem solving is needed to provide the means to access, monitor and direct what is known and what can be done. Teaching and learning THROUGH problem solving is needed to provide a context for further learning and to exercise the application of the knowledge, skills, and processes required as a result of the first two approaches. (p. 26)

TEACHING PROBLEM SOLVING

If the exhortation from Siemon and Booker is taken seriously it should cause a change in the whole approach to the teaching of problem solving - in the way classrooms are organised, the content of the mathematics being studied, the context in which the mathematics is studied, the skills that are deemed important for problem solving, and the way problem-solving performance is evaluated. In this section the studies reviewed focus on changes in classroom practices which reflect the changes above.

One approach which a number of researchers have hypothesised will assist teachers in conventional classrooms to adopt a problem-solving orientation is the use of tasks called "good" questions (Sullivan, 1989; Sullivan & Clarke, 1987; Sullivan & Clarke, 1988). Good questions are defined as those which "require more than simply recall of information, allow the pupils to learn from the act of completing the task, and are open, that is, there are multiple possible correct answers" (Sullivan, Clarke, & Wallbridge, 1991, p. 1) .

In an exploratory study with Years 6 and 10 students in Australian classrooms and Year 10 students in Malaysian classrooms, Sullivan, Clarke and Wallbridge (1991) found the following:

> There was no apparent difference between the responses of pupils working in pairs and those working individually on solving ``good'' questions. .... Year 10 students in both Australian and Malaysian classrooms gave more multiple and general responses than year 6 pupils. .... Instruction did not result in more pupils giving multiple responses, but rephrasing the questions to make the requirement for multiple responses more explicit did. ... Pupils can give multiple

> responses to "good" questions, pupils can learn from answering these questions, and the questions used require more than recall of facts and procedures. (p. 20)

Traditional mathematics texts do not encourage teachers to change their teaching practices in relation to problem solving. However, a group of 65 Filipino mathematics teachers undertaking a specially designed course in problem solving at Curtin University in Perth, Western Australia in 1989/1990, sought ways to enhance two such texts used at the Years 9 and 10 levels in the Philippines (Malone, Johnston, & Kandl, 1990; Malone, Kissane, & Johnston, 1990). These teachers investigated aspects of problem solving which were applicable to the Filipino school setting and then prepared a booklet to accompany each text. The booklets described teaching strategies, provided worksheets, and gave suggestions designed to assist teachers to enhance the lack of problem-solving content in each text. These teachers then used the booklets in training workshops on their return to the Philippines.

A number of studies have been conducted into the effects of long term exposure to problem-solving instruction. Stacey (1989a) reported on a study with 12 and 13 year olds at a wealthy private school in Melbourne, Victoria. The teachers used a problem-solving program of one lesson per week for a year taken from Stacey and Groves (1985). This program was designed to give students experience in learning about simple problem-solving strategies and good habits as well as experience at reflection on what they were doing. In a second contrasting study Stacey (1990b) investigated a class at a working-class school which had been taught for 3 years from age 12 to 15 by a young teacher very committed to teaching through a problem-solving approach and to demonstrating the everyday usefulness of mathematics. This teacher designed programs which emphasised real world applications of new content but lacked any component of reflection on the tasks such as that in the first study (Stacey, 1989a). In each of these studies the experimental classes were compared with similar classes whose instruction was regarded as traditional with occasional problem solving for amusement purposes. Despite the contrasting teaching approaches, there was a marked similarity in the behaviours of the experimental groups from the problem-solving classes compared with the control classes in each study. Both groups of experimental students worked more deliberately and methodically and kept more helpful written records of their work which allowed them to organise the information, to keep track of where they were at, and to guide them in what to do next. The experimental students did not grab at surface relationships in a problem and then close quickly on an answer as was the behaviour of control student who "closed quickly on an answer by combining numbers in the questions in a superficial way" (Stacey, 1991b, p. 12).

A further study was conducted by Stacey (1989b) with single-sex groups of three students to explore why the performance in group problem solving was found to be slightly worse than individual performance. Four groups of girls

from a private school for girls were videotaped solving a collection of problems which included three of the problems used by Stacey (1989a) in her study on patterns with linear generalising problems. Three groups of boys from a coeducational private school were also videotaped while solving the same set of problems. The seven groups covered Years 7 to 9 with both sexes represented at Years 8 and 9 and only girls at Year 7.

Analysis of the videotape protocols revealed that all groups readily generated ideas and hypotheses. Choices made between the ideas, however, were often not considered carefully. All except one of the groups proposed a correct method for solution of each of the three problems, but not all groups were able to select the correct method from those proposed and then follow with the solution. In fact, groups tended to pass over correct methods after they were proposed rather than actively reject these correct methods. Of interest in terms of the differences in performance was the checking behaviour exhibited by the groups. The checking which proved most effective in eliminating wrong methods and ideas compared predictions with specific known cases. Wrong ideas which were dismissed using general arguments tended to resurface later in the problem when larger numbers were involved. Stacey (1991b, 1991c) cautioned teachers to be careful when using small groups in teaching mathematical problem solving. Group behaviour where students demand clear explanations and where ideas are carefully considered and evaluated should be reinforced, and teachers should be certain to provide feedback on correct solutions.

Dossel and Pegg (1990) investigated the procedures used by a cooperative pair and a non-cooperative pair of primary (12 year old) students of above average ability when solving non routine problems, and the influence of co-operation on problem-solving behaviours. The observations made of the two pairs revealed that it was the quality of the interaction that was important in the effectiveness of the cooperative learning situation. This quality was characterised on the part of the cooperative pair by a willingness to listen to each other and to share information, a spirit of co-operation rather than competition, and observation and monitoring of each other's actions and their progress towards a solution. The homogeneity of the ability of the cooperative pair also appeared to be a deciding factor. The authors claimed that more research is needed to determine the variables that enhance the quality of group interaction. Teachers should also be made aware of the difficulty of identifying students who will work well in pairs because, even for the cooperative pair, changes in the interaction occurred from one problem to another and from one time to another.

Innovative Practices

During the 1980s emphasis was placed on the teaching of problem solving with the results that practices have changed right across the school spectrum from primary to senior high school in the teaching of mathematics. Hand in hand with these changes in teaching came the need for appropriate practices and

procedures for assessing problem solving, particularly in secondary schools. One approach which was reported is the development and trialing of a bank of problems for Year 11 mathematics students (Headlam and Stacey, in press). The bank contained 23 problem-solving tasks each of which can be done by an individual student in less than 20 minutes. These tasks were classified as either algebra, geometry or arithmetic and each was calibrated for difficulty using a Rasch procedure. Teachers therefore know the relative difficulty of the tasks and this allows them to gain a measure of a student's problem-solving ability, to match problems to students' ability, to measure a student's problem-solving progress over a year as he/she works through a range of the problems, and to measure the effectiveness of a teaching program for a class.

PERFORMANCE IN PROBLEM SOLVING

Gender Differences

Stacey (1991a) reported on a study which sought to examine the changes across Years 7 to 10 in factors such as knowledge of mathematical content and problem-solving strategies, attitudes to mathematics and beliefs about how it is done, metacognitive factors and aspects of control and affective factors. These factors are regarded in the literature as being related to problem-solving performance. Fourteen mixed-ability classes from a suburban lower-middle class government school in Melbourne were used to obtain data on student achievement, attitudes, metacognitive knowledge and preferred learning styles, especially relating to learning by rote and learning with understanding.

Of particular interest in this study were gender differences on each of these measures. There was no gender difference on either of the achievement measures, one of which was a problem-solving test which involved non routine problems. Stacey (1991a) seemed to expect differences in favour of the boys on the problem-solving test from previous findings on gender differences in mathematics achievement. Girls consistently showed greater metacognitive awareness of mathematical processes. Younger girls attached greater importance than boys to learning styles that are related to the teacher, effort and memorising. By Year 10, however, the values placed on these by the boys had risen to the same level as that of the girls. Neither boys nor girls valued high activity methods even though Stacey indicated that one expects the boys to favour activity more than the girls. Only on confidence and, to a lesser extent, enjoyment did the sex differences consistently favour boys.

Data from this study also yielded insight into the differences between students who can and cannot use mathematical knowledge in unfamiliar situations when the students are judged as having similar mathematical capacities by their teacher. Two Year 10 students who were judged by their teacher as being of similar ability and the weakest in the class, were rated as D (on a scale of A to E) for their overall work in mathematics over the year. Stacey (1990a) investigated each student's capacity to use his/her mathematical

knowledge in relatively unfamiliar problem-solving situation. The female student performed at the median level for the class on two measures of problem solving while the male student was joint lowest in the class on the same measure.

A third male student was rated by the teacher as quite bright and an A student was also investigated because his score was equal to the first male studied above. The female D level student out-performed the male A student on the unfamiliar problems. She had a well developed intuitive strategy of "guess and check" which, coupled with her positive attitude and her self-reliant ways to monitor her progress, allowed her to make optimal use of her somewhat limited mathematical knowledge. Stacey concluded that students' understanding and mastery of mathematical ideas interact with their attitudes and beliefs about learning mathematics, their willingness to think for themselves, and their level of metacognitive knowledge to cause success or failure in using mathematical knowledge in unfamiliar situations.

Cognitive Processes

A useful outline of recent developments in theories of human problem solving was presented by Long (1989), in which she highlighted the computer simulation of cognitive performance behaviour. The importance of domain-specific knowledge and the acquisition and maintenance of higher-order thinking and skills that are necessary to perform the task were stressed. As well she traced the development of mathematical problem-solving research this century and emphasised the importance of a constructivist view of learning and teaching in school mathematics. Long further claimed that there are three types of knowledge which should be considered within the problem-solving process, namely, conceptual, procedural and metacognitive. She argued that classrooms which reflect current knowledge about problem solving ought to incorporate all three types of mathematical knowledge in order that students become mathematical thinkers where the thinking is guided by a well-coordinated, multifaceted knowledge base. In this section a number of studies will be reviewed which deal with different aspects of the cognitive processes used by students when they are solving mathematical problems.

Studies by Sweller and his colleagues were reported in the 1988 review of problem-solving research. One study concluded that studying worked examples instead of solving a large number of problems was a more efficient means for the acquisition of schema in learning algebra (Sweller & Cooper, 1985). These studies found that strategies used by expert and novice problem solvers differ and that the means-ends strategies used by novices place a heavy load on cognitive processing capacity which inhibits schema acquisition (Sweller, Mawer & Ward, 1983; Sweller, 1984; Owen & Sweller, 1985; Sweller, 1988) .

Another source of heavy cognitive load which militated against the benefits of worked examples was guidance in the form of subgoals or worked examples which require attention to two or more sources of information.

Tarmizi and Sweller (1988) found that, in geometry, if information were presented in a format that did not split the learners' attention, their performances using worked examples were superior in solving conventional problems. They stressed that the guidance provided by worked examples or subgoals must not require students to attend to multiple sources of information as this imposes a heavy cognitive load which interferes with schema acquisition (p. 435).

In discussing mathematics and science learning Sweller (1989) claimed there are two pillars to skilled problem-solving performance - schema acquisition and rule automation. A schema was defined as a cognitive construct that permits problem solvers to classify problems into particular categories which require particular moves for solution, and an automated rule as one that can be used without conscious processing (p. 458). He claimed that a heavy emphasis on conventional problem solving is not the best way to acquire schemata or facilitate rule automation. Furthermore, alternatives to conventional problem solving such as goal-free problems or worked examples must be carefully chosen in order to minimise the cognitive load on the student. This claim is substantiated in five experiments using geometric optics and kinematics under junior high classroom conditions (Ward & Sweller, 1990). They found that ineffective worked examples in their experiments could be converted to effective ones by eliminating, as far as possible, the need for students to split their attention among multiple sources of information (p. 38).

The ongoing work of Sweller and his colleagues continues to challenge one conventional mode of mathematics teaching which relies on presentation of new material followed by worked examples and a large number of practice problems or exercises. It also questions the emphasis placed on problem solving and heuristics in the 1980s by pointing out that superior problem-solving performance does not derive from superior heuristics but from domain specific skills (Owen and Sweller, 1989).

Low and Over (1989) focused on algebraic story problems to examine whether text editing skills (identifying what information is missing from problems and what information is irrelevant to problems) are predictive of success in solving these problems. They also examined the relationship between text editing skills and traditional measures of schematic knowledge such as memory and problem categorisation. Seventy-two Year 10 students (approximately equal numbers of males and females) were tested on five tasks over two days. On the first day the tasks were identification of missing information, deletion of irrelevant information, and problem solution. On the second day the tasks were recognition, memory and categorisation through selecting the odd one out of a group of four problems. Results indicate a significantly high correlation between students' ability to edit the text of algebraic story problems and their ability to solve these problems. Text editing also correlated highly with students' ability to categorise problems as being similar to or different from each other. Low and Over indicate that the results

lend support to the claim that knowledge of problem structure is necessary for solution of algebraic story problems. These results appear to support the findings of Sweller and his colleagues regarding schema acquisition.

Research into cognitive processing load of mathematical representations and strategies used by teachers and 5-8 year old children was reported by Boulton-Lewis and Halford (1990) and Boulton-Lewis (1991). Halford's (1988, in press) structure mapping theory of cognitive development was used to determine the level of complexity of two key numerical concepts - subtraction and place value. The additional processing load of typical mathematical analogs (representations) and procedures used by the children and their teachers was also determined. The sample of 29 children, equally spread across Years 1, 2, and 3, was interviewed before and/or after instruction by their teacher on the two numerical concepts in order to assess content knowledge, representation and procedures. All interviews were videotaped.

Following the instruction each teacher was asked to describe the representations and procedures he or she used. Children were tested in the interview on a range of subtraction tasks from basic number facts to 2 digits from 2 digits with and without regrouping, and a knowledge of place value covering the teen numbers (12 - 18), multiples of ten and 2 digit numbers less than 100 and two numbers between 100 and 200. A range of materials was provided for children to use. When the interviewer showed them the task the children were asked to read it, to represent it with materials, and to explain the process.

The results showed complex interactions of knowledge and use of representations, content and procedures in the responses given by each child to the problems (Boulton-Lewis & Halford, 1990, p. 10). For subtraction, the procedure which caused the most incorrect responses on the posttest was counting backwards which the authors demonstrated makes the task of subtraction more difficult because there is a greater memory and processing load.

A surprising result in place value was that 25% of the sample of Year 1 and Year 2 children changed from using more efficient to less efficient materials when representing tens in numbers. The researchers argued that this is a result of these children being introduced to a mixture of materials and therefore not knowing one representation well enough. They claimed that any concrete teaching aid is only useful if children grasp the correspondence between the structures of the material and the concept, and that some representations and procedures which are supposed to assist learning make the task more difficult.

In a study of six year olds solving arithmetic word problems with materials provided, Langford (1988) sought information about strategies and errors that were present. He also manipulated the conditions under which children were allowed to operate in order to compare his model of how young children perform arithmetical word problems (Langford, 1986) with three other

models by Riley, Greeno, and Heller, (1983), Kintsch and Greeno (1985), and Briars and Larkin (1984). The models by Riley et al. (1983) and Kintsch and Greeno (1985) assume that, when young children are presented with a word problem, initial information is represented internally in the form of a mental schema, a strategy is used to find the answer, and then the child builds the model on the table to find out the size of the answer (Langford, 1989). The models by Briars and Larkin (1984) and Langford (1986) assume that, when young children solve arithmetical word problems using concrete materials, they do not form a mental schema to represent the information because the representation with the materials "on the table" serves this function (Langford, 1988).

In Langford's (1988) first condition each child was read the initial information twice before use of materials or counting on fingers was permitted. They were not given any further information. In the second condition each child was permitted to begin modelling as soon as desired and further information could be requested as the model was being constructed. The subjects in the experiment were 20 boys and 20 girls (6.0 - 7.0 years) in Year 1. Both the models by Briars and Larkin (1984) and Langford (1986) which espoused "thinking on the table" as the chief solution method were supported under the second condition with 36 of 40 children choosing to model the problem concurrently with the provision of information. Under the first condition when concurrent modelling was prevented, solution attempts and errors suggested that children were pushed into the internal schema models of Riley et al. (1983).

While the word problems used by Langford (1988) were addition and subtraction, a longitudinal study of children's solutions to multiplication and division word problems was reported on by Mulligan (1989, 1990). She followed 70 children from Year 2 into Year 3 and found that 75% of the children were able to solve these problems even though they had not received formal instruction in multiplication or division number facts or word problems for most of the 2-year period. Eight problem structures were given resulting in three levels of problem-solving strategies - direct modelling with fingers or physical objects, sophisticated counting without direct modelling, and use of known facts. Performance and strategies used were affected by the semantics of the problem, the use of physical objects and the number size. The analysis of solution strategies revealed the intuitive models which appeared to overarch the method of solution. For multiplication problems the predominant model was a two-step process of making equivalent sets and adding them together, although the children were not constrained by this one underlying model. Three distinct models were found for division: (a) sharing one-by-one, (b) building-up, and (c) building-down (Mulligan, 1991). Widespread preference was found for the building-up model although the other two were used prior to instruction. These data have implications for the teaching of division which traditionally relies on the partitive and quotitive models for division in the upper primary school.

The data obtained in a pilot study (Mulligan, 1988) prior to the longitudinal study just mentioned, were analysed by Watson and Mulligan (1990) using a mapping procedure based on the SOLO Taxonomy (Biggs and Collis, 1982). The Taxonomy was expanded to include a procedure for mapping solutions to problems (Chick, Watson, & Collis, 1988; Collis & Watson, 1991). Analysis of the responses of children in Years K - 2 on a simple multiplication problem which was most easily solved by repeated addition, yielded "an eloquent demonstration of the transition which takes place when children move from the Ikonic to the Concrete Symbolic mode of reasoning" (Watson and Mulligan, 1990, p. 41). This represented an extension of the range of application of the SOLO Taxonomy in analysing children's understanding of mathematical concepts and problem-solving abilities. Previous work spanned primary to tertiary levels (eg. Chick, 1988; Pegg & Davey, 1989; Watson, Chick, & Collis, 1988; Watson, 1988b) . The implications of these data for early childhood teachers are important, since children in the lower grades could be operating at as many as six levels some of which may not need ikonic support materials to solve problems

Another study used the SOLO Taxonomy to determine performance quality in problem solving for a group of 10 students, 15 years old in South Australia (Bennett, 1987). The Taxonomy was used to determine the quality of think aloud responses to mathematical word problems. The author developed a framework based on the dimensions of the Taxonomy for analysing and scoring the responses and claimed this refinement of the Taxonomy could be of value to classroom teachers.

Isaacs (1990c) also used the SOLO Taxonomy and a modified form of the Hollowell coding scheme when analysing written solutions and audio-taped responses of Years 7 to 11 secondary students whilst solving non routine proportion problems. He found that both instruments were in substantial agreement in identifying expert and novice problem solvers, but that the SOLO Taxonomy was difficult to interpret when used to analyse students' responses which indicated mixtures of thinking at different levels.

Strategies

Research into strategies has sought to identify the thought processes of students while solving problems. Watson (1988a) used a problem about three hungry men with students from Year 3 through to university and discovered a range of strategies which included working backwards with and without fractions, drawing a picture, ratio, and algebraic equations. She also examined the errors made by students of all ages and drew conclusions from the data for the teaching of problem solving and algebra and the testing of problem solving.

Baillie (1987) investigated the extent to which normal and intellectually handicapped children used visual imagery to assist in problem solving. Twenty children from both primary and special schools in Queensland solved a series of problems covering logical, spatial and numerical thinking and graded according to their amenability for solution using visual imagery. Results

indicated that visual imagery was not used spontaneously in solving the problems. Retrospective data in the form of both unstructured and structured written reports were collected by Owens (1990a) from 52 second year preservice teachers who were required to pose themselves questions to explore with a seven piece tangram puzzle and to construct squares using matches. Results indicated a limited thinking about spatial concepts as indicated by the small number of unprompted comments on imagery. They did lead to a system for classification of the data. Four categories of processing were evident from the data: conceptual - dealing with shape names and properties; imagery - dealing with turning and flipping shapes; heuristic - dealing with planning or strategies; and affective - dealing with feelings. Owens concluded that retrospective written reports did lead to some disclosure of students' thinking while posing problems. Observation and video replay data were also used by Owens (1990b) to determine the spatial and problem thinking processes of twelve children in Years K - 6 and 19 children in Year 4 when solving three spatial problems using manipulatives - pentominoes, a tangram puzzle and a matches puzzle. She concluded that students most likely consider shape as the only property of space to be investigated, and that the three problems aided children's development of spatial concepts and problem-solving skills.

In a study of perseverance when solving difficult or unfamiliar number problems Taplin (1990a) found that many of the Year 6 students (10 male and 10 female) in her study, invented their own strategies to solve the problems. Furthermore, some students were hampered by continually attempting to use "taught" strategies in contexts where they did not recognise them as being inappropriate. She urged teachers to encourage students to be flexible and to try their own strategies when the one they were taught did not work. Taplin (1990b) also found that, when task analysis maps were used to analyse interview protocols, the students who persevered and were ultimately successful were more likely to be flexible in trying different approaches with the same data, while the quitters were more likely to repeat the same approach.

Isaacs (1990d) investigated developments in the quantity and quality of methods used to solve problems by students of Years 7, 9, 10 and 11 from two schools where strategies were taught at all year levels. He hypothesised that Year 7 students would use intuitive methods with a gradual progression in the use of mainly taught methods by Year 11. He found that Year 9 students were beginning to adopt taught methods as well as use intuitive methods whereas the Year 10 students appeared to ignore taught methods and to revert to intuitive ones. There was a reluctance also on the part of the Year 11 students to use taught methods because they did not connect these methods with the type of problem in the study. These findings challenge teachers to help students see the links between techniques taught in school mathematics and problem solving in general.

A process tracing study employing a stimulated recall technique was conducted by Putt and Pountney (1989) to determine the mental processes of ten Year 7 and 8 students while solving word problems and process problems.

Data revealed that all students exhibited some type of mediating process. Mental computation and metacognition were exhibited by all except one of the students while devising a plan and by all students when evaluating their solutions. A number of mediating processes were not evident in any of the students and the authors observed that Year 7 and 8 students found it difficult to monitor their own thought processes while solving problems.

Siemon (1989) conducted a year long teaching experiment with a Year 4 class to examine the role of metacognition in the children's' mathematical problem solving. Protocols from 12 students together with written responses designed to sample attitudes and problem solving yielded the data for the study. She reported on some of the misconceptions revealed by these written responses and the rich insights they provide into students' mathematical thinking as they construct their own mathematics in an attempt to make sense of what is being taught.

ASSESSMENT OF PROBLEM SOLVING

Recent state curriculum documents and *A National Statement on Mathematics for Australian Schools* (ACE, 1991) have stressed the importance of problem solving as a long-term goal. Stacey and Bourke (1988; Bourke & Stacey, 1988) argued that "the provision of a well-researched and reasonably convenient means of assessing goals and charting student progress is an important step to making the teaching of problem solving a credible part of the mathematics curriculum" (p. 12). Stacey and Bourke developed a test of mathematical problem solving for the upper primary school which assessed five problem-solving processes. These were (a) correctness of results, (b) extracting information, (c) method used in solving the problem, (d) accuracy in computation, and (e) quality of exploration (p. 73). When this test was used with Year 4, 5, and 6 students in government schools in Melbourne it was found that:

1. Year 4 performance was generally lower than that of the other students.
2. The differences were small on most items and processes between Years 5 and 6.
3. Most of the differences between the sexes, while not significant, showed higher scores for the girls.
4. A '"lock-step" method of administration was favoured over "free" format and led to better performance especially for the girls.
5. The marking schema which was developed allowed teachers and researchers to make highly reliable judgements.

Two studies from an information processing perspective developed taxonomies of students' problem-solving abilities/behaviours. Fong's (1990) hierarchical taxonomy was based on students' differing abilities to retrieve information in a problem and to apply rules to solve the problem. He hypothesised five levels based on an information processing model of learning and described a pilot study which sought to validate the taxonomy through in depth interviews while students solved problems constructed on the basis of the taxonomy. Yee (1990) developed a taxonomy of problem-solving behaviours. all possible observable behaviours were classified into five categories. These were (a) problem-orientation heuristics, (b) problem-solution heuristics, (c) domain-specific knowledge, (d) metacognition, and (e) affective behaviours. This taxonomy was used to examine in-depth individual problem-solving performance, to examine globally episodes of behaviours such as metacognition and as the basis for a scoring schema to evaluate problem solving performance in terms of observable behaviours.

As assessment instrument which has been published recently is called the Collis-Romberg Mathematical Problem Solving Profiles (Collis & Romberg, 1992). The instrument comes in a kit which includes a manual and a master copy of two profiles - the Junior version suitable for 9 to 13 year olds, and the Senior version suitable for 13 to 17 year olds. Unlike some test manuals, this one explains very succinctly what the Profiles are, why they were developed, and how they are administered and interpreted, before it discusses the detailed research and development which let to the final product.

The Profiles are the product of research in the 1970s by Romberg and his colleagues (e.g., Wearne & Romberg, 1977) on test of mathematical problem solving using "superitems" (Cureton, 1965) coupled with assessment based on the SOLO Taxonomy (Biggs & Collis, 1982). There are five items in each of the Profiles covering the content categories of number, algebra, space, measurement, and chance and data. Each item contains four questions which are designed so that a correct response indicates the ability to respond at the particular SOLO level designated by that question. The different structural levels corresponding to the four questions are unistructural, multistructural, relational, and extended abstract. Once a student's SOLO level has been determined for each of the five items, a profile can be prepared and an overall SOLO level for five students ascertained. Instructions are also given in the manual for preparing a group diagnostic profile based on the SOLO level assigned to each individual.

Collis and Romberg (1992) outline some projects in which the SOLO Taxonomy's theoretical approach is being used in different countries. While the results obtained with these Profiles need to be compared with different measures of problem-solving performance and the quality of students' thinking when solving problems, the Collis-Romberg Mathematical Problem Solving Profiles are a valuable addition to the instruments available to mathematics education researchers working in the area of problem-solving.

Research in this area of assessment is likely to increase in the next five years as the various Australian state education systems move towards the inclusion of problem-solving tasks and investigations in the mathematics curriculum and the forms of assessment move more towards those that are criterion based.

MATHEMATICAL MODELLING AND INVESTIGATIONS

In the previous MERGA review, Stacey and Groves (1988) concluded that during the years 1984-1988 "Problem solving [including applications and modelling] has changed from a fashionable, fringe activity which interested a small number of teachers to a formally recognised part of the curriculum aims of most schools and almost all school systems" (p. 2). While this situation is true, Groves (1989) made a valid point that there needs to be genuine curriculum development in modelling and problem solving if they are to become more than isolated and idiosyncratic activities. She further claimed that "Modelling should be viewed as a natural part of *using* mathematics to solve real problems" (p. 16).

One example of curriculum support materials which allows teachers to explore the teaching of modelling was that produced by the curriculum Development Centre (1988). It is known as the Mathematics Curriculum and Teaching Program (MCTP) and was refereed to in the 1988 review. An example of a two year course with a focus on mathematical modelling rather than on the use of particular models was described by Galbraith and Clatworthy (1989). The course was part of a subject accredited as an entry for tertiary studies in science and engineering. A qualitative research methodology was used with the authors as participant observers for the evaluation of the course. Structured and unstructured data on mathematical performance and the teaching approach were gathered over the two years via oral, written and audio-visual records. The authors concluded that:

1. The course facilitated the performance of quality modelling using the modelling procedure in the Open University (1978) course.
2. The teaching approach of individual, small group and large group activity was highly effective.
3. The system of assessment is a valid and worthwhile one.
4. Personal growth occurred in the students as a result of participating in the course.

CONCLUSIONS

During the period since 1988, state education systems in Australia have incorporated problem solving activities in their primary and secondary

mathematics curricula. The state of Victoria has gone the furthest in requiring students in Year 12 to have executed "Common Assessment Tasks" which involve them in projects, investigations and/or modelling activities. Most of the other states encourage and advise schools to include such problem solving activities in their school based assessment. Schools and teachers are free to decide how they teach the problem-solving skills required to successfully complete these assignments. At present there are two widely differing approaches advocated in Australia, namely, a heuristic approach based on the Polya model and the approach advocated by Sweller.

The work of Sweller and his colleagues suggests that, at the secondary level, teachers should be focusing on domain specific strategies which will enhance the development of multiple schemata to meet differing problematic situations of school mathematics. Such an approach to problem solving will be much easier to pass on to teachers of mathematics operating in the conventional role of imparters of knowledge and skills as it requires only small changes to their current practice. The Polya model on the other hand requires radical changes in the belief systems of both teachers and students as well as substantial role changes on the part of the teacher. This is more likely to fail in a school system not supported by ongoing in-service training at the tertiary and school levels. Experience over the last 30 years suggests that innovation and radical change can only be brought about gradually. It therefore seems likely that the modest proposals of Sweller might represent the realistic first steps of the "journey of a thousand miles".

The research reviewed in this chapter has shown that it is possible to develop problem-solving behaviours in primary and secondary mathematics classes using open questions, innovative real world applications, and small discussion groups. The success of all of these approaches depends however upon the teacher. Teacher Education institutions have not yet developed the innovative in-service (or preservice) programs which will ensure that a majority (or even a substantial minority) of the primary teachers and secondary mathematics teachers will introduce and maintain these practices in their everyday classrooms. Action research studies to develop such professional development programs are needed. Since there is now an abundance of curriculum materials to use in classrooms the studies should focus on ways to change teachers' perceptions of their roles as mathematics teachers and their belief systems about the nature of school mathematics, as well as creating classroom climates that are conducive to learning mathematics through problem solving.

Concurrently there is also a need to develop valid and reliable observational instruments which can be used in these quasi-naturalistic studies to clearly indicate the change factors that operate in different educational settings. There is a parallel need to continue working on instruments to monitor and measure students' thinking processes and problem-solving achievement. Promising starts have been made and the signs are propitious

that, if all goes well, valid and reliable change indicators will be attained by the end of the century.

REFERENCES

Australian Education Council. (1991). *A national statement on mathematics for Australian schools*. Melbourne, Victoria: Curriculum Corporation.

Baillie, M. J. (1987). *A descriptive study exploring children's use of visual imagery in mathematical problem solving*. Unpublished master's thesis, Brisbane College of Advanced Education, Brisbane, Queensland.

Bennett, G. P. (1987). *The quality of problem solving in mathematics: An application of the SOLO Taxonomy to think aloud solutions of mathematical word problems*. Unpublished master's thesis, Flinders University of South Australia, Adelaide.

Biggs, J. B., & Collis, K. F. (1982). *Evaluating the quality of learning: The SOLO Taxonomy*. New York: Academic Press.

Blane, D. D., & Leder, G. C. (Eds.). (1988). *Mathematics Education in Australia: A selection of recent research*. Melbourne, Victoria: Mathematics Eduction Research Group of Australasia and Monash University.

Boulton-Lewis, G. M. (1991, July). *The demands of some representations and strategies used in teaching mathematics to young children*. Paper presented at the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia.

Boulton-Lewis, G. M., & Halford, G. S. (1990). Young children and teachers negotiating mathematics. In M. Bezzina & J. Butcher (Eds.), *The changing face of professional education* (pp. 99-103). New South Wales: Sydney University.

Bourke, S., & Stacey, K. (1988). Assessing problem solving in mathematics: Some variables related to student performance. *Australian Educational Researcher, 15*(1), 73-83.

Briars, D. J., & Larkin, J. H. (1984). An integrated model of skill in solving elementary word problems. *Cognition and Instruction, 1*, 245-296.

Chick, H. L. (1988). Student response to a polynomial problem in light of the SOLO Taxonomy. *Australian Senior Mathematics Journal, 2*, 91-110.

Chick, H. L., Watson, J. M., & Collis, K. F. (1988, May). Using the SOLO Taxonomy for error analysis in mathematics. *Research in Mathematics Education in Australia*, 34-47.

Clarke, D., & McDonough, A. (1989). The problems of the problem solving classroom. *The Australian Mathematics Teacher, 45*(2), 20-24.

Clarke, D. J., & Sullivan, P. (1990). Is a question the best answer? *The Australian Mathematics Teacher, 46*(3), 30-33.

Collis, K. F., & Watson, J. M. (1991). A mapping procedure for analysing the structure of mathematics responses. *Journal of Structural Learning, 11*, 65-87.

Collis, K. F., & Romberg, T. A. (1992). *Collis-Romberg mathematical problem solving profiles*. Melbourne, Victoria: Australian Council for Educational Research.

Cureton, E. F. (1965). Reliability and validity: Basic assumptions and experimental designs. *Educational and Psychological Measurement, 25*, 326-346.

Curriculum Development Centre. (1988). The mathematics curriculum and teaching program. Canberra, Australian Capital Territory: Author.

Dossel, S., & Pegg, J. (1990, July). *An analysis of problem-solving performance of a cohesive and a non-cohesive pair of primary students*. Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

Fong, H-K. (1990, July). *Information processing taxonomy: An evaluation technique for assessing a pupil's ability to solve mathematical problems*. Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

Galbraith, P. L., & Clatworthy, N. J. (1989, July). *Learning, teaching: A case study in mathematical modelling*. Paper presented at the twelfth annual conference of the Mathematics Education Research Group of Australasia, Bathurst, New South Wales.

Groves, S. (1989). Problem-solving and modelling - years 7 to 12. In B. Doig (Ed.), *Every one counts* (pp. 10-17). Parkville, Victoria: Mathematical Association of Victoria.

Halford, G. S. (1988). A structure-mapping approach to cognitive development. In A. Demetriou (Ed.), *The neo-Piagetian theories of cognitive development: Towards an integration*. Amsterdam, Netherlands: Elsevier Science Publishers for North Holland.

Halford, G. S. (in press). *Children's understanding: The development of mental models*. Hillsdale, N. J. Erlbaum.

Headlam, W., & Stacey, K. (in press). Preparing a bank of problems for year eleven students. *Australian Senior Mathematics Journal*.

Isaacs, I. (1990a, March). *Problem solving is not a new branch of mathematics*. Paper presented at the annual conference of the Mathematics Teachers' Association of the Darwin, Northern Territory.

Isaacs, I. (1990b, July). *Teaching problem solving: An opportunity missed to synthesize school mathematics*. Paper presented at the thirteenth biennial conference of the Australian Association of Mathematics Teachers, Hobart, Tasmania.

Isaacs, I. (1990c, July). *A comparison of the SOLO Taxonomy and a problem-solving protocol coding schema for identifying novice and expert problem solving behaviour*. Paper presented at the thirteenth annual conference of

the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

Isaacs, I. (1990d). The persistence of intuitive methods in the development of mathematical problem solving skills. *Mathematics Education Research Journal, 2*(1), 23-44.

Kintsch, W., & Greeno, J. G. (1985). Understanding and solving word arithmetic problems. *Psychological Review, 92*, 109-129.

Langford, P. (1986). Arithmetical word problems: Thinking in the head versus thinking on the table. *Educational Studies in Mathematics, 17*, 193-199.

Langford, P. (1988, June). Arithmetical word problems: Evidence for thinking on the table. *Research in Mathematics Education in Australia*, 1-7.

Langford, P. (1989). Research on solving arithmetical word problems. *The Australian Mathematics Teacher, 45*(4), 24-25.

Long, E. (1989, July). *Recent developments in theories of human problem solving and their application to classroom mathematics.* Paper presented at the twelfth annual conference of the Mathematics Education Research Group of Australasia, Bathurst, New South Wales.

Low, R., & Over, R. (1989). Detection of missing and irrelevant information within algebraic story problems. *British Journal of Educational Psychology, 59*, 296-305.

Malone, J. A., Johnston, J., & Kandl, T. (Eds.). (1990). *Solve it, too. Problem solving in Mathematics IV.* Perth, Western Australia: Curtin University of Technology.

Malone, J. A., Kissane, B. V., & Johnston, J. (Eds.). (1990). *Solve it. Problem solving in Mathematics III.* Perth, Western Australia: Curtin University of Technology.

Meyer, R. A. (1975). *A study of the relationship of mathematical problem solving performance and intellectual abilities of fourth-grade children.* Unpublished doctoral dissertation, University of Wisconsin, Madison, Wisconsin.

Meyer, R. A. (1975). *A study of the relationship of mathematical problem solving performance and intellectual abilities of fourth-grade children.* (Working paper No. 160) Madison, Wisconsin: Wisconsin Research and Development Center.

Mulligan, J. (1988, July). *An analysis of children's solutions to multiplication and division word problems - a pilot study.* Paper presented at the eleventh annual conference of the Mathematics Education Research Group of Australasia, Deakin University, Victoria.

Mulligan, J. (1989, July). *Children's solutions to multiplication and division word problems.* Paper presented at the twelfth annual conference of the Mathematics Education Research Group of Australasia, Bathurst, New South Wales.

Mulligan, J. (1990, July). *Children's solutions to multiplication and division word problems: A longitudinal study.* Paper presented at the thirteenth

annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

Mulligan, J. (1991, July). *The role of intuitive models in young children's solutions to multiplication and division word problems.* Paper presented at the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia.

Open University, The (1978). *Mathematics foundation course, mathematical modelling units 1 to 5.* Milton Keynes: Author.

Owen, E., & Sweller, J. (1985). What do students learn while solving mathematics problems. *Journal of Educational Psychology, 77*(3), 272-284.

Owen, E., & Sweller, J. (1989). Should problem solving be used as a learning device in mathematics? *Journal of Research in Mathematics Education , 20*(3), 322-328.

Owens, K. (1990a, July). *Getting inside the problem solver's head: Using retrospection and observation to access spatial and problem thinking processes.* Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

Owens, K. (1990b). When qualitative and quantitative analysis are complementary: An example on the use of visual imagery by primary school children. In M. Bezzina & J. Butcher (Eds.), *The changing face of professional education* (pp. 737-742). New South Wales: Sydney University.

Pegg, J., & Davey, G. (1989). Clarifying level descriptors for childrens' understanding of some basic 2-D geometric shapes. *Mathematics Education Research Journal, 1*(1), 16-27.

Putt, I., & Pountney, G. (1989, July). *Mental activity during mathematical problem solving.* Paper presented at the twelfth annual conference of the Mathematics Education Research Group of Australasia, Bathurst, New South Wales.

Riley, M. S., Greeno, J. G., & Heller, J. I. (1983). Development of childrens' problem-solving ability in arithmetic. In H. G. Ginsburg (Ed.), *The development of mathematical thinking.* New York: Academic Press.

Siemon, D. F. (1989, July). *Learning from children's mathematics - a proactive approach to mathematics education.* Paper presented at the twelfth annual conference of the Mathematics Education Research Group of Australasia, Bathurst, New South Wales.

Siemon, D. F., & Booker, G. (1990). Teaching and learning FOR, ABOUT, and THROUGH problem solving. *Prime Number, 5*(3), 15-27.

Stacey, K. (1989a). Finding and using patterns in linear generalising problems. *Educational Studies in Mathematics, 20*(2), 147-164.

Stacey, K. (1989b, July). *Problem solving in groups: Are two heads better than one?* Paper presented at the twelfth annual conference of the Mathematics Education Research Group of Australasia, Bathurst, New South Wales.

Stacey, K. (1990a). Making optimal use of mathematical knowledge. *Australian Journal of Remedial Education, 22*(4), 6-10.

Stacey, K. (1990b, July). *What does teaching about problem solving teach?* Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

Stacey, K. (1991a). Attitudes and meta-cognitive aspects of learning mathematics: An analysis by gender. In L. Rennie, L. Parker, & G. Hildebrand (Eds.), *Action for equity - the second decade. Proceedings of the GASAT6 Conference* (pp. 337-346). Melbourne, Victoria.

Stacey, K. (1991b). Linking application and acquisition of mathematical ideas through problem solving. *Zentralblatt fur Didaktik der Mathematik, 91*(1),8-14.

Stacey, K. (1991c). *Mathematical problem solving in groups: Are two heads better than one?* Manuscript submitted for publication.

Stacey, K. (1991d, July). The effects on students' problem-solving behaviour of long-term teaching through a problem-solving approach. *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education* (pp. 278-285). Assisi, Italy.

Stacey, K., & Bourke, S. (1988, December). Testing mathematical problem solving. *Research in Mathematics Education in Australia* 12-28.

Stacey, K., & Groves, S. (1985). *Strategies for Problem Solving.* Melbourne, Victoria: Latitude Publications.

Stacey, K., & Groves, S. (1988). Problem solving: An annotated bibliography. In D. Blane & G. Leder (Eds.), *Mathematics Education in Australia: A selection of recent research.* Melbourne, Victoria: Mathematics Education Research Group of Australasia and Monash University.

Stacey, K., & Groves, S. (1990). The teaching of applications, modelling and problem solving in Australia: 1984-1988. In W. Blum, M. Niss, & I. Huntley (Eds.), *Modelling; applications and applied problem solving* (pp. 242-248). Chichester: Ellis Horwood.

Sullivan, P. (1989). One way of catering for all abilities in your mathematics class. In B. Doig (Ed.), *Every one counts* (pp. 134-138). Parkville, Victoria: Mathematical Association of Victoria.

Sullivan, P., & Clarke, D. J. (1987). Good teaching and good questions. In W. Caughey (Ed.), *From now to the future* (pp. 114-120). Parkville, Victoria: Mathematical Association of Victoria.

Sullivan, P., & Clarke, D. J. (1988). Asking better questions. *Journal of Science and Mathematics Education in South East Asia, 11*(1), 14-18.

Sullivan, P., Clarke, D., & Wallbridge, M. (1991, July). *Problem solving with conventional mathematics content: Responses of pupils to open mathematical tasks.* Paper presented at the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia.

Sweller, J. (1984, August). *Factors affecting schema acquisition during mathematical problem solving.* Paper presented at the fifth International Congress on Mathematical Education, Adelaide, South Australia.

Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science, 12*, 257-285.

Sweller, J. (1989). Cognitive technology: some procedures for facilitating learning and problem solving in mathematics and science. *Journal of Educational Psychology, 81*(4), 457-466.

Sweller, J., & Cooper, G. A. (1985). The use of worked examples as a substitute for problem solving in learning algebra. *Cognition and Instruction, 2*(1), 59-89.

Sweller, J., Mawer, R., & Ward, M. (1983). Development of expertise in mathematical problem solving. *Journal of Experimental Psychology: General, 112*, 634-656.

Taplin, M. (1990a, July). *Invented strategies for problem solving.* Paper presented at the thirteenth biennial conference of the Australian Association of Mathematics Teachers, Hobart, Tasmania.

Taplin, M. (1990b, July). *Perseverance on solving number problems.* Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

Tarmizi, R. A., & Sweller, J. (1988). Guidance during mathematical problem solving. *Journal of Educational Psychology, 80*(4), 424-436.

Ward, M., & Sweller, J. (1990). Structuring effective worked examples. *Cognition and Instruction, 1*(1) ,1-39.

Watson, J. (1988a). Three hungry men and strategies for problem solving. *For the Learning of Mathematics, 8*(3), 20-26.

Watson, J. M. (1988b, July). *Using a SOLO Taxonomy mapping procedure for error analysis in geometry.* Paper presented to the eleventh annual conference of the Mathematics Education Research Group of Australasia, Geelong, Victoria.

Watson, J. M., Chick, H. L., & Collis, K. F. (1988). Applying the SOLO Taxonomy to errors on area problems. In J. Pegg (Ed.), *Mathematical interfaces* (pp. 260-281). Armidale, New South Wales: Australian Association of Mathematics Teachers.

Watson, J. M., & Mulligan, J. (1990). Mapping solutions to an early multiplication word problem. *Mathematics Education Research Journal, 2*(2), 28-44.

Wearne, D. C. (1976). *Development of a test of mathematical problem solving which yields comprehension, application, and problem solving scores.* Unpublished doctoral dissertation, University of Wisconsin, Madison, Wisconsin.

Wearne, D. C., & Romberg, T. A. (1977). *DMP accountability test.* (Working Paper No. 217). Madison, Wisconsin: Wisconsin Research and Development Center.

Whitaker, D. (1976). A study of the relationahip between selected non-cognitive factors and theproblem solving performance of furth-grade children. (Technical Report No. 396). Madison, Wisconsin: Wisconsin Research and Development Center.

Yee, F. P. (1990, July). A *taxonomy of mathematical problem-solving behaviours*. Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania.

RESEARCH IN GEOMETRY AND MEASUREMENT

Geoff. Davey
Division of Education
Griffith University

John E Pegg
Department of Science, Technology and Mathematics Education
University of New England

There are two main incentives for studying geometry at the school level. One is to assist children to obtain a better grasp of shape and space and enhance their ability to use shape and space in a variety of problem solving tasks. The second is to address more formal aspects of geometry and so develop better appreciation of mathematical structure and proof. Recent Australian research into teaching and learning geometry has tended to focus, in the main, on the first aspect, that is, on the way in which students perceive, comprehend and use basic geometric objects. In particular, it has studied the development in their understanding of concepts, their misconceptions and their spatial abilities and thinking processes. Other areas of research have included interpretation of representations and some formal aspects of geometry related to proof and theorems. The research included in this chapter has been summarised under the following headings, Visual abilities and drawing, Research based on the generative learning model, Research related to formal geometry and proof, Levels of Growth of understanding: van Hiele and SOLO Taxonomy, and Measurement.

VISUAL ABILITIES AND DRAWING

Spatial ability has long been part of the psychological domain and a large number of tests have been devised to measure its various components. With the study of space and geometry firmly entrenched within the primary school syllabus, mathematics educators have seen the need to investigate children's spatial abilities. In particular, this has included those that relate to conventional geometric activities and problem solving, although the relationship between spatial abilities and geometry is not at all simple. How children solve spatial problems and geometric problems is becoming an important research topic. The psychological literature reveals that the construct of spatial ability has considerable intuitive appeal and appears to be an obvious necessity for everyday functioning, but its description and

assessment are complex tasks. Mathematics education researchers have found these tasks equally challenging even though they have a more limited focus. Australian research has concentrated mainly on the skills, strategies and mental processes that children use to solve problems that are related to conventional school geometry.

Leeson (1990) worked mainly with children at the kindergarten level in order to identify those spatial abilities that seemed to relate to geometric understanding and problem solving with 2D and 3D shapes. These included constructing shapes similar to prototypes and diagrams, naming objects in the environment that resembled given shapes, estimating numbers of shapes required to cover outlines and the like. He found that even young children possessed a wide range of abilities and that many were able to solve spatial problems often assigned to much older children. The ability of young children to solve tangram-like problems was investigated by Mansfield and Scott (1990). Their purpose was not so much to determine whether the children were successful or not or to study relative success rates at different age ranges but, rather, to determine what actual procedures were employed by the successful solvers compared with the less successful children. While there was a greater level of persistence with the older children and a greater range of procedures, in the main, it was children who rotated and turned the pieces over who were more successful. These children seemed to recognise when a shape would lead to a solution or whether it was necessary to reposition the pieces. Two interesting questions that remain unanswered are whether these problem solving actions were spontaneously learned and whether they can be taught.

In various studies, Owens (1990, 1991b) also investigated the use of visual imagery and other mental processes that primary level children use while working on problems with manipulative materials. She collected data by means of a combination of observation, retrospection and recall stimulated by video replays. Analysis using various learning models including the SOLO Taxonomy, generative learning model, and categories of visual imagery suggested by Presmeg (1986) revealed that the subjects made use of a limited range of mathematical concepts and thinking processes. Shape appeared to be the only aspect of figures used with global estimation and analytic procedures being preferred to rotation and turning. Owens' work revealed the difficulty of using the retrospection procedures with young children, since they seemed to be unable or unwilling to elaborate on their thinking, especially when it concerned visual imagery.

Emerging from this work were a number of "paper and pencil" tasks to test a range of mathematico-spatial thinking processes used by children in Years 2 to 5. The test items were initially prepared as concrete items for individual testing, but Owens (1991a) developed these into a format that allows group administration in two sessions totalling about 90 minutes. The test requires a careful introduction with demonstrations to ensure that children understand the intent of the items and thus are able to relate the pencil and paper items to concrete examples. Owens was careful to include test items

which relate to activities and thinking expected in the mathematics classroom and at the same time correspond to standard spatial ability tests. The results measure the early conceptualisation of attribute of 2D shapes at the recognition level and the use of visual imagery in making decisions about shapes. The range of spatial thinking processes include embedded figures, recognition of shapes, reproduction and analysis of shapes and the mental transformation of 2D into 3D figures by folding. The test proved to have good internal consistency except for those items on folding nets into 3D figures. She also claimed that in addition to its function for assessment, its use in the general classroom situation provided a valuable learning experience.

Observations of children in class settings and clinical interviews were the basis of a study by Zevenberger (1991) into the language used in geometry lessons in Years 1, 3, and 5. She found that the younger children often used 2D terms, such as triangle, square, or circle to signify solid figures when answering a question in school. She argued that while this appears to support the stage theories of Piaget and van Hiele, it is rather the result of the syllabus used and discourses to which the children had been exposed. In school the word "shape" is used mainly in reference to plane figures so it causes children to direct their attention to the dominant plane shape associated with the solid.

Research on Drawings

Cooper and Sweller (1989) investigated students' ability to interpret various representations of simple 3D objects including verbal descriptions, representations based on coordinates and different types of drawing such as plans, elevations, and perspective. Their studies showed that real life representations were much more easily processed than more technical procedures, such as layer plans, coded plans, elevations and coordinate descriptions. Of the real life representations, perspective drawings and solid models were more easily interpreted than verbal descriptions. Children found it just as easy to copy a perspective drawing as a prototype. They concluded that perspective drawings were internalised, in the same way as the objects themselves, by both children and adults. This study has significance for text book writers and for teachers of technical drawing units in which orthogonal drawing is used. Skill in interpreting this kind of drawing may not be acquired by studying 3D geometry, perspective and actual prototypes. It may well require much more intensive teaching and specific practice.

In another study, Cooper and Krainer (1990) sought to discover if children were better able to identify right triangles in orientations in which they had training or in the conventional horizontal - vertical orientation. This question is also of some importance to text book writers and, of course, teachers. If a particular figure is presented in a standard orientation, children may form a mental image or schema of the figure in this standard orientation which means that figures in other orientations will require mental rotation before they can be judged congruent or otherwise. The results indicated that even trained children generally recognise right triangles in the horizontal - vertical orientation better

than all others. It appeared that there was a natural bias towards this format which overrides the training given. Of the non-standard orientations, "upward pointing" triangles tend to be better recognised than "downward pointing" triangles.

In a recent paper, Mitchelmore (1991) summarised his work, of the past several years, concerning the difficulties that younger children experience with drawing parallel lines. His work stated that the difficulties were not perceptual since the children can actually use parallelism in alignment tasks, nor were they due to failure to notice parallels. While there was a bias towards drawing intersecting lines perpendicular, which caused distortion of parallelism, Mitchelmore put forward additional hypotheses. He suggested that children may have a view of "an exact copy" of a figure that is different from adults and that preservation of the length property may have greater salience than parallelism. The low salience could be attributed to the fact that the concept of parallelism is rarely used overtly in everyday life except as two lines perpendicular to a third line. The word "parallel" also has no simple English counterpart. Investigation of these conjectures is currently under way.

RESEARCH BASED ON THE GENERATIVE LEARNING MODEL

The research of Mansfield and Happs (1989, 1990, 1991a, 1991b, 1991c) is based on the Generative Learning Model of Osborne and Wittrock (1983) which is probably more widely known in science education than in mathematics education. Aspects of this model appear to be similar to those found in constructivist theory in that they describe how individual students process incoming information and the mechanisms they employ to generate meaning. Some assumptions made by this model are:

1. Students often have views on mathematical concepts prior to instruction. These views are often different from the accepted mathematical view.
2. Teachers are often unaware of students' views.
3. Students' views seem sensible and useful to them.
4. Students' views can remain uninfluenced by or influenced in unanticipated ways by instruction.
5. Students must actively generate their own understanding.

The work of Mansfield and Happs (1989, 1990) has shown that complex factors act as mediators in the teaching/learning environment which interferes with student learning. What students learn as a result of formal instruction is often not the understanding intended by their teachers. This occurs because input is selected and attended to by the learners themselves. The learners filter the input, admitting only those features that they consider relevant to their own

pre-existing schemata. This process enables certain misconceptions to take hold as "concepts" develop and individual interpretation takes root. Some misconceptions are tenaciously held and difficult to correct because they are relevant to and actually make sense to the individual learner.

Students' conceptual frameworks based on misconceptions are sometimes difficult to detect by teachers and researchers. Three interesting aspects were noted in this regard. In some cases, different stimulus materials may produce different outcomes; for example, some students fail to recognise reflex angles in line drawings but accept the notion easily when geostrips provide the stimulus to distinguish between reflex and acute angles. In other cases, students may give a correct response to a certain question even when they have serious misconceptions. The third complicating factor is the real possibility that students spontaneously generate ideas while they are being interviewed.

Mansfield and Happs (1990, 1991c) have also investigated the long term effects of teaching strategies which were planned specifically to bring about conceptual change by linking new information to prior knowledge and by challenging students' misconceptions. The pre-existing frameworks were first determined through the techniques of concept maps, individual interviews, questionnaires and survey tests. Their research so far has considered the concepts of parallel lines and angles. Commonly occurring misconceptions include that curved lines can be parallel and that parallel lines must be aligned. Other students may believe that a straight angle is not an angle at all or that orientation is a factor in comparing angles and testing them for equality.

After pre-testing, the students participated in a teaching program which was based on the principles of generative learning. These promoted an on-going exchange of ideas between groups of students and between students and teacher. In this way, opportunities to compare the students' views with the mathematicians' views were provided. This enabled students to modify their own views when they found the mathematicians' views to be more useful. The students were tested again at the conclusion of the teaching program, six months later and again after two years.

It has become clear that individual students' learning outcomes could differ significantly in the long term. Some students made substantial modifications to their views and maintained these views for over a year, while others showed only a short term shift in their thinking, only to regress to their original misconceptions. A third group displayed a long term mix of their own misconceptions and the conventional views of standard mathematics. In general, the experimental group performed better than the control group over the long term, yet for a number of individual students this was not the case, nor was it the case on all test items (Mansfield & Happs, 1991a).

The source of students' misconceptions has not been clearly identified in these various studies. What has stood out has been the tenacity of students' beliefs. This has thrown into question the efficacy of conventional teaching strategies that emphasise exposition by the classroom teacher and which do not confront students' misconceptions. Without this confrontation, teaching seems

merely to encourage students to adopt the teacher's perspective in the short term within the context of the mathematics classroom.

The important task remaining is for the researchers to tease out what student factors are responsible for the different ways students process the same information and what other factors have an impact on students' long term construction of ideas.

Finally, mention should be made of the use of concepts maps by Mansfield and Happs (1991c) in a variety of teaching situations. Their use has not been limited to evaluating teaching outcomes or monitoring students' progress, but they have been shown to be a useful device to foster discussion and debate in a lively, challenging manner within groups of students.

RESEARCH RELATED TO FORMAL GEOMETRY AND PROOF

In most Australian states, the study of proof oriented formal geometry has been restricted to students in the "more able" classes. However, there is still concern at students' ability to construct a proof and students' beliefs about what they are actually doing while engaged in such a task. Kemp (1990) undertook a study to look closely at conceptions of geometric proof of a small group of students and relate this to the curriculum to which they had been exposed. She interviewed four able but not exceptional students at regular intervals over a five week period and monitored their attempts at a special proof task on each occasion. The students saw proof primarily as a means of explaining rather than validating a proposition. They had a limited understanding (i) of the need to prove a proposition true in all cases and (ii) that a deductive proof establishes the truth of a proposition for all cases. To the students, a special case appeared to be a less complicated way of proving a result. An examination of texts and syllabuses indicated the courses that the students had followed over the previous three years had failed to give them opportunities to make conjectures which may or may not be true and to see why deductive proof is superior in establishing validity. The emphasis given in the curriculum guides appeared to be on the form of the proof thus giving the impression of "proof for its own sake".

Motivated by what they termed the consistently poor performance of students in Year 12 exams in geometry, Chinnapan and Lawson (1990) designed a study to explore problem-solving performance in formal geometry. In particular, they chose to investigate the differences in processing behaviours of successful and unsuccessful problem-solvers and to develop a useful and practical coding schedule to describe these behaviours. Other questions focused on the relative importance of the students' domain-specific-knowledge base and the role of strategic behaviours.

Fifteen student volunteers in Year 11, who had studied geometry in the previous three years of high school mathematics, undertook three training

sessions in "think aloud" procedures so that their behaviour could be monitored as accurately as possible on the four problems that formed the basis of the study. Each of these called for the computation of an angle size or length of a line in triangles and circles. Students' behaviours, such as the generation of new information, labelling, extending diagrams and correcting errors were recorded on custom-built coding schedules. The high achievers tended to produce more coding events in all categories, except *error*, than low achieving students. Similarly, high achievers produced significantly more than low achievers in the areas of generation of information and identification of what is given or what has been generated. While both groups devoted most attention to these two activities, the high group spent most of the time on generation and the low group on identification and management.

The study showed that the more successful students were better at accessing their *domain specific knowledge*, not just once but several times during a single problem and using it to generate new information, than were the lower achieving group. The investigators concluded that it was domain specific knowledge rather than the more general strategic knowledge that enabled the higher group to be more successful.

One original approach to looking at formal geometric skills was undertaken by Watson (1988). She considered student attempts at a series of relatively difficult geometric questions by using a mapping procedure based on the SOLO Taxonomy of Biggs and Collis (1982). Before describing her work it is useful to provide a brief overview of the SOLO Taxonomy as it is also referred to by researchers in two following sections.

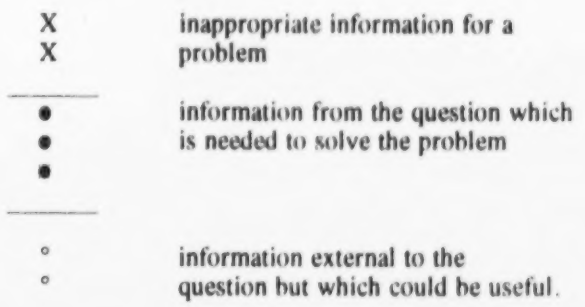
The SOLO Taxonomy is a categorisation system for coding student responses to questions. It has been used in a number of subject areas such as English, science and history as well as several topic areas within mathematics. The Taxonomy arose as a reaction to the stages of development ideas of Piaget which were seen to be at variance with many findings associated with learning in the classroom. In particular, the notion of the irreversibility of the sequence of stages was not seen to be true.

The SOLO Taxonomy, however, has many Piagetian elements and these are most notable in the series of modes of functioning. The Taxonomy identifies five modes of functioning. They are *sensori-motor* (from birth), *ikonic* (from 18 months), *concrete-symbolic* (from 6 years), *formal* (from 15 years), *post formal* (from about 20 years). The ages are meant to be approximate only and indicate when a person first has access to that mode. A major difference between the modes and the stages of Piaget is that more than one mode can be available to a person and it is possible to have multimodal functioning.

In addition, within each mode is a series of levels of understanding. Hence, given a question within a given mode, a student can respond at any one of five levels. Three of the levels occur within the mode and are referred to as: *unistructural* - a focus on one element, *multistructural* - a focus on several independent elements, *relational* - a focus on several elements where the

relationships between the elements are taken into account. The remaining two levels lie outside of the targeted mode. They are *prestructural* - a response that does not take into account any relevant information, and *extended abstract* - a response that brings into play relevant material that lies outside of the question or problem. This last level represents a major jump in thought and is equated to the unistructural level in the next mode.

One tool that has been used to explore the SOLO Taxonomy has been a mapping procedure first suggested in Chick, Watson and Collis (1988). Here a metaphor template has been designed to incorporate the possible responses to problems associated with some target mode.


X
X
inappropriate information for a problem
information from the question which is needed to solve the problem
information external to the question but which could be useful.

This basic mapping procedure has since been refined and modified (Watson, 1988; Collis & Watson, 1991). This later version includes additional features which attempt to show more clearly the data processing which is believed to take place. The map now includes cues from the question (most notably the data), the concepts/processes used and the responses (both intermediate and final). The mapping of cognitive functioning appears to have enormous potential. It has the ability to help the analysis of tasks, to identify error patterns and to explore more deeply the SOLO Taxonomy itself.

As mentioned previously, Watson (1988) used the new mapping approach to explore student responses to a series of geometric properties. She was able to provide response maps for all correct and incorrect attempts at answers provided by the students. As a result, groups of error patterns were identified. It is clear that this information, in the hands of a teacher, would lead to a greatly enhanced understanding of students' attempts at questions and the potential for more considered and relevant remediation.

LEVELS OF GROWTH OF UNDERSTANDING

Research into analysing and explaining students' understanding in geometry has been a major focus of the work of Davey and Pegg over the past several years. Their work is clearly in the Piagetian mould and has drawn heavily on the ideas expressed in the van Hiele Theory (1986) and the SOLO Taxonomy of Biggs and Collis (1982). Before describing the direction and results of their

work, it is appropriate to outline briefly some key aspects of the van Hiele Theory in order that the following discussion might more easily be understood.

van Hiele Theory

The van Hiele Theory has been under fairly intensive investigation in the western literature since about 1980 as the ideas have struck a resonant chord with many researchers and teachers throughout the world. The two basic ideas associated with the theory are:

1. student growth in geometry can be identified in terms of a series of five *levels*; and,
2. student growth through the levels can be achieved if the instructional process proceeds through five *phases*.

The levels, commonly numbered 1 to 5, represent a logical hierarchy of growth which begins with students focusing on the global nature of figures (Level 1). From these experiences the properties of the figures emerge, initially as independent entities (Level 2). Later, the student can see relationships between the properties themselves and between various figures (Level 3). At Level 4 the need and value of deduction are understood and at Level 5 students are able to go beyond deduction by challenging and modifying basic assumptions.

The five phases also represent a logical sequence. In this case, however, the hierarchy represents a teaching/learning plan. The phases are: *information* - the area to be investigated is clarified; *directed orientation* - simple tasks are undertaken; *explicitation* - the language used by the student is clarified; *free orientation* - more difficult tasks with multiple solution paths are undertaken; and, *integration* - the purpose of the study is clarified and summarised.

Both aspects of this theory have been addressed. Davey (1988) detailed some experiences with primary-aged students with both 2-D and 3-D tasks. His findings were considered within the first two van Hiele levels and in terms of the descriptions: language, perception, reasoning, and drawing. Two useful findings that emerged from this work were that (i) the van Hiele levels were not discrete structures like a series ascending plateau, and (ii) the way students structured their language was linked to their level on a topic. Pegg (1990) discussed the phases aspect of the van Hiele model and drew attention to four issues which arose through a series of interviews with teachers and students. The issues were: timing, language, teacher and student roles, and the sequence of phases.

SOLO Taxonomy

While the work on phase-based instruction appears to have enormous potential in helping teachers structure their teaching, the implications of the level descriptions have prompted the most research into the van Hiele theory. One approach to this form of investigation was to compare the level descriptions of

the van Hiele theory with the SOLO Taxonomy. There were many broad similarities between the theories, including the Piagetian basis and the fact that both frameworks evolved out of a dissatisfaction with some of Piaget's findings when these were applied to school situations.

Pegg and Davey (1989) undertook a comparison of these theories by interpreting the responses of over 270 students on their descriptions of four common 2D shapes. The results confirmed a close relationship between the two theories but it was found that the SOLO Taxonomy was more useful as a research tool as it offered greater specificity of results. First, many students tested and interviewed were only able to spontaneously provide one property - a feature not highlighted in the van Hiele theory but indicative of the unistructural level in the SOLO Taxonomy. Second, many students used ikonic (SOLO) or Level 1 (van Hiele) support to add more detail to their answers which also included references to properties. Again this is consistent with the multi-model ideas of the SOLO Taxonomy but seems to contradict the van Hiele idea that persons at a given level work only at that level. Finally, the term ikonic for a mode of functioning in the SOLO Taxonomy (although consistent with van Hiele's ideas) seemed to be a more appropriate description label for the understanding associated with students' early views on geometry - views that are heavily image-laden.

The ideas in this paper were re-examined and extended (Davey & Pegg, 1989). In particular the issue of class inclusion and minimum properties were investigated. The results of this study also provided useful information on how students interpreted the word "describe". It appears the most that can be expected by asking students for a description is a multistructural response in the concrete-symbolic mode (SOLO) or a Level 2 response (van Hiele). If higher levels of performance are to be identified then more specific questions need to be included. Typical examples of such items, in a format that is easy for teachers to implement, have been provided in Pegg and Davey (1991).

Two further large scale investigations have been undertaken by the writers. The studies focused on underpinning aspects of 2D shapes, namely, parallelism (Davey & Pegg, 1990) and angles (Davey & Pegg, 1991). The study into the students' understanding of parallelism (Davey & Pegg, 1990) involved 70 students in a testing and interviewing programme on a broad range of questions. These questions included a language focus as well as 23 specific items which involved identifying and constructing parallel lines and the solving of problems in which parallel lines were the underlying concept. A number of broad findings were identified. First, a range of student responses to descriptions of parallel lines was received and these could be grouped into several categories. They ranged from relevance on some mental image like "looks like a road", to a focus on lines, often with an attempt at some qualification, to statements which specifically included the equidistant property. Second, these responses were very similar in style to those received about 2D figures in the 1989 paper and as a result were clearly interpretable in terms of the SOLO Taxonomy. Further, the similarity even went as far as

having significant sub-levels at both the unistructural and multistructural levels in the concrete-symbolic mode. Third, the description/language used by the student reflected, within broad parameters, the type of questions and answers the student was able to provide to the test items. And lastly, there was a strong link between the student knowledge of parallelism and angles. Poor understanding of angles seemed to provide a barrier to student growth in parallelism.

The study into students' understanding of angles (Davey & Pegg, 1991) involved 40 students also in a testing and interview programme. Initially the students were asked to undertake a Piagetian drawing task which involved constructing as accurately as possible a given angle. This was followed by language/description exercises and, finally, a number of tasks involving angle concepts.

Once again there was a clear hierarchy of student descriptions. These descriptions ranged from an angle being a corner which is pointy or sharp to being where two lines meet at a point to being the distance or area between two lines. There was also a close connection between this language and the way students attempted both the angle construction and the various activities. What is more, there appeared to be relatively clear sets of activities that were appropriate for each of the various Piagetian stages. Most strikingly it was not until Piaget's Level IIIB that students were consistently able to compare angles and increase and decrease the size of an angle.

MEASUREMENT

Four papers are reviewed in this section. While each study had a distinct approach, they all share many common features. Three of the articles (Outhred & Mitchelmore, 1991; Tierney, Boyd, & Davis, 1990; Watson, Chick & Collis, 1988) focused on area concepts while the fourth (Campbell, Watson & Collis, 1990) considered growth in understanding volume. Two of the papers (Tierney, Boyd, & Davis, 1990; Watson, Chick & Collis, 1988) investigated the thinking and understanding of tertiary students and two (Campbell, Watson, & Collis, 1990; Watson, Chick & Collis, 1988) used the SOLO Taxonomy as a theoretical basis.

Two of the papers dealing with area had a common theme, namely, area concepts are more difficult to acquire than is generally thought. The work of Tierney et al. (1990) arose from concerns that many prospective primary teachers might not have a sufficient understanding of area concepts to support their teaching of these concepts to young children. The study involved a series of teaching sessions lasting six hours in all on tasks related to perimeter and area concepts. At the end of each session, students were asked to write about what they had done during the sessions and what they had learnt. Several interesting findings emerged from an analysis of the students' written records. Students often confused area and perimeter concepts. Conservation of area was

a critical notion but it seemed possible that what is often referred to as conservation of area might simply be "equality of figure". Finally, the authors believed that they were seeing, in their tertiary students, a resurfacing of misconceptions that these students had never properly resolved and which had lain dormant for a number of years. Two possible reasons for such misconceptions were hypothesised to be (i) a lack of concentration on the perception of area when students were young or (ii) some gaps between the concept of unit area and the multiplicative approach. Interestingly it was this last feature that was the focus of the Outhred and Mitchelmore (1991) study.

In the Outhred and Mitchelmore study 37 children from Year 2 to Year 5 were interviewed in order to identify the skills involved in covering rectangular areas by squares. The tasks involved the children counting, drawing and measuring in order to tap possible numerical, spatial and measurement levels associated with growth in area concepts. The results identified the difficulty many children have in drawing what they were able to physically construct. Further, it seemed that all three activity types might be needed if there is to be a linking of the concrete experiences with the evolution of a formula. While further investigation is warranted it seems possible that the lack of these activities in schools could well be a factor in the poor showing of much older students, as for example found by Tierney et al. (1990).

The third study (Watson et al., 1988) analysed the responses of tertiary students to a number of questions involving more difficult area concepts. These questions involved the use of algebraic formulae on various 2-D and 3-D shapes. The students' responses were analysed in terms of response maps based on the SOLO Taxonomy. The procedure proved to be a useful tool to pinpoint the types of errors students can make and the level of difficulty of a given question. Such analyses could prove useful for teachers and a practical way in which the SOLO Taxonomy might help improve the quality of teaching and learning.

The paper by Campbell et al. (1990) reported a two-part investigation into intellectual growth in the understanding of volume measurement. The first part involved the individual testing of 48 primary school children on how they calculated the volume of rectangular solids. The solids were both physical objects and drawn figures with some made up of individual cubes while others were undivided. The second part involved 83 secondary and tertiary students who were given a series of problems involving the calculation of volumes of solid figures. The results were interpreted within the framework of the SOLO Taxonomy. The results of the study identified growth in student understanding which allowed for examples of thinking from ikonic through to formal mode thinking. In addition, the study identified more than one unistructural-multistructural-relational sequence within the concrete-symbolic mode, i.e., there appeared to be a growth or maturity in understanding developed without recourse to a higher mode of thinking. Further the writers found evidence of multimodal functioning with some students using ikonic support along with their concrete-symbolic answers.

CONCLUSIONS

Research into geometry and measurement has continued to be a significant aspect of the general research effort of Australian mathematics educators. There has been a clear emergence of a concern with how students acquire and use both fundamental concepts and certain basic skills. Among the skills investigated during the last four years were drawing, the interpretation and identification of diagrams, language use by students and particularly solving spatial problems. It was not at all surprising that different researchers considered aspects of problem solving in various grades from Year 1 through to Year 12 with problems ranging from tangram type problems to more formal examples such as the determination of angle size. It was interesting to note that in most cases this research was focused on cognitive functioning and the processing of information. Specifically researchers investigated how students tackled the problem, what strategies they employed and what errors they made. Another fruitful avenue of research compared the methods employed by successful and unsuccessful problem solvers.

The other main focus was the acquisition, development and/or retention of basic concepts. These concepts included parallelism, the angle, minimum definition, class inclusion, proof, area and volume. In most cases it was suggested that these ideas were not as straight forward as commonly believed and as a result full understanding may take much longer than previously thought. Indeed some research indicated that certain students develop misunderstandings about various concepts that are most tenaciously held and are resistant to change by conventional teaching methods.

Most of the studies reviewed in this chapter had a strong theory base which underpinned the investigations undertaken. The most commonly used theories included the generative learning model, the SOLO Taxonomy and the van Hiele Theory. The generative learning model relies heavily on constructivism, while the SOLO and van Hiele models are based upon the stage theory of Piaget. The last two, have been successfully divorced from the notion of age related development. It is worth noting that there was no evidence that these theories are in any way incompatible. While some might argue that findings interpreted in terms of theories offer the potential for greater advances in our knowledge base, other researchers did not see the need to relate their studies to any general theory.

Clearly much is still to be learned about teaching and learning geometry and measurement. It would seem important to both the research community and to practising teachers that the trends identified above be continued. In particular, more work needs to be directed at exactly how students process spatial information and use this information to solve problems. There is also a need to know whether and how teachers can assist students to improve these skills. In addition, concept development requires further study so that teachers

are better able to overcome and prevent the formation of misconceptions. Both the generative learning model and the phases of the van Hiele Theory may be helpful in such studies.

There is also the question of the place of theories in geometry education research. Are the theories that have been advanced so far incompatible or are they but different approaches in attempting to understand a very complex area? Is there a better theory, that is, one that models growth more accurately and possibly encompasses current theories within its scope? Finally, mention should be made of two areas where little or no research has been undertaken in Australia. The first is with children in pre-school and the first few years of formal school while the second concerns the place of computers in the learning of geometry and measurement. Clearly both are important topics for future study.

REFERENCES

Biggs, J. & Collis, K. (1982). *Evaluating the quality of learning: SOLO Taxonomy*. New York: Academic Press.

Campbell, J., Watson, J., & Collis, K. (1990, July). *Volume measurement and the SOLO Taxonomy*. Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, University of Tasmania, Hobart.

Chick, H., Watson, J., & Collis, K. (1988). Using the SOLO taxonomy for error analysis in mathematics. *Research in Mathematics Education in Australia* , June, 34-46.

Chinnapan, M., & Lawson, M. (1990, July). *Activity during geometric problem solving*. Paper presented to the thirteenth annual Mathematics Education Research Group of Australasia conference, University of Tasmania, Hobart.

Collis, K. F., & Watson, J. M. (1991). A mapping procedure for analysing the structure of mathematics responses. *Journal of Structural Learning and Intelligent Systems, 11*, 65-87.

Cooper. M. and Krainer. K. (1990). Children's recognition of right angled triangles in unlearned positions. In G. Booker, P. Cobb, & T. deMendicuti (Eds.), *Proceedings of the fourteenth International Group of Psychology of Mathematics Education Conference* (pp. 227-234), Mexico. .

Davey, G. (1988). Primary school geometry-the van Hiele challenge. In J. E. Pegg (Ed.), *Mathematical Interfaces*, (pp. 149-153), Australian Association of Mathematics Teachers: University of New England, Armidale, New South Wales.

Davey, G., & Pegg, J. (1989a). Clarifying level descriptors for children's understanding of some basic 2D geometric shapes. *Mathematics Education Research Journal, 1*(1), 16-27.

Davey, G., & Pegg, J. (1989b, July). *Relating descriptions of common 2D shapes to underlying geometric concepts*. Paper presented at the twelfth annual conference of the Mathematics Education Research Group Australasia, Charles Sturt University. Bathurst, New South Wales.

Davey, G., & Pegg, J. (1990, July). *Students' understanding of parallelism*. Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, University of Tasmania, Hobart.

Davey, G., & Pegg, J. (1991, July). *Angles on angles: students' perceptions*. Paper presented at the fourteenth annual conference of the Mathematics Education Research Group of Australasia, University of Western Australia, Perth.

Happs, J. C. & Mansfield, H. (1989, March). *Students and Teachers' perceptions of the cognitive and affective outcomes of some lessons in geometry*. Paper presented at the annual meeting of the American Educational Research Association (Special Interest Group: Mathematics Education), San Francisco, USA.

Kemp, M. (1990, July). *Adolescents' conception of proof in geometry*. Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, University of Tasmania, Hobart.

Leeson, N. (1990, 1990). *Spatial abilities of kindergarten and year 6 pupils: a preliminary study*. Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, University of Tasmania, Hobart.

Mansfield, H., & Happs, J. (1989). Using concepts maps to explore students' understanding in geometry. In *Conference proceedings of the Psychology of Mathematics Education*, Paris, France.

Mansfield, H., & Happs, J. (1990, July). *Learning theory, misconceptions and teaching in geometry*. Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, University of Tasmania, Hobart.

Mansfield, H. & Happs, J. (1991a, July). *Difficulties in achieving long term conceptual change in geometry*. Paper presented at the fourteenth annual conference of the Mathematics Education Research Group of Australasia, University of Western Australia, Perth.

Mansfield, H., & Happs, J. (1991b, July). *How can we examine students' conceptions in geometry*. Paper presented at the working group on Geometry at the conference of the International Group of the Psychology of Mathematics Education, Assisi, Italy.

Mansfield, H., & Happs, J. (1991c). Concepts maps. *The Australian Mathematics Teacher*, *47* (3), 30-33.

Mansfield, H. & Scott, J. (1990). Young children solving spatial problems. In G. Booker, P. Cobb, & T. deMendicuti (Eds.), *Proceedings of the*

fourteenth International Group of Psychology of Mathematics Education Conference (pp.275-282), Mexico.

Mitchelmore, M. (1991, July). *Children drawing parallels.* Keynote address at the fourteenth annual conference at the Mathematics Education Research Group of Australasia, University of Western Australia, Perth.

Osborne, R. & Wittrock, M. (1983). Learning science: A generative process. *Science Education*, 67(4), 489-508.

Outhred, L. & Mitchelmore, M. (1991, July). *The transition from concrete to pictorial representation of area.* Paper presented at the fourteenth annual conference of the Mathematics Education Research Group of Australasia, University of Western Australia, Perth.

Owens, K. (1990, July). *Getting inside the problem solver's head: using introspection and observations.* Paper presented to thirteenth annual conference of the Mathematics Education Research Group of Australasia, University of Tasmania, Hobart.

Owens, K. (1991a, July). *Children's responses to a "paper and pencil" test of spatial thinking processes.* Paper presented at the fourteenth annual conference of the Mathematics Education Research Group of Australasia, University of Western Australia, Perth.

Owens, K. (1991b, September). Assisting young children with the spatial aspects of mathematics. Paper presented at the conference of the Mathematical Association of New South Wales, Wollongong, New South Wales.

Pegg, J. (1990). Teaching Implications of the van Hiele Theory: The Phases. In K. Milton & H. McCann (Eds.), *Mathematical Turning Points* (pp. 427-434). Australian Association of Mathematics Teachers: University of Tasmania, Hobart.

Pegg, J., & Davey, G. (1991). Levels of geometric understanding. *The Australian Mathematics Teacher*, *47*(2), 20-23.

Presmeg, N. (1986). Visualisation in school mathematics. *For the Learning of Mathematics*, *6*(3), 42-46.

Sweller, J., & Cooper, M. (1989). Secondary school students' representation of solids. *Journal of Research in Mathematics Education*, *20*(2), 202-212.

Tierney, C., Boyd, C., & Davis, G. (1990). Prospective primary teachers' conception of area. In G. Booker, P. Cobb, & T. deMendicuti (Eds.), *Proceedings of the fourteenth International Group of Psychology of Mathematics Education Conference*, (pp. 227-234),.Mexico,

Watson, J. (1988, July). *Using a SOLO Taxonomy mapping procedure for error analysis in geometry.* Paper presented at the eleventh annual conference of the Mathematics Education Research Group of Australasia, Deakin University, Geelong, Victoria.

Watson, J., Chick, H., & Collis, K. (1988). Applying the SOLO Taxonomy to Errors on Area Problems. In J. E. Pegg, (Ed.) *Mathematical Interfaces* (pp. 260-281). Australian Association of

Mathematical Teachers: University of New England, Armidale, New South Wales.

van Hiele, P. M. (1986). *Structure and insight*. New York: Academic Press.

Zevenbergen, R. (1991, July). *Children's construction of shape*. Paper presented to fourteenth annual conference of the Mathematics Education Research Group of Australasia, University of Western Australia, Perth.

RESEARCH IN TEACHING AND LEARNING ALGEBRA

Mollie E. MacGregor,
Institute of Education
University of Melbourne

Cyril R. Quinlan,
Australian Catholic University

There have been several important Australian studies in the area of algebra learning. The work of Booth during the 1980s (see Booth, 1988) has enlightened teachers about children's difficulties and also provided a useful foundation for further empirical research studies. Booth traced many common errors in elementary algebra to four sources of difficulty: the belief that mathematical activity leads to an "answer", the interpretation of algebraic notation and operation symbols, the meaning of letters and variables, and the use of informal methods in arithmetic and misunderstanding of certain arithmetical conventions. Booth's insights into children's reasoning and the methodology used in her studies have had considerable influence on other researchers in the field of elementary algebra learning.

The main thrust of recent and current projects by Australian researchers is an evaluation of various approaches to the teaching of algebra in the early years of secondary school and the development of theories to explain how children acquire or fail to acquire algebraic skills and understanding. Most of the work is closely tied to classroom practice and teaching experiments, and involves pencil-and-paper testing in schools. This chapter summarizes studies on teaching experiments, studies designed to evaluate certain aspects of curriculum or practice, studies of the interaction of language and algebra learning, exploratory studies of students' misconceptions and errors, and psychological studies linking students' behaviour on mathematical tasks with theories of cognition.

TEACHING EXPERIMENTS AND MATERIAL

Long (1989, 1990, in press) developed a teaching program for Year-9 algebra that encouraged the construction of knowledge by talking and reasoning in a problem-solving context. Its effectiveness was tested in six classes. The program advocated continual formative monitoring of learning by the class teacher, and the lesson materials advised teachers how they might observe, question and challenge learners during activities. Sample lesson plans

presented guidelines for shared activities and discussion between students supported by intervention and monitoring by the teacher. An evaluation of the program showed that the general directions to teachers for reviewing learning and adapting activities to learners' needs were not always translated into practice. The findings provide useful information about the nature and degree of support required by teachers in developing and using new approaches in the classroom.

In the state of New South Wales (NSW), the Algebra Research Group was founded in 1986 in order to develop an approach to the teaching of early algebra that made use of concrete manipulatives as aids for clarifying basic concepts. The group employed an action research program which involved the writing of student worksheets and teachers' notes and trailing them in a variety of classrooms. The feedback obtained from the teachers who took part in the trials led to redrafting and retrialing until the material was ready for publication as four booklets entitled *A Concrete Approach to Algebra* (Quinlan, Low, Sawyer & White, 1989). Many of the teaching suggestions were incorporated into the 1989 NSW mathematics syllabus for the first two years of secondary school (Years 7 and 8). They include activities, such as building geometric patterns out of toothpicks, followed by small group discussion of the numerical relationships seen in the patterns. Students describe these patterns first in ordinary English and later with algebraic symbols. Algebraic functions in the form of linear expressions are represented by manipulatives so that the value of the variable is modelled by a number of square centimetres of area or by a number of small similar objects inside a container. Rearranging their models helps students to understand that an algebraic generalization may be described by different algebraic expressions. Appropriate models are suggested for other topics in elementary algebra such as second degree expressions, positive integral indices and the solving of linear equations.

EVALUATION STUDIES

Quinlan has written a series of reports (Quinlan 1990a, 1990b, 1991a, 1991c; Quinlan & Collis 1990a, 1990b) based on his research data collected during 1990 from 517 students across Years 7 to 12. The emphasis in the early reports was largely on various aspects of introducing algebra with the aid of manipulatives. For instance, in Quinlan (1990b) the focus was on particular applications of the concrete approach recommended by the NSW Algebra Research Group. These were the use of models for:

1. representing zero and non-integral values of variables;
2. studying relationships such as $3x = y$ or $2x + y + 6 = 3x + 6$; and
3. supporting the expression of generalizations in algebraic form.

Quinlan and Collis (1990a, 1990b) investigated a rationale for a concrete approach to algebra, using theories such as Halford's (1987) Structure Mapping Theory, and found indications that students taught by the manipulative approach were more likely than those taught by the traditional textbook approach to view algebraic symbols as representing numbers rather than objects. Quinlan (1990a) reported how the following errors rapidly diminished with experience: use of alphabetic number codes, ignoring letters as having no meaning, replacing letters by arbitrary numbers, and refusing to write algebraic letters in answers. Misconceptions found to be more persistent were that letters stood for objects or people rather than numbers of these things, and that letters were like storehouses for numbers-to-be-discovered. The view of symbols which appeared to be most demanding cognitively was that they represented numerical variables. The level of development of this view was positively and significantly ($p < .001$) correlated with success on algebraic tasks such as substitution and acceptance of lack of closure. Attention was drawn to the visual impact of modelling algebraic structures with concrete materials as a possible advantage over modelling from arithmetic examples. Interviews some six months after manipulative models were used to introduce algebra showed that students were not dependent on these models but could still use them. They were found to be effective for self-correction. Quinlan concluded that mapping from models to algebra made explicit the power of algebra to express generalizations.

Pegg and Redden (1990a, 1990b) carried out an evaluation of one aspect of the pattern-based approach to algebra set down in the NSW syllabus (New South Wales Board of Secondary Education, 1989). Working with Year 7 and Year 8 pupils in six secondary schools, they investigated the quality of written descriptions of the rule for describing a pattern. The pupils' activities included completing and generating number patterns, and describing the rule that determines a pattern. Advocates of this approach expect that algebraic symbolism will emerge as a consequence of students' need to write their verbal rules succinctly. Pegg and Redden pointed out that although the logic of this approach is appealing, very little is known about its effects on students' understanding. The study showed that the quality of students' descriptions did not vary across schools, and there was a clear division of the scripts into two broad areas: those that focused on a relationship between two variables, and those that described a counting procedure for generating successive values of one variable. Verbal descriptions ranged from sophisticated sentences relating dependent and independent variables to ambiguous statements that reflected the confusion of the students who wrote them.

INTERACTION OF LANGUAGE AND ALGEBRA LEARNING

Cuevas and Dawe (1988) carried out an exploratory case study investigating the ways in which novice and expert teachers in Miami (USA) and Sydney (Australia) introduced algebra to pupils. The teachers were video-taped, enabling their uses of language in negotiating meaning to be analysed in detail.

Miller (1990) has written on the effectiveness of brief in-class writing in mathematics courses, including algebra, for helping students to clarify their thinking and for alerting teachers to any misconceptions. A specific example of the "writing to understand" approach in the context of algebra is seen in a study by MacGregor (1990).

MacGregor reported that the writing of sentences of English to describe a relation between two variables shown graphically helped tertiary level students to write correct equations. Students in the sample who were not required to write sentences wrote more incorrect equations. The most successful students were those who expressed the relation in familiar idiomatic forms that were grammatically complex. The results showed that the complexity of syntax that a student used for the sentence had no effect on success in constructing an equation. For example, given a diagram showing a 6:1 ratio of cooks to engineers, many students wrote the sentence "There are six times as many cooks as engineers" and almost all of them converted this correctly to an algebraic equation such as $c = 6e$. Others who wrote "For every engineer there are six cooks" were also successful in writing equations. It is clear that students were not using their sentences as material for direct translation of key words to symbols. It was concluded that any form of paraphrase is equally helpful as a prompt for the writing of a correct equation. Reasons for this could be that the writing of a sentence delays an immediate, intuitive response, or that the act of writing encourages reflection on the meaning of both sentence and equation. MacGregor pointed out that although it is widely believed that talking and writing in ordinary language are important activities in the mathematics classroom, there is very little research into the interaction of language and mathematics learning in Australian secondary schools.

STUDENTS' ERRORS

Stacey (1989) investigated students' ability to recognise a linear pattern and express it as a general rule. The students were in Years 4 to 8. It was found that most students were able to recognise a constant difference and hence to calculate values in the extension of a pattern. However when numbers became too large to deal with easily, many students resorted to incorrect methods based on direct proportion. They tended to grab at a relationship without subjecting it to checking and critical thinking. Students' tendency to generalize using a

simple but wrong rule was likely to affect their success in understanding relations between variables and writing formulas.

Coady & Pegg (1991a) interviewed and tested five first-year tertiary mathematics students on their understanding of basic algebra. The students had been selected to represent a wide range of mathematical backgrounds from 2-unit "Mathematics in Society" to 3-unit Mathematics. The four aspects of algebra tested were: expressing generalization, interpreting algebraic letters, solving equations, and understanding indices. Like the younger students in Stacey's (1989) study, the tertiary students with a 2-unit mathematics background perceived the constant difference between successive numbers in a linear pattern or looked for a ratio, and were unable to formulate a general rule. Three of the students considered only integer values of a variable. All were able to deal with routine equations that were in a familiar format. The authors concluded that students entering first-year courses have a wide range of abilities, and their misconceptions and errors are very similar to those of junior secondary students. It appears that a six-year study of algebra in school does not necessarily lead to a high level of competence.

Fundamental errors preventing progress in elementary algebra were revealed in MacGregor's (1991) study of Year 9 students in a representative sample of Victorian schools. They included the belief that, for example, ab means "a plus b"; the belief that an equation must have a single "answer" on the right hand side; use of the equals sign to link associated but unequal values; and the writing of powers when products are intended, for example, interpreting x^5 to mean "x multiplied by 5". An additional source of difficulty was found to be a partly developed concept of multiplication still tied to addition. Associated with this was uncertainty about the difference in meaning between "times as many" and "more than", and difficulty in understanding ratio. A few students, even some at tertiary level, interpreted a 6:1 ratio as meaning "six times more than", and hence as equivalent to "seven times". These findings supported the conclusion of Booth (1988) that some students lack a foundation of arithmetic structures on which to build algebraic understanding.

Using data from 1400 students in three high schools in Bunbury, Western Australia, Perso (1991) identified and classified 19 common misconceptions in algebra. These misconceptions include ways of using and interpreting algebraic letters, misinterpretation of the equals sign, disregard for order of operations and parentheses, and misuse of rules of algebraic manipulation. The fact that the same errors and misconceptions have been reported by researchers in Britain, USA and elsewhere suggests that they are not caused by particular teaching styles or curriculum materials. Perso's classification of misconceptions and their possible causes was accompanied by suggestions to teachers on ways to prevent their occurrence.

Coady and Pegg (1991b) carried out an assessment of the levels of algebraic understanding in 116 students who were about to commence an Applied Science degree programme. Test items used were drawn from a set

developed by Kuchemann (1980), and students were graded according to the Kuchemann levels. Fifteen students from the sample were also interviewed. The results showed that either the test was not appropriate for identifying levels of understanding in older students or about one-third of the students had not progressed beyond the level at which algebraic letters are assigned specific values or are interpreted as the names of objects. As the authors commented, the results "reflect badly on the algebraic learning that has occurred in the secondary school" (p. 4).

In Quinlan's (1990b) later work, characteristics of Year 7 students who progressed in algebra during six months were compared with those who stayed at the lowest standard throughout that time. It was found that those who were destined to improve were faster in learning such aspects of basic algebra as substitution skills, which showed that they understood the conventions for writing algebraic expressions, and recognizing that algebraic symbols stood for numbers rather than objects and that these numbers could vary. The poor achievers were more inclined to conjoin symbols for addition and were more persistent with prestructural views, which showed that they did not understand the implications of several of the test items used in Quinlan's study.

COGNITIVE FOUNDATIONS

The work of Pegg and Redden (1990b) included a theoretical study of levels of thinking, and the relation of these levels to the SOLO Taxonomy developed by Biggs and Collis (1982). Pegg and Redden (1990b) investigated students' performance on developing algebraic rules to describe number patterns, and found that written responses could be categorized into two broad areas equivalent to the concrete-symbolic and ikonic modes respectively of the SOLO Taxonomy. Each of these two areas was divided into subdivisions or levels, interpreted as unistructural, multistructural or relational. Responses at the unistructural level-ikonic mode focused on counting to extend a pattern and made no mention of a relationship between two variables. An important finding of the study was that the spontaneous use of the dependent/independent variable in describing patterns indicated understanding in the concrete symbolic mode. It was not until students reached the multistructural level within this mode that they were able to write algebraic sentences. There appeared to be a strong link between the use of a written rule and the use of algebraic symbolism.

Quinlan's series of reports from December 1991 onwards (1991a, 1991b, 1991c) was on the levels of difficulty experienced by students in developing an understanding of algebraic symbols. Some of the propositions, based on psychological principles and found to be supported by empirical evidence, were that:

1. Students with better levels of understanding of the meaning of algebraic symbols are more likely to have higher degrees of success with algebraic tasks.
2. Success in interpreting algebraic expressions is a "prelude" (defined by stated statistical criteria) to success in comparing the values of two expressions or comparing the values of two variables within the one equation.
3. Success in solving simple linear equations is a prelude to success in identifying the conditions for equality of two expressions or for the equality of two variables within the one equation.
4. Success in identifying the conditions for equality of two algebraic expressions (or two variables within the one equation) is a prelude to success in identifying the conditions for one expression (or variable) to be greater than the other.

Quinlan's data supported the proposition that mastery of the meanings of algebraic expressions forms the first unistructural stage of a new learning cycle through multistructural to relational levels.

MacGregor's (1991) work linked research findings in cognitive psychology and psycholinguistics to current issues in algebra learning. A study involving graduate and undergraduate students and secondary school students used the well-known reversal error as a means of exploring difficulties and misinterpretations associated with the reading and writing of linear equations. Several explanations of the reversal error have been put forward in the literature. They include: the misinterpretation of algebraic letters as abbreviated words; attempting to translate directly from words to mathematical symbols, from left to right without concern for meaning; the misleading influence of mental pictures of groups of objects; and selection of an inappropriate cognitive frame. MacGregor's study showed that these explanations were inadequate: the reversal error was still very common in basic algebraic tasks that had been designed to eliminate the possibility of error from the four causes listed above. One of these tasks was the following:

> "The number y is eight times the number z". Write this information in mathematical symbols.

Only one-third of a representative sample of 235 Year 9 students who were given this item in a pencil-and-paper test wrote correct equations. Half the responses were reversed equations or expressions (e.g., $z = 8y$, $8y = z$ or $8y \times z$). The high number of reversals for this and similar items indicated that students had not tried to write an algebraic equation by simply looking for key words in a statement of English and replacing them with mathematical symbols. If they had done so their responses would have been correct. The forms of responses suggested that the majority of students had tried to represent meaning, disregarding the syntactic form of the given statements;

their reversed equations were attempts to represent on paper cognitive models of compared unequal quantities. This hypothesis is supported by current theories of language processing and comprehension in the psycholinguistic literature. MacGregor suggested that psycholinguistic factors should be taken into account in the design of algebra teaching materials.

REFERENCES

Biggs, J. B., & Collis, K. F. (1982). *Evaluating the quality of learning: the SOLO taxonomy*. New York: Academic Press.

Booth, L. R. (1988). Children's difficulties in beginning algebra. In A. F. Coxford & A. P. Shulte (Eds.), *The ideas of algebra, K-12* (pp. 20-32). Reston: National Council of Technical Mathematics.

Coady, C., & Pegg, J. (1991a). An investigation of five first-year tertiary students' understanding of basic algebraic concepts. *Australian Senior Mathematics Journal, 5*(2), 111-120.

Coady, C., & Pegg, J. (1991b, July). *A study of the Kuchemann algebra test items as related to first year university students*. Paper presented at the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia. (Available from J. Pegg, Department of Science, Technology and Mathematics Education, University of New England, Armidale NSW 2351, Australia.)

Cuevas, G., & Dawe, L. (1988). *Case studies of the language functions of teachers in negotiating meaning in algebra in Australia and the United States* (Research Report). Miami: University of Miami. (Available from L. Dawe, Institute of Education, University of Sydney, NSW 2006, Australia.)

Halford, G. S. (1987). A structure-mapping approach to cognitive development. *International Journal of Psychology, 22*, 609-642.

Kuchemann, D. E. (1980). *The understanding of generalised arithmetic by secondary school children*. Unpublished doctoral thesis. London: University of London.

Long, E. M. (1989). *Talking maths: helping you to understand algebra*. Adelaide, South Australia: University of Adelaide Press.

Long, E. M. (1990). *Problem solving in high school algebra: A theory-based approach to classroom practice*. Unpublished doctoral thesis. Adelaide, South Australia: University of Adelaide.

Long, E. M. (in press). Secrets of the grandmasters: pattern-making in mathematics. *Australian Mathematics Teacher*.

MacGregor, M. (1990). Writing in natural language helps students construct algebraic equations. *Mathematics Education Research Journal, 2*(2), 1-11.

MacGregor, M. (1991). *Making sense of algebra: cognitive processes influencing comprehension*. Geelong, Victoria: Deakin University.

Miller, L. D. (1990). *Writing to learn algebra*. Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania. (Available from author, Science and Mathematics Education Centre, Curtin University of Technology, GPO Box U 1987, Perth WA 6001, Australia.)

New South Wales Board of Secondary Education (1989). *Mathematics syllabus years 7-8*. North Sydney, New South Wales: Board of Secondary Education.

Pegg, J., & Redden, E. (1990a). Procedures for and experiences in introducing algebra in New South Wales. *Mathematics Teacher, 83*(5), 386-391.

Pegg, J., & Redden, E. (1990b, July). *Student experiences in developing algebraic symbolism using number patterns*. Paper presented at the thirteenth annual conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania. (Available from authors, Department of Science, Technology and Mathematics Education, University of New England, Armidale NSW 2351, Australia.)

Perso, T. (1991, July). *Misconceptions in algebra*. Paper presented at the fourteenth annual conference of the Mathematics Education Research Group of Australasia, Perth, Western Australia. (Available from author, Science and Mathematics Education Centre, Curtin University of Technology, GPO Box U 1987, Perth WA 6001, Australia.)

Quinlan, C. (1990a). Addressing difficulties with early algebra: Theory and practice. In M. Bezzina & J. Butcher (Eds.), *The changing face of professional education* (pp. 774-779). Sydney, New South Wales: Australian Association for Research in Education.

Quinlan, C. (1990b, July). *Refining a concrete approach to algebra*. Workshop presented at the thirteenth biennial conference of the Australian Association of Mathematics Teachers, Hobart, Tasmania.

Quinlan. C. (1991a). Algebra with meaning. *Australian Mathematics Teacher, 47*(3), 28-29.

Quinlan, C. (1991b, December). *Developing an understanding of algebraic symbols*. Paper presented at the annual general meeting of the Mathematical Association of Tasmania, Hobart, Tasmania. (Available from author, ACU-NSW, PO Box 201, Castle Hill NSW 2154, Australia.)

Quinlan, C. R. E. (1991c). Structure mapping with manipulatives for algebra. In F. Furinghette (Ed.), *Proceedings of the fifteenth Psychology of Mathematics Education Conference:Vol. 1* (p. lxxv). Assisi, Italy.

Quinlan, C , & Collis, K. (1990a, July). Investigating a rationale for a concrete approach to algebra. In K. Milton & H. McCann (Eds.), *Mathematical turning points: strategies for the 1990s* (pp. 435-454). Hobart, Tasmania: Mathematical Association of Tasmania.

Quinlan, C , & Collis, K. (1990b, July). Some cognitive foundations for a concrete approach to algebra. In K. Milton & H. McCann (Eds.), *Mathematical turning points: strategies for the 1990s* (pp. 73-74). Hobart, Tasmania: Mathematical Association of Tasmania.

Quinlan, C., Low, B., Sawyer, T., & White, P. (1989). *A concrete approach to algebra*. Sydney, New South Wales: Mathematical Association of New South Wales.

Stacey, K. (1989). Finding and using patterns in linear generalising problems. *Educational Studies in Mathematics, 20*, 147-164.

CONTRIBUTORS

Editors

Bill Atweh is a lecturer in mathematics education and a principal member of the Centre for Mathematics and Science Education at the Queensland University of Technology in Brisbane. He has been a teacher and lecturer for 13 years in the areas of primary and secondary mathematics education and computer studies. He lectures at the undergraduate and masters levels and supervises doctoral students. His main areas of research interest are the affective factors in mathematics education, gender and social class factors and contexts of mathematics education. He is currently the Vice President (Publications) of the Mathematics Education Research Group of Australasia.

Jane Watson is a senior lecturer in mathematics education at the University of Tasmania in Hobart. Her recent research interests include the understanding of common decimal fractions, the assessment of cognitive functioning during problem solving, and the mapping of solutions to mathematics problems. She is now moving into the area of teaching and learning of probability and statistics. Currently, Jane is the Vice President (Research) for the Mathematics Education Research Group of Australasia. Until recently, she edited a research column for the journal of the Australian Association of Mathematics Teachers.

Authors

Marjorie Carss is a senior lecturer at the Department of Education of the University of Queensland in Brisbane. Her research interests include the study of the attitudes and perceptions of students in a retraining program in mathematics and science. He has special interests in problem solving, critical thinking and girls and mathematics.

Philip Clarkson is the Head of the Science Department on the Mercy Campus of the Australian Catholic University - Victoria. He has taught mathematics, chemistry and science in secondary schools, and has been involved in teacher training and educational research both in Australia and Papua New Guinea for a number of years. Philip's research interests include language and mathematics learning, spatial ability, student attitudes and perceptions, and curriculum development and assessment. He is the foundation editor of the *Mathematics Education Research Journal*.

Tom Cooper is an associate professor in mathematics education and the head of the school of Mathematics, Science and Technology Education at the Queensland University of Technology in Brisbane. Currently, Tom is the President of the Queensland Association of Mathematics Teachers and a representative on the council of the Australian Association of Mathematics

Teachers. His research interests include the social context of mathematics education, problem solving and teaching and learning of the fundamental concepts of mathematics.

Geoff Davey is a lecturer in mathematics education within the Division of Education at Griffith University in Brisbane, Queensland. His main research interests are the development of geometric concepts and skills and the use of geometric patterns in teaching. Geoff is a past Editor of the *Australian Mathematics Teacher* and is a member of both the Queensland Association of Mathematics Teachers and the Mathematics Education Research Group of Australasia.

Brian Doig is a former primary school teacher and lecturer in teacher education. Currently, he is a research fellow at the Measurement Division of the Australian Council for Educational Research, in Hawthorn, Victoria. His interests include assessment of conceptual understanding, particularly in mathematics and science, and the use of calculators and computers in learning mathematical concepts. Brian's other interests include the preparation of classroom materials to improve the learning of mathematics.

Nerida Ellerton is an associate professor and chair of the Mathematics Teaching and Research Group in the Faculty of Education at Deakin University - Geelong Campus in Victoria. She has a doctoral degree in physical chemistry form the University of Adelaide and a doctorate in mathematics education from Victoria University of Wellington in New Zealand. Nerida has taught at primary, secondary and tertiary levels for over 25 years, has served on numerous occasions as a mathematics education consultant in Malaysia, and has published extensively in mathematics education and scientific journals. Her research interests include the influence of language, culture and affective factors on mathematic learning.

Lyn English-Halford, is a senior lecturer in mathematics education at the Centre for Mathematics and Science Education of the Queensland University of Technology in Brisbane. Lyn's teaching and supervision responsibilities mainly focus on masters' and doctoral students. Her major research projects are: the cognitive competence of low, average, and high achieving children; the general thinking processes and knowledge/process interactions; the relationship between cognitive style, problem solving, and scientific reasoning; and the development of children's deductive reasoning and their ability to recognise situations of undecidability. Lyn is presently the Queensland coordinator for the Philosophy for Children Program sponsored by the Australian Council for Educational Research.

Helen Forgasz is a former high school teacher of mathematics, physics and computer studies. She has recently worked on a project sponsored by the

Australian International Development Aid Bureau in the Philippines. Helen is currently a doctoral candidate and a research assistant at Monash University in Victoria. Her research interests include mathematics and physics education, gender issues and women in education.

Peter Galbraith is a reader at the Department of Education at the University of Queensland in Brisbane. His major research interests include the study of perceptions of school mathematics within the community, and with teachers of mathematics and student teachers. He has developed teaching materials on mathematical modelling.

Tony Herrington is a lecturer in mathematics education at Edith Cowan University, Perth, Western Australia, where he teaches primary and secondary preservice teacher education courses. He has been involved in research into mathematics education for over 10 years beginning at Monash University where he completed his masters degree in 1983. Tony has a special interest in research on memory. He is presently completing a doctoral degree at the University of Western Australia in the area of strategies for learning mathematical intellectual skills.

Ian Isaacs is a senior lecturer in mathematics education in the Faculty of Education at the Northern Territory University, in Darwin. His research interests are in problem solving at the secondary level, in particular the assessment and measurement of problem solving abilities in mathematics among secondary students. Ian is the secretary of the Problem Solving Special Interest Group of the Mathematics Education Research Group of Australasia.

Clive Kanes is a lecturer in mathematics education at Griffith University in Brisbane, Queensland. He lectures in undergraduate and masters courses for secondary teachers. Clive's research interests include the pedagogic interactions in mathematics classrooms, the nature of understanding in mathematics, and the relationship between teacher pedagogic content knowledge and student learning. He uses theoretical frameworks based on cognitive psychology and philosophy of language and discourse.

Gilah Leder started her working life as a high school teacher of mathematics. She is currently an associate professor at the Graduate School at Monash University in Melbourne. Her research interests include the impact of affective variables on mathematics learning, gender differences, exceptionality, and the interaction between teaching, assessment and learning of mathematics. Gilah is the co-editor of *Educating girls: Practice and research* and *Mathematics and gender*. Her forthcoming publications include a chapter in the *Handbook of research in mathematics teaching and learning* on "Mathematics and girls: Changing perspectives".

Mollie MacGregor taught secondary school mathematics and science for many years. She has special interests in the problems of low achievers and the interaction between language skills and mathematics learning. Mollie has followed these interests in post-graduate study and research. Presently, Mollie is a research fellow at the Institute of Education, University of Melbourne in Victoria working on a project to investigate the cognitive and linguistic demands of learning algebra funded by the Australian Research Council.

Judith Mousley teaches mathematics education in the Division of Education, Deakin University - Geelong Campus, Victoria. She has fifteen years experience as a primary and middle-school teacher. Her research interests include the development of students' mathematical theories, teachers' theories of action research, school based professional development through action research, and the perceptions of excellence in mathematics teaching.

Joanne Mulligan is a senior lecturer in mathematics education at Macquarie University in New South Wales. She has recently completed 10 years of masters and doctoral studies and research in mathematics education. Her main interest is the acquisition of number processes. Joanne's doctoral thesis was on children's performance on multiplication and division problems. Joanne has pursued her enthusiasm for mathematics education in her roles as primary teacher, assistant principal, coordinator and inservice leader. She is an author and consultant, and an active contributor to the Mathematics Education Research Group of Australasia, the Mathematics Education Lecturers Association, and the Mathematics Association of New South Wales. Joanne won the 1990 Mathematics Education Research Group of Australasia award for research achievement by a young academic.

John Pegg is a senior lecturer in mathematics education in the Department of Science, Technology and Mathematics Education at the University of New England, Armidale, New South Wales. His research interests include the analysis of students' understanding in various mathematical topics and the development of effective inservice programs. Currently, John holds the positions of Secretary for the New England Mathematical Association and the Mathematical Education Lecturers' Association and is the Editor of the *Australian Senior Mathematics Journal*.

Robert Perry is an associate professor of education at the University of Western Sydney, Macarthur Campus in New South Wales. He has worked as a mathematics educator in universities and colleges for 21 years and now holds the post of Head, Division of Teacher Education within the Faculty of Education. Bob has been actively involved with the Mathematics Education Research Group of Australasia since its formation. His current research interests include the development of measurement concepts in young children and the relationships between secondary mathematics teachers and support

teachers. Bob writes in the areas of mathematics education and teacher education.

Ian Putt is a senior lecturer in mathematics education in the School of Education at James Cook University in Townsville, Queensland. He lectures in undergraduate mathematics and mathematics education subjects to primary preservice and inservice teachers and supervises masters' and doctoral students. His research interests include tertiary distance students' thinking while studying from text, primary and lower secondary students' thinking while solving mathematical problems, the development of an instrument to measure affective characteristics of primary students, and the monitoring of mathematics anxiety and attitudes of student teachers. Ian is chairperson of the Problem Solving Special Interest Group of the Mathematics Education Research Group of Australasia.

Cyril Quinlan, is a member of the Congregation of the Marist Brothers of the Schools. He has taught secondary school mathematics for 27 years, with some work in the primary school. Since 1979 he has been lecturing in mathematics and mathematics curriculum to preservice mathematics teachers at the Australian Catholic University - New South Wales. He is currently completing his doctoral thesis on developing understanding of algebraic symbols.

Jan Thomas is a lecturer in mathematics education at the Victoria University of Technology in Melbourne. Her current research includes an investigation into the linguistic and cultural background of first year mathematic students in Victorian universities. She is also the director of a project investigating the English needs of Australian residents from non-English speaking background who are graduates in mathematics, science and technology. Currently, Jan is a foundation member and the Secretary of the Australian Mathematical Sciences Council and a former Vice-president of the Mathematical Association of Victoria. Jan is the chairperson of the Politics Special Interest Group of the Mathematics Education Research Group of Australasia.

Khoon Yoong Wong is a lecturer in distance education at the External Studies Unit, Murdoch University in Western Australia. His main responsibility consists of designing external courses for the School of Mathematical and Physical Sciences. He has extensive experience in training mathematics teachers in Malaysia, Singapore, the Philippines and Australia. His current research interests include: the development and evaluation of a Hypercard stack for teaching calculus at matriculation and first year university level, learning of angles using Logo, learning strategies in mathematics, and comparative mathematics education between Australia and the Southeast Asia region.

Robert Wright is a senior lecturer in mathematics education at the University of New England - Northern Rivers Campus in New South Wales. His main

research interests are in the area of early childhood mathematics. Bob's current research projects focusing on early mathematics learning and teaching include: teachers' implicit theories, children's numerical development, psychological and social analyses of classroom processes, and early intervention for "at-risk" learners.

Solve all your maths problems.

Here's a new, unique way for you to find out how your students solve problems. ACER's **Mathematical Problem-Solving Profiles** - devised by internationally respected maths educators, Kevin Collis and Tom Romberg - are a breakthrough.

You can:

- ❒ diagnose your students' strengths and weaknesses in problem solving in about 40-50 minutes
- ❒ assess your students' responses using SOLO - an easy-to-use taxonomy
- ❒ get lots of good ideas on what to do next after assessment - one of the manual's very special features
- ❒ save by making a once only purchase of the Kit, then you use the blackline masters to photocopy the profiles as often as you like.

The authors' clever profiling enables you to use:
- the Junior version with your students 9-13 years
- the Senior version with your students 13-17 years.

Topics: Algebra, Chance and Data, Measurement, Number, Space.

COLLIS-ROMBERG MATHEMATICAL PROBLEM SOLVING PROFILES

Send your order for the **Problem-Solving Profiles** *($99 plus $5 post), or ask to be sent our free brochure. Quote code 990ED-1 on your order.*

ACER Australian Council for Educational Research
Frederick Street, Hawthorn, Vic 3122, Australia
Tel: (03)819 1400 Fax: (03)819 5502

AD192

Maths books from Deakin University Press

FOSTERING AND SUSTAINING MATHEMATICS THINKING THROUGH PROBLEM SOLVING
(J. Mason and J. Davis)
ISBN 0 7300 1264 6 106pp 350g **RRP: $16.00 plus p & h**

MATHS IN A CULTURAL CONTEXT ABORIGINAL PERSPECTIVES ON SPACE, TIME AND MONEY
(Pam Harris)
ISBN 0 7300 1209 3 168pp 400g **RRP: $23.00 plus p & h**

BILINGUALISM AND MATHEMATICS LEARNING (P.C. Clarkson)
ISBN 0 7300 1292 1 60pp 250g **RRP: $16.00 plus p & h**

MAKING SENSE OF ALGEBRA: COGNITIVE PROCESSES INFLUENCING COMPREHENSION
(Mollie E. MacGregor)
ISBN 0 7300 1282 4 139pp 350g **RRP: $23.00 plus p & h**

MATHEMATICS IN LANGUAGE: A REVIEW OF LANGUAGE FACTORS IN MATHEMATICS LEARNING *(N.F. Ellerton and M.A. [Ken] Clements)*
ISBN 0 7300 1356 1 170pp 400g **RRP: $22.50 plus p & h**

MODELLING WITH MATHEMATICS IN PRIMARY AND SECONDARY SCHOOLS
(J. Mason and J. Davis)
ISBN 0 7300 1265 4 78pp 264g **RRP: $20.00 plus p & h**

THESE AND OTHER TITLES AVAILABLE FROM:
Deakin University Press A.C.N. 005 382 954
Deakin University Tel: (052) 271156
Geelong Vic 3217 Australia Fax: (052) 272020
Catalogues mailed free on request **Bankcard, Mastercard & Visa accepted**

POSTGRADUATE STUDIES

IN

EDUCATION AND TRAINING

AT

VICTORIA UNIVERSITY OF TECHNOLOGY

The Department of Education, Victoria University of Technology offers postgraduate studies in Education and Training leading to a Master of Education. The graduate program is a linked award that can lead to a:

* Graduate Certificate in Education and Training;
* Graduate Diploma of Education in Education and Training; or
* Master of Education in Education and Training.

Participants undertake Core Units and Applied Studies. The Applied Studies include **Program Designer in Mathematics** for those with responsibility for program development in schools, TAFE and industry settings.

For further information on this and other graduate programs in education, contact Professor John Wilson on (03) 688 4786 or (03) 688 4409. The facsimile number is (03) 688 4646 and the postal address is Department of Education, Victoria University of Technology, Ballarat Road, Footscray 3011.

National Key Centre for School Science and Mathematics

curtin
University of Technology
Perth Western Australia

Postgraduate Studies in Science, Mathematics and Technology Education

Full-Time, Part-Time or Distance Education Options

Background Information
The national Key Centre for Teaching and Research in School Science and Mathematics at Curtin University is funded by the Commonwealth government as a centre of excellence. It is the only Key Centre in Australia in the area of education, and it has the largest group of postgraduate students specifically in science and mathematics education in any Australian tertiary institution. The Key Centre sponsors extensive postgraduate, research, publication and national workshop programs. The postgraduate programs provide a unique opportunity for teachers and teacher educators in universities, secondary and primary schools, and TAFE to pursue postgraduate study towards the following awards:

Postgraduate Diploma in Science/Mathematics/Technology Education
This consists of coursework plus minor project and occupies the equivalent of one year of full-time study. It involves science, mathematics and technology education units, science/mathematics content units and education options. Most people entering the Postgraduate Diploma have four years of recognised qualifications consisting of a Bachelor of Education or a Bachelor's degree plus a teaching qualification. Other qualifications and experience could satisfy entry requirements. All units may be completed by distance education methods without attending Curtin University.

Master of Science (Science/Mathematics/Technology Education)
The equivalent of a minimum of another one year of full-time study beyond the Postgraduate Diploma leads to the Master's degree. It may be completed by coursework plus project or by research thesis only. Qualifications equivalent to the Key Centre's Postgraduate Diploma could be recognised, but additional coursework may be required. The Master's degree can be completed by distance education.

Doctor of Philosophy or Doctor of Science Education
The Doctorate involves the equivalent of a minimum of two years of full-time study. A relevant Master's degree or a good honours degree plus a teaching qualification provide the most usual entry.

- The degree by thesis only (PhD) is for those who wish to pursue an area of applied research in depth.
- The degree by coursework plus thesis (ScEdD) involves a thesis of an applied nature together with coursework units covering the Key Centre's areas of research specialisation, including gender issues, teaching and learning, classroom environments and curriculum issues in science, mathematics and technology education.

For both the PhD and the ScEdD some time must be spent in residency at Curtin. All coursework units and most of the thesis work can be completed by distance education.

Modes of Study
All programs are available on a full-time study, part-time study or distance education mode.

Tax-Free Scholarships
APRA and Curtin Scholarships for full-time Master's-by-thesis, or Doctoral study. Commonwealth government Australian Postgraduate Research Awards (APRAs) and Curtin University Postgraduate Awards provide a tax-free indexed living allowance ($14,260-$18,403 p.a. in 1992) and a relocation and thesis allowance. In 1991, most applicants holding an appropriate Master's or first class honours degree received scholarships.

Key Centre 'Top Up' Scholarships for full-time Doctoral study. Scholarships worth an additional $4,000 p.a. can be held in conjunction with an APRA or Curtin Scholarship.

HECS Exemption Scholarships. In 1992, all Curtin doctoral and Master's-by-thesis students are exempted from tuition costs under the Higher Education Contribution Scheme (HECS).

Contact for Additional Information

Key Centre for School Science and Mathematics
Curtin University of Technology
GPO Box U1987
PERTH Western Australia 6001

Fax: (09) 351 2503
Tel: (09) 351 7896

PRENTICE HALL AUSTRALIA *presents . . .*

The Student Edition of MATLAB™

TURN ON THE POWER

"SERIOUS" NUMBERS DEMAND SERIOUS SOFTWARE. That's why professionals have come to depend on MATLAB for handling numerical linear algebra and matrixes in scientific and engineering calculations. Now ***The Student Edition of MATLAB™*** gives your students an opportunity to harness the numerical horsepower of MATLAB for less than the price of a standard textbook.

Because the basic data element in MATLAB is a matrix that does not require dimensioning, the system can express problem solutions almost exactly as they are written mathematically – *with no programming required.* Users can solve may numerical problems in a fraction of the time it would take to write a comparable program in a language like FORTRAN/ BASIC or C, and they can plot their results in revealing two- and three- dimensional graphics.

To find out more about ***The Student Edition of MATLAB™*** contact

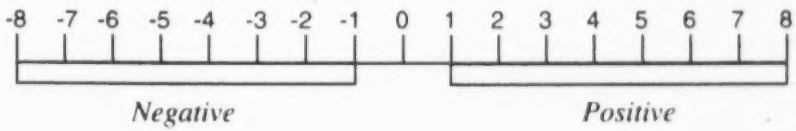
College Marketing
PRENTICE HALL AUSTRALIA
7 Grosvenor Place, Brookvale NSW 2111
Phone: (02) 939 2133 Fax: (02) 905 7934

A.C.N. 000 383 406

Dear Megan,
I'm sorry to hear that you have been ill and don't have your maths textbook. I'll try to explain what we have been doing, so here it goes.

A negative number is a number that is below zero and it's not a decimal.

A number line might help.



If I had this sum 0 + ⁻2 =

I'd start at zero and as you are adding a ⁻2, you move in the direction of the negative numbers. It's the same if you are saying 0 + ⁺2 = ⁺2 because the answer is moving in the direction of positives.

Subtracting is or can be quite hard to understand as I don't really understand it.

I don't understand how to do it and I can't seem to pick it up. It just doesn't make sense to me. I don't want to say the wrong thing, so I won't try and explain. If I knew I'd try, but I don't, so bye for now.

Angela (Age 12)